OCR Core Maths 2

Past paper questions
Trigonometry

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Trigonometry

- We define
$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}.$$
This identity is very useful in solving equations like $\sin\theta - 2\cos\theta = 0$ which yields $\tan\theta = 2$. The solutions of this in the range $0° \leqslant \theta \leqslant 360°$ are $\underline{\theta = 63.4°}$ and $\underline{\theta = 243.4°}$ to one decimal place.

- Know the following (or better yet, learn a couple and be able to derive the rest, quickly, from your knowledge of the trigonometric functions):

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$0°$	0	1	0
$30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45°$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90°$	1	0	undefined
$180°$	0	-1	0

- Be able to sketch $\sin\theta$, $\cos\theta$ and $\tan\theta$ in both degrees and radians.

- By considering a right angled triangle (or a point on the unit circle) we can derive the important result $\sin^2\theta + \cos^2\theta \equiv 1$. This is useful in solving certain trigonometric equations. Worked example; solve $1 = 2\cos^2\theta + \sin\theta$ for $0° \leqslant \theta \leqslant 360°$.
$$\begin{aligned}
1 &= 2\cos^2\theta + \sin\theta \\
1 &= 2(1 - \sin^2\theta) + \sin\theta && \text{get rid of } \cos^2\theta, \\
0 &= 1 - 2\sin^2\theta + \sin\theta && \text{quadratic in } \sin\theta, \\
0 &= 2\sin^2\theta - \sin\theta - 1 && \text{factorise as normal,} \\
0 &= (2\sin\theta + 1)(\sin\theta - 1).
\end{aligned}$$
So we just solve $\sin\theta = -\frac{1}{2}$ and $\sin\theta = 1$. Therefore $\underline{\theta = 210°}$ or $\underline{\theta = 330°}$ or $\underline{\theta = 90°}$.

- The above relation is also useful in converting between the different trigonometric functions. For example if $\cos\theta = \frac{6}{7}$ then, to find $\sin\theta$, do **not** use "$\cos^{-1}$" on your calculator

and then "sin" the answer. Instead
$$\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1, \\
\sin^2\theta + \frac{36}{49} &= 1, \\
\sin\theta &= \pm\sqrt{\frac{13}{49}} = \pm\frac{\sqrt{13}}{7}.
\end{aligned}$$

Without further information you must keep both the positive and negative solution.

- If a question tells you that the angle is 'acute', 'obtuse' or 'reflex' then you must visualise the appropriate graph and interpret. For example given that $\sin\theta = \frac{1}{3}$ and that θ is obtuse, find the value of $\cos\theta$. By the argument above you will find that

$$\cos\theta = \pm\frac{\sqrt{8}}{3} = \pm\frac{2\sqrt{2}}{3}.$$

  However, given an obtuse angle ($90° < \theta < 180°$) the cosine graph is negative, so the final answer should be $\cos\theta = -\frac{2\sqrt{2}}{3}$.

- You must be careful when you see things like $2\tan x \sin x = \tan x$. It is SO tempting to divide both sides by $\tan x$ to yield $2\sin x = 1$. But you must bring everything to one side and factorise;

$$2\tan x \sin x - \tan x = 0 \qquad \Rightarrow \qquad \tan x(2\sin x - 1) = 0.$$

  The full set of solutions can then be by solving $\tan x = 0$ <u>and</u> $2\sin x - 1 = 0$. [It is completely analogous to $x^2 = x$. If we divide by x we find $x = 1$, but we know this has missed the solution $x = 0$. However when we factorise we find $x(x-1) = 0$ and both solutions are found.]

- Given a trigonometric equation it is always best first to isolate the trigonometric function on its own; for example

$$9\cos(\dots) + 2 = 7 \qquad \Rightarrow \qquad \cos(\dots) = \tfrac{5}{9}.$$

- For complicated trigonometric equations where you are not just 'cos'ing, 'sin'ing or 'tan'ing a single variable (x, θ, t or the like), it is often easiest to make a substitution.

