OCR Core Maths 4

Past paper questions
Vectors

Edited by K V Kumaran

Email: kvkumaran@gmail.com

Phone: 07961319548

Vectors

- The vector $\begin{pmatrix}3\\4\end{pmatrix}$ can be written $3\mathbf{i}+4\mathbf{j}$ and represents a vector going 3 right and 4 up. By Pythagoras' Theorem it can be shown that the magnitude of this vector is $\sqrt{3^2+4^2}=5$ and by trigonometry the direction is $\tan^{-1}\frac{4}{3}$ above the horizontal.
- Two vectors are equal if their magnitudes and directions are the same. Two vectors are parallel if one is a scalar multiple of the other. For example show that $\begin{pmatrix}2\\3\end{pmatrix}$ is parallel to $\begin{pmatrix}3\\4.5\end{pmatrix}$; so show that $1.5\times\begin{pmatrix}2\\3\end{pmatrix}=\begin{pmatrix}3\\4.5\end{pmatrix}$.
- When multiplying a vector by a positive scalar it changes the length of the vector but not the direction. If the scalar is negative then it also reverses the direction of the vector. For example $3\begin{pmatrix}2\\-3\end{pmatrix}=\begin{pmatrix}6\\-9\end{pmatrix}$.
- When adding vectors, you just add the x components and add the y components. For example $\begin{pmatrix}2\\-3\end{pmatrix}+\begin{pmatrix}2\\5\end{pmatrix}=\begin{pmatrix}4\\2\end{pmatrix}$.
- A unit vector is a vector with a magnitude 1. A unit vector in a given direction can be constructed by dividing a vector by its magnitude. For example the unit vector from $\begin{pmatrix}2\\3\end{pmatrix}$ is $\frac{1}{\sqrt{2^2+3^2}}\times\begin{pmatrix}2\\3\end{pmatrix}=\frac{1}{\sqrt{13}}\times\begin{pmatrix}2\\3\end{pmatrix}$.
- You must know the geometric interpretation of $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-\mathbf{b}$. Also know that in general if you have position vectors $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$ then $\overrightarrow{AB}=\mathbf{b}-\mathbf{a}$.
- It cannot be stressed enough that *subtraction* is the most important operation with vectors. If you wish to travel *from* one point ($\mathbf{a}$) *to* another ($\mathbf{b}$) then we use subtraction: $\overrightarrow{AB}=\mathbf{b}-\mathbf{a}$.

  I will repeat that! If you wish to travel *from* $\mathbf{a}$ *to* $\mathbf{b}$ then use subtraction: $\overrightarrow{AB}=\mathbf{b}-\mathbf{a}$.

- If you wish to calculate a length in 3D space then you merely need to calculate the magnitude of the vector that travels between the two points (i.e. $|\mathbf{b} - \mathbf{a}|$).

- A line can be written in vector form. If you know a line goes through a point (a, b) and has the gradient m then its vector form is $\mathbf{r} = \begin{pmatrix} a \\ b \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ m \end{pmatrix}$ where λ is a scalar that takes different values on different points on the line. The vector $\begin{pmatrix} 1 \\ m \end{pmatrix}$ can be re-written to make the components 'nicer'. For example $\begin{pmatrix} 1 \\ 2/3 \end{pmatrix} \Rightarrow \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. (These vectors are not equal, but they have the same direction.) The most general form is

$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{d}$$

  where $\mathbf{a}$ is the point it passes through and $\mathbf{d}$ is the direction vector (i.e. the vector that points *along* the line).

- We can therefore show that the equation of the line through $\mathbf{a}$ and $\mathbf{b}$ is given by $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$, because $\mathbf{b} - \mathbf{a}$ is the vector that travels *from* $\mathbf{a}$ *to* $\mathbf{b}$ along the line. For example find the line that passes through $(2, 3, 1)$ and $(3, 6, -1)$. This gives

$$\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \quad \text{or} \quad \mathbf{r} = \begin{pmatrix} 3 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix}.$$

- To find the angle between two vectors we use the scalar product result

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$$

  where $|\mathbf{u}|$ represents the magnitude of vector $\mathbf{u}$. From this we can see that two vectors are perpendicular if their scalar product is zero.