  For example to solve $\cos(2x+30) = \frac{1}{4}$ in the range $0° \leqslant x \leqslant 360°$ the desired substitution is clearly $u = 2x + 30$, but you **must** remember to also convert the range also (many students forget this) so:

$$\begin{aligned} \cos(2x+30) &= \tfrac{1}{4} \qquad & 0° \leqslant x \leqslant 360°, \\ \cos u &= \tfrac{1}{4} \qquad & 30° \leqslant u \leqslant 750°, \\ u &= \dots°, \dots°, \dots°, \dots°. \end{aligned}$$

  However, we don't want solutions in u, so we need to use $x = \frac{u-30}{2}$ on each u solution to get

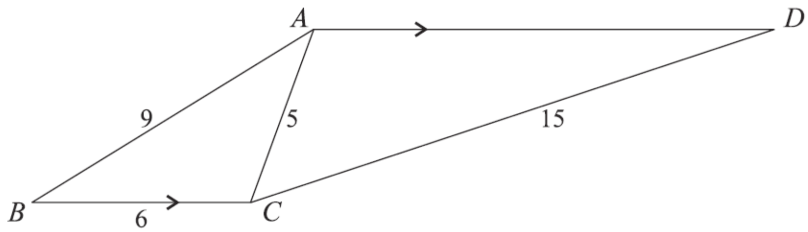
$$x = \dots°, \dots°, \dots°, \dots°.$$

Sine & Cosine Rules

- The sine rule states for any triangle $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.
- The cosine rule states that $a^2 = b^2 + c^2 - 2bc\cos A$. Practice both sine and cosine rules on page 293.
- By considering half of a general parallelogram we can show that the area of any triangle is given by $A = \frac{1}{2}ab\sin C$.
- You must be good at bearing problems which result in triangles. Remember to draw lots of North lines and remember also that they are all parallel; therefore you can use Corresponding, Alternate and Allied angle theorems. . . revise your GCSE notes! Bearings are measured clockwise from North and must contain three digits. For example

$$12.2^\circ \quad \Rightarrow \quad 012.2^\circ.$$

1.



In the diagram, $ABCD$ is a quadrilateral in which AD is parallel to BC. It is given that $AB = 9$, $BC = 6$, $CA = 5$ and $CD = 15$.

(i) Show that $\cos BCA = -\frac{1}{3}$, and hence find the value of $\sin BCA$. [4]

(ii) Find the angle ADC correct to the nearest $0.1°$. [4]

Q4 June2005

2.

(a) **(i)** Write down the exact values of $\cos \frac{1}{6}\pi$ and $\tan \frac{1}{3}\pi$ (where the angles are in radians). Hence verify that $x = \frac{1}{6}\pi$ is a solution of the equation

$$2\cos x = \tan 2x.$$ [3]

(ii) Sketch, on a single diagram, the graphs of $y = 2\cos x$ and $y = \tan 2x$, for x (radians) such that $0 \leqslant x \leqslant \pi$. Hence state, in terms of π, the other values of x between 0 and π satisfying the equation

$$2\cos x = \tan 2x.$$ [4]

(b) **(i)** Use the trapezium rule, with 3 strips, to find an approximate value for the area of the region bounded by the curve $y = \tan x$, the x-axis, and the lines $x = 0.1$ and $x = 0.4$. (Values of x are in radians.) [4]

(ii) State with a reason whether this approximation is an underestimate or an overestimate. [1]

Q9 June2005

3.

Triangle ABC has $AB = 10$ cm, $BC = 7$ cm and angle $B = 80°$. Calculate

(i) the area of the triangle, [2]

(ii) the length of CA, [2]

(iii) the size of angle C. [2]

Q2 Jan 2006

4.

(i) Sketch, on a single diagram showing values of x from $-180°$ to $+180°$, the graphs of $y = \tan x$ and $y = 4\cos x$. [3]

The equation

$$\tan x = 4\cos x$$

has two roots in the interval $-180° \leqslant x \leqslant 180°$. These are denoted by α and β, where $\alpha < \beta$.

(ii) Show α and β on your sketch, and express β in terms of α. [3]

(iii) Show that the equation $\tan x = 4\cos x$ may be written as

$$4\sin^2 x + \sin x - 4 = 0.$$

Hence find the value of $\beta - \alpha$, correct to the nearest degree. [6]

Q9 Jan 2006

5.

Solve each of the following equations, for $0° \leqslant x \leqslant 180°$.

(i) $2\sin^2 x = 1 + \cos x$. [4]

(ii) $\sin 2x = -\cos 2x$. [4]

Q5 June 2006

6.

In a triangle ABC, $AB = 5\sqrt{2}$ cm, $BC = 8$ cm and angle $B = 60°$.