- The scalar product is most easily calculated as follows; $\begin{pmatrix} a_x \\ a_y \end{pmatrix} \cdot \begin{pmatrix} b_x \\ b_y \end{pmatrix} = a_x b_x + a_y b_y$. (It is just a number, *not* a vector!)

- The following table sums up the 3D equivalents of the 2D results we have already found:

2D	3D
$\mathbf{i}, \mathbf{j}$	$\mathbf{i}, \mathbf{j}, \mathbf{k}$
$\lvert\mathbf{a}\rvert = \sqrt{a_x + a_y}$	$\lvert\mathbf{a}\rvert = \sqrt{a_x + a_y + a_z}$
$\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y$	$\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z$

  Most of the results from the 2D section (above) still hold true for 3D vectors.

- Obviously in 2D provided lines have different gradient then they *must* intercept somewhere. However in 3D it is possible for two lines to have different direction vectors (i.e. not be parallel) and still not cross: these lines are called *skew*. This example shows how to discover if lines in 3D intercept or are skew.

$$\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \qquad \text{and} \qquad \mathbf{r} = \begin{pmatrix} 0 \\ -6 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$

Firstly we note the different direction vectors, so they cannot be parallel. Equate the x and y components[4] of both lines and solve for λ and μ:

$$\begin{aligned} (x): \qquad & 4 + \lambda = 2\mu, \\ (y): \qquad & -1 - \lambda = -6 + \mu. \end{aligned}$$

These solve to $\lambda = 2$ and $\mu = 3$. Put these values back into the original lines and compare z-coordinates: if they are the same then they intercept, if different then skew.

$$\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 8 \end{pmatrix} \qquad \text{and} \qquad \mathbf{r} = \begin{pmatrix} 0 \\ -6 \\ 5 \end{pmatrix} + 3 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 8 \end{pmatrix}.$$

Therefore the lines cross at $(6, -3, 8)$. (You should find that the x and y-coordinates are *always* the same, it is only the z-coordinate that might be different; a nice little check!)

- To find the angle between two lines then *dot their direction vectors*. Using the two lines in the above example we find:

$$\begin{aligned} \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} &= \sqrt{1^2 + 1^2 + 4^2}\sqrt{2^2 + 1^2 + 1^2} \cos\theta \\ 2 - 1 + 4 &= \sqrt{18}\sqrt{6}\cos\theta \\ \theta &= 61.2^\circ \text{ (to 3s.f.)}. \end{aligned}$$

If you get an answer $90 < \theta \leqslant 180$ then give $180^\circ - \theta$ as your answer (Between any two lines there are two possible angles between them; think about it. The acute angle tends to be 'nicer').

- When working out angles in 3D you must be very careful that you are 'dotting' the right vectors! For example if $A = (1, 2, -2)$, $B = (3, 1, -4)$ and $C = (7, 5, 1)$ find the angle $A\hat{B}C$. Draw a sketch! We require the angle at B so we need to dot $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

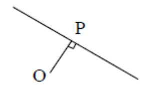
Now $\overrightarrow{BA} = \mathbf{a} - \mathbf{b} = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$ and $\overrightarrow{BC} = \mathbf{c} - \mathbf{b} = \begin{pmatrix} 4 \\ 4 \\ 5 \end{pmatrix}$. Therefore dotting we find:

$$\begin{aligned} \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 4 \\ 5 \end{pmatrix} &= \sqrt{9}\sqrt{57}\cos\theta \\ -8 + 4 + 10 &= 3\sqrt{57}\cos\theta \\ \frac{2}{\sqrt{57}} &= \cos\theta \qquad \Rightarrow \qquad \theta = 74.6^\circ \text{(to 3s.f.)}. \end{aligned}$$

- Some tough problems involve the use of

$$\text{“two vectors are at perpendicular”} \quad \Leftrightarrow \quad \text{“the dot product is zero”}.$$

For example find the point (P) on the line l: $\mathbf{r} = \begin{pmatrix}1\\2\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\2\end{pmatrix}$ closest to the origin.

Firstly draw a sketch of a line running some distance past an origin. 