(i) Find the exact area of the triangle, giving your answer as simply as possible. [3]

(ii) Find the length of AC, correct to 3 significant figures. [3]

Q4 Jan 2007

7.

(i) **(a)** Sketch the graph of $y = 2\cos x$ for values of x such that $0° \leqslant x \leqslant 360°$, indicating the coordinates of any points where the curve meets the axes. [2]

(b) Solve the equation $2\cos x = 0.8$, giving all values of x between $0°$ and $360°$. [3]

(ii) Solve the equation $2\cos x = \sin x$, giving all values of x between $-180°$ and $180°$. [3]

Q7 Jan 2007

8.

(i) Show that the equation

$$3\cos^2\theta = \sin\theta + 1$$

can be expressed in the form

$$3\sin^2\theta + \sin\theta - 2 = 0. \qquad [2]$$

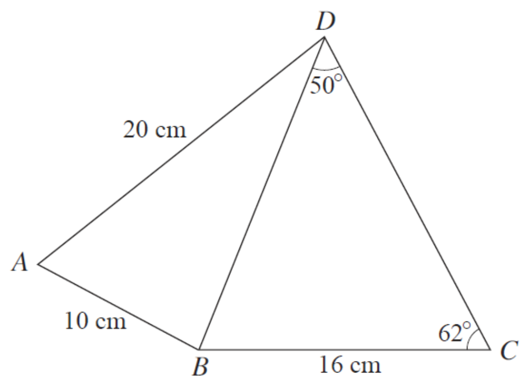
(ii) Hence solve the equation

$$3\cos^2\theta = \sin\theta + 1,$$

giving all values of θ between $0°$ and $360°$. [5]

Q5 June 2007

9.



In the diagram, angle $BDC = 50°$ and angle $BCD = 62°$. It is given that $AB = 10$ cm, $AD = 20$ cm and $BC = 16$ cm.

(i) Find the length of BD. [2]

(ii) Find angle BAD. [3]

Q4 Jan 2008

10.

(i)

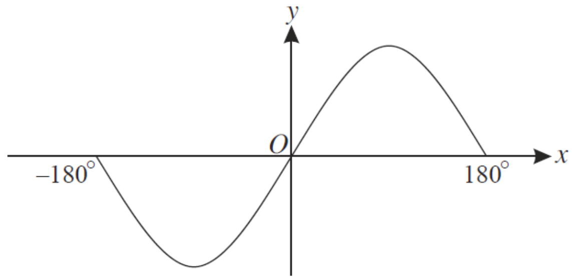


Fig. 1

Fig. 1 shows the curve $y = 2\sin x$ for values of x such that $-180° \leqslant x \leqslant 180°$. State the coordinates of the maximum and minimum points on this part of the curve. [2]

(ii)

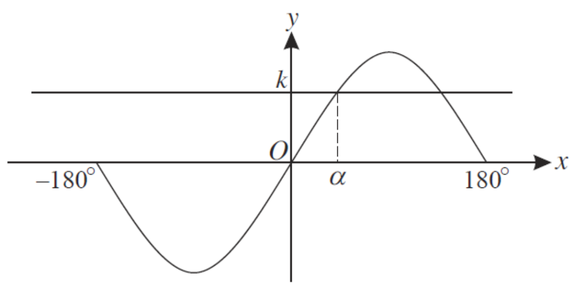


Fig. 2

Fig. 2 shows the curve $y = 2\sin x$ and the line $y = k$. The smallest positive solution of the equation $2\sin x = k$ is denoted by α. State, in terms of α, and in the range $-180° \leqslant x \leqslant 180°$,

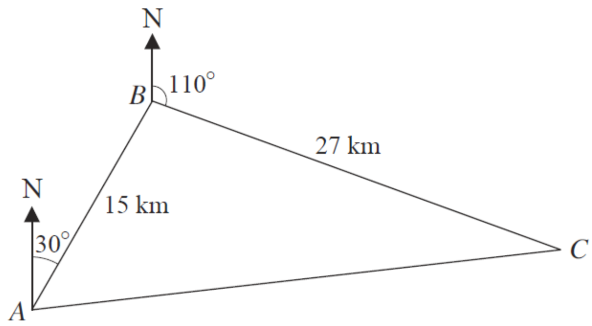
(a) another solution of the equation $2\sin x = k$, [1]

(b) one solution of the equation $2\sin x = -k$. [1]

(iii) Find the x-coordinates of the points where the curve $y = 2\sin x$ intersects the curve $y = 2 - 3\cos^2 x$, for values of x such that $-180° \leqslant x \leqslant 180°$. [6]

Q9 Jan 2008

11.