At the point P the vector $\overrightarrow{OP}$ must be perpendicular to the line. The direction vector is the vector *along* the line l, so we need

$$(\overrightarrow{OP}).\begin{pmatrix}1\\-1\\2\end{pmatrix} = 0.$$

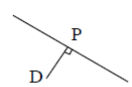
The point P is some point on the line, so $P = \begin{pmatrix}1+\lambda\\2-\lambda\\2\lambda\end{pmatrix}$ for some λ. So $\overrightarrow{OP} = \begin{pmatrix}1+\lambda\\2-\lambda\\2\lambda\end{pmatrix} - \begin{pmatrix}0\\0\\0\end{pmatrix} = \begin{pmatrix}1+\lambda\\2-\lambda\\2\lambda\end{pmatrix}$. Therefore

$$\begin{pmatrix}1+\lambda\\2-\lambda\\2\lambda\end{pmatrix}.\begin{pmatrix}1\\-1\\2\end{pmatrix} = 0.$$

This gives $1+\lambda-2+\lambda+4\lambda = 0$ which solves to $\lambda = \frac{1}{6}$. Putting this λ back into l we find

$$P = (\tfrac{7}{6}, \tfrac{11}{6}, \tfrac{1}{3}).$$

- Another tough example done in two ways: Find the shortest distance from point $D = (2,-1,3)$ to the line $\mathbf{r} = \begin{pmatrix}1\\0\\1\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\1\end{pmatrix}$.

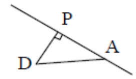
  - **Method I**: First method similar to above. Draw a sketch! 

 Let the point on the line closest to D be P. So $P = \begin{pmatrix}1+2\lambda\\-\lambda\\1+\lambda\end{pmatrix}$. We require $\overrightarrow{DP}$ to be perpendicular to the line if it is the closest point. Thus

$$\overrightarrow{DP}\cdot\begin{pmatrix}2\\-1\\1\end{pmatrix} = 0 \quad \Rightarrow \quad \left(\begin{pmatrix}1+2\lambda\\-\lambda\\1+\lambda\end{pmatrix} - \begin{pmatrix}2\\-1\\3\end{pmatrix}\right)\cdot\begin{pmatrix}2\\-1\\1\end{pmatrix} = 0.$$

This solves to $\lambda = \frac{5}{6}$. Therefore $P = (\frac{8}{3}, -\frac{5}{6}, \frac{11}{6})$. Therefore the distance is

$$\text{dist.} = |\overrightarrow{DP}| = |\mathbf{p}-\mathbf{d}| = \sqrt{(2-\tfrac{8}{3})^2 + (\tfrac{5}{6}-1)^2 + (3-\tfrac{11}{6})^2} = \frac{\sqrt{66}}{6}.$$

– **Method II**: Again, draw a sketch. 

Let P be the point closest to D. This time also include the point that we know the line passes through $A = (1, 0, 1)$. We have therefore created a right angled triangle APD with a right angle at P. Length AD is just the magnitude of $\mathbf{d} - \mathbf{a}$; $|\mathbf{d} - \mathbf{a}| = \sqrt{(2-1)^2 + (-1-0)^2 + (3-1)^2} = \sqrt{6}$. Angle $D\hat{A}P$ can be worked out by $\overrightarrow{AD} \cdot$ (direction vector). So

$$(\mathbf{d} - \mathbf{a}) \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = |\mathbf{d} - \mathbf{a}| \left| \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \right| \cos\theta$$

$$\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \sqrt{6}\sqrt{6} \cos D\hat{A}P.$$

So $\cos D\hat{A}P = \frac{5}{6}$.

By right angled trigonometry $\sin D\hat{A}P = \frac{DP}{\sqrt{6}}$. To convert a sin into a cos we use $\sin^2\theta + \cos^2\theta = 1$ which gives $\sin D\hat{A}P = \frac{\sqrt{11}}{6}$. Therefore $DP = \sqrt{6} \times \frac{\sqrt{11}}{6} = \frac{\sqrt{66}}{6}$, just as before.

1.

The line L_1 passes through the points $(2, -3, 1)$ and $(-1, -2, -4)$. The line L_2 passes through the point $(3, 2, -9)$ and is parallel to the vector $4\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$.

(i) Find an equation for L_1 in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [2]

(ii) Prove that L_1 and L_2 are skew. [5]

Q3 June 2005

2.

ABCD is a parallelogram. The position vectors of *A*, *B* and *C* are given respectively by

$$\mathbf{a} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}, \qquad \mathbf{b} = 3\mathbf{i} - 2\mathbf{j}, \qquad \mathbf{c} = \mathbf{i} - \mathbf{j} - 2\mathbf{k}.$$

(i) Find the position vector of *D*. [3]

(ii) Determine, to the nearest degree, the angle *ABC*. [4]

Q5 June 2005

3.

Two lines have vector equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix} + t\begin{pmatrix} -8 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} -2 \\ a \\ -2 \end{pmatrix} + s\begin{pmatrix} -9 \\ 2 \\ -5 \end{pmatrix},$$

where a is a constant.

(i) Calculate the acute angle between the lines. [5]

(ii) Given that these two lines intersect, find a and the point of intersection. [8]

Q9 Jan 2006

4.

The position vectors of three points A, B and C relative to an origin O are given respectively by

$$\overrightarrow{OA} = 7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k},$$
$$\overrightarrow{OB} = 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$$
$$\text{and} \qquad \overrightarrow{OC} = 5\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}.$$

(i) Find the angle between AB and AC. [6]

(ii) Find the area of triangle ABC. [2]

Q4 June 2006

5.

Two lines have vector equations

$$\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} + a\mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = -8\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} - \mathbf{k}),$$

where a is a constant.

(i) Given that the lines are skew, find the value that a cannot take. [6]

(ii) Given instead that the lines intersect, find the point of intersection. [2]

Q7 June 2006

6.

The points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ relative to an origin O, where $\mathbf{a} = 4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = -7\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$.

(i) Find the length of AB. [3]

(ii) Use a scalar product to find angle OAB. [3]

Q3 Jan 2007

7.

The position vectors of the points P and Q with respect to an origin O are $5\mathbf{i} + 2\mathbf{j} - 9\mathbf{k}$ and $4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$ respectively.

(i) Find a vector equation for the line PQ. [2]

The position vector of the point T is $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

(ii) Write down a vector equation for the line OT and show that OT is perpendicular to PQ. [4]

It is given that OT intersects PQ.

(iii) Find the position vector of the point of intersection of OT and PQ. [3]

(iv) Hence find the perpendicular distance from O to PQ, giving your answer in an exact form. [2]

Q10 Jan 2007

8.

Lines L_1, L_2 and L_3 have vector equations

$$L_1:\ \mathbf{r} = (5\mathbf{i} - \mathbf{j} - 2\mathbf{k}) + s(-6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}),$$
$$L_2:\ \mathbf{r} = (3\mathbf{i} - 8\mathbf{j}) + t(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}),$$
$$L_3:\ \mathbf{r} = (2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + u(3\mathbf{i} + c\mathbf{j} + \mathbf{k}).$$

(i) Calculate the acute angle between L_1 and L_2. [4]

(ii) Given that L_1 and L_3 are parallel, find the value of c. [2]

(iii) Given instead that L_2 and L_3 intersect, find the value of c. [5]

Q9 June 2007

9.

Find the angle between the vectors $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $2\mathbf{i} + \mathbf{j} + \mathbf{k}$. [4]

Q1 Jan 2008

10.

The vector equations of two lines are

$$\mathbf{r} = (5\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}) + s(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) \quad \text{and} \quad \mathbf{r} = (2\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}) + t(2\mathbf{i} - \mathbf{j} - 5\mathbf{k}).$$

Prove that the two lines are

(i) perpendicular, [3]

(ii) skew. [5]

Q5 Jan 2008

11.

Relative to an origin O, the points A and B have position vectors $3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ respectively.

(i) Find a vector equation of the line passing through A and B. [2]

(ii) Find the position vector of the point P on AB such that OP is perpendicular to AB. [5]

Q4 June 2008

12.

Two lines have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 12 \\ 0 \\ 5 \end{pmatrix} + s\begin{pmatrix} 1 \\ -4 \\ -2 \end{pmatrix}.$$

(i) Show that the lines intersect. [4]

(ii) Find the angle between the lines. [4]

Q6 June 2008

13.

(i) Show that the straight line with equation $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + t\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$ meets the line passing through (9, 7, 5) and (7, 8, 2), and find the point of intersection of these lines. **[6]**

(ii) Find the acute angle between these lines. **[4]**

Q7 Jan 2009

14.

(i) The vector $\mathbf{u} = \frac{3}{13}\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ is perpendicular to the vector $4\mathbf{i} + \mathbf{k}$ and to the vector $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$. Find the values of b and c, and show that $\mathbf{u}$ is a unit vector. **[6]**

(ii) Calculate, to the nearest degree, the angle between the vectors $4\mathbf{i} + \mathbf{k}$ and $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$. **[3]**

Q7 June 2009

15.

Points A, B and C have position vectors $-5\mathbf{i} - 10\mathbf{j} + 12\mathbf{k}$, $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $3\mathbf{i} + 6\mathbf{j} + p\mathbf{k}$ respectively, where p is a constant.

(i) Given that angle $ABC = 90°$, find the value of p. **[4]**

(ii) Given instead that ABC is a straight line, find the value of p. **[2]**

Q2 Jan 2010

16.

The equation of a straight line l is $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$. O is the origin.

(i) The point P on l is given by $t = 1$. Calculate the acute angle between OP and l. **[4]**

(ii) Find the position vector of the point Q on l such that OQ is perpendicular to l. **[4]**

(iii) Find the length of OQ. **[2]**

Q9 Jan 2010

17.

Lines l_1 and l_2 have vector equations

$$\mathbf{r} = \mathbf{j} + \mathbf{k} + t(2\mathbf{i} + a\mathbf{j} + \mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = 3\mathbf{i} - \mathbf{k} + s(2\mathbf{i} + 2\mathbf{j} - 6\mathbf{k})$$

respectively, where t and s are parameters and a is a constant.

(i) Given that l_1 and l_2 are perpendicular, find the value of a. **[3]**

(ii) Given instead that l_1 and l_2 intersect, find

(a) the value of a, **[4]**

(b) the angle between the lines. **[3]**

Q6 June 2010

18.

The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} + s\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$. The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$.

(i) Find the acute angle between l_1 and l_2. **[4]**

(ii) Show that l_1 and l_2 are skew. **[4]**

(iii) One of the numbers in the equation of line l_1 is changed so that the equation becomes $\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ a \end{pmatrix} + s\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$. Given that l_1 and l_2 now intersect, find a. **[2]**

Q6 Jan 2011

19.

Find the unit vector in the direction of $\begin{pmatrix} 2 \\ -3 \\ \sqrt{12} \end{pmatrix}$. **[3]**

Q2 June 2011

20.

The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} + s\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

respectively.

(i) Show that l_1 and l_2 are skew. **[3]**

(ii) Find the acute angle between l_1 and l_2. **[4]**

(iii) The point A lies on l_1 and OA is perpendicular to l_1, where O is the origin. Find the position vector of A. **[3]**

Q5 June 2011

21.

(i) Find, in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, an equation of the line l through the points (4, 2, 7) and (5, –4, –1). **[3]**

(ii) Find the acute angle between the line l and a line in the direction of the vector $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. **[4]**

Q2 Jan 2012

22.

The equation of a straight line l is

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}.$$

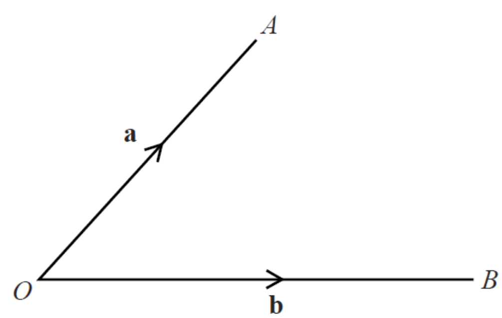
O is the origin.

(i) Find the position vector of the point P on l such that OP is perpendicular to l. **[3]**

(ii) A point Q on l is such that the length of OQ is 3 units. Find the two possible position vectors of Q. **[3]**

Q7 Jan 2012

23.



In the diagram the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ with respect to the origin O. Given that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 4$ and $\mathbf{a}.\mathbf{b} = 6$, find

(i) the angle AOB, **[2]**

(ii) $|\mathbf{a} - \mathbf{b}|$. **[3]**

Q5 June 2012

24.

Lines l_1 and l_2 have vector equations

$$\mathbf{r} = -\mathbf{i} + 2\mathbf{j} + 7\mathbf{k} + t(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \quad \text{and} \quad \mathbf{r} = 2\mathbf{i} + 9\mathbf{j} - 4\mathbf{k} + s(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$$

respectively. The point A has coordinates $(-3, 0, 6)$ relative to the origin O.

(i) Show that A lies on l_1 and that OA is perpendicular to l_1. **[3]**

(ii) Show that the line through O and A intersects l_2. **[4]**

(iii) Given that the point of intersection in part **(ii)** is B, find the ratio $|\overrightarrow{OA}| : |\overrightarrow{BA}|$. **[3]**

Q10 June 2012

25.

The equations of two lines are

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 6\mathbf{i} + 8\mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}).$$

(i) Show that these lines meet, and find the coordinates of the point of intersection. **[5]**

(ii) Find the acute angle between these lines. **[3]**

Q4 Jan 2013

26.

The points A (3, 2, 1), B (5, 4, −3), C (3, 17, −4) and D (1, 6, 3) form a quadrilateral $ABCD$.

(i) Show that $AB = AD$. **[2]**

(ii) Find a vector equation of the line through A and the mid-point of BD. **[3]**

(iii) Show that C lies on the line found in part **(ii)**. **[1]**

(iv) What type of quadrilateral is $ABCD$? **[1]**

Q8 Jan 2013

27.

Determine whether the lines whose equations are

$$\mathbf{r} = (1 + 2\lambda)\mathbf{i} - \lambda\mathbf{j} + (3 + 5\lambda)\mathbf{k} \quad \text{and} \quad \mathbf{r} = (\mu - 1)\mathbf{i} + (5 - \mu)\mathbf{j} + (2 - 5\mu)\mathbf{k}$$

are parallel, intersect or are skew. **[6]**

Q3 June 2013

28.

Points A (2, 2, 5), B (1, −1, −4), C (3, 3, 10) and D (8, 6, 3) are the vertices of a pyramid with a triangular base.

(i) Calculate the lengths AB and AC, and the angle BAC. **[4]**

(ii) Show that $\overrightarrow{AD}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **[3]**

(iii) Calculate the volume of the pyramid $ABCD$. **[3]**

[The volume of the pyramid is $V = \frac{1}{3} \times$ base area $\times$ perpendicular height.]

Q7 June 2013

29.

The points $O(0, 0, 0)$, $A(2, 8, 2)$, $B(5, 5, 8)$ and $C(3, -3, 6)$ form a parallelogram $OABC$. Use a scalar product to find the acute angle between the diagonals of this parallelogram. **[5]**

Q2 June 2014

30.

The equations of three lines are as follows.

Line A: $\mathbf{r} = \mathbf{i} + 4\mathbf{j} + \mathbf{k} + s(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$

Line B: $\mathbf{r} = 2\mathbf{i} + 8\mathbf{j} + 2\mathbf{k} + t(\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$

Line C: $\mathbf{r} = -\mathbf{i} + 19\mathbf{j} + 15\mathbf{k} + u(2\mathbf{i} - 4\mathbf{j} - 4\mathbf{k})$

(i) Show that lines A and B are skew. **[4]**

(ii) Determine, giving reasons, the geometrical relationship between lines A and C. **[2]**

Q5 June 2014

31.

A triangle has vertices at A (1, 1, 3), B (5, 9, −5) and C (6, 5, −4). P is the point on AB such that $AP:PB = 3:1$.

(i) Show that $\overrightarrow{CP}$ is perpendicular to $\overrightarrow{AB}$. **[4]**

(ii) Find the area of the triangle ABC. **[2]**

Q2 June 2015

32.

Two lines have equations

$$\mathbf{r} = 3\mathbf{i} + 5\mathbf{j} - \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + \mathbf{k}) \text{ and } \mathbf{r} = 4\mathbf{i} + 10\mathbf{j} + 19\mathbf{k} + \mu(\mathbf{i} - \mathbf{j} + \alpha\mathbf{k}),$$

where α is a constant.

Find the value of α in each of the following cases.

(i) The lines intersect at the point (7, 7, 1). **[3]**

(ii) The angle between their directions is 60°. **[4]**

Q9 June 2015