In the diagram, a lifeboat station is at point A. A distress call is received and the lifeboat travels 15 km on a bearing of 030° to point B. A second call is received and the lifeboat then travels 27 km on a bearing of 110° to arrive at point C. The lifeboat then travels back to the station at A.

(i) Show that angle ABC is 100°. [1]

(ii) Find the distance that the lifeboat has to travel to get from C back to A. [2]

(iii) Find the bearing on which the lifeboat has to travel to get from C to A. [4]

Q6 June 2008

12.

(a) **(i)** Show that the equation

$$2\sin x \tan x - 5 = \cos x$$

can be expressed in the form

$$3\cos^2 x + 5\cos x - 2 = 0.$$ [3]

(ii) Hence solve the equation

$$2\sin x \tan x - 5 = \cos x,$$

giving all values of x, in radians, for $0 \leqslant x \leqslant 2\pi$. [4]

Q9 June 2008

13.

Some walkers see a tower, T, in the distance and want to know how far away it is. They take a bearing from a point A and then walk for 50 m in a straight line before taking another bearing from a point B. They find that angle TAB is 70° and angle TBA is 107° (see diagram).

(i) Find the distance of the tower from A. **[2]**

(ii) They continue walking in the same direction for another 100 m to a point C, so that AC is 150 m. What is the distance of the tower from C? **[3]**

(iii) Find the shortest distance of the walkers from the tower as they walk from A to C. **[2]**

14.

(i) The polynomial f(x) is defined by

$$f(x) = x^3 - x^2 - 3x + 3.$$

Show that $x = 1$ is a root of the equation $f(x) = 0$, and hence find the other two roots. **[6]**

(ii) Hence solve the equation

$$\tan^3 x - \tan^2 x - 3\tan x + 3 = 0$$

for $0 \leqslant x \leqslant 2\pi$. Give each solution for x in an exact form. **[6]**

Q9 Jan 2009

15.

The lengths of the three sides of a triangle are 6.4 cm, 7.0 cm and 11.3 cm.

(i) Find the largest angle in the triangle. **[3]**

(ii) Find the area of the triangle. **[2]**

Q1 June 2009

16.

Solve each of the following equations for $0° \leqslant x \leqslant 180°$.

(i) $\sin 2x = 0.5$ **[3]**

(ii) $2\sin^2 x = 2 - \sqrt{3}\cos x$ **[5]**

Q5 June 2009

17.

(i) Show that the equation

$$2\sin^2 x = 5\cos x - 1$$

can be expressed in the form

$$2\cos^2 x + 5\cos x - 3 = 0.$$ **[2]**

(ii) Hence solve the equation

$$2\sin^2 x = 5\cos x - 1,$$

giving all values of x between 0° and 360°. **[4]**

Q1 Jan 2010

18.

(i) Show that $\dfrac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} \equiv \tan^2 x - 1$. **[2]**

(ii) Hence solve the equation

$$\frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} = 5 - \tan x,$$

for $0° \leqslant x \leqslant 360°$. **[6]**

Q7 June 2010

19.

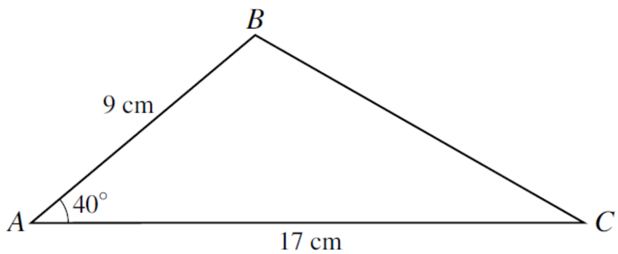
Solve each of the following equations for $0° \leqslant x \leqslant 180°$.

(i) $3\tan 2x = 1$ **[3]**

(ii) $3\cos^2 x + 2\sin x - 3 = 0$ **[5]**

Q7 Jan 2011

20.



The diagram shows triangle ABC, with $AB = 9$ cm, $AC = 17$ cm and angle $BAC = 40°$.

(i) Find the length of BC. **[2]**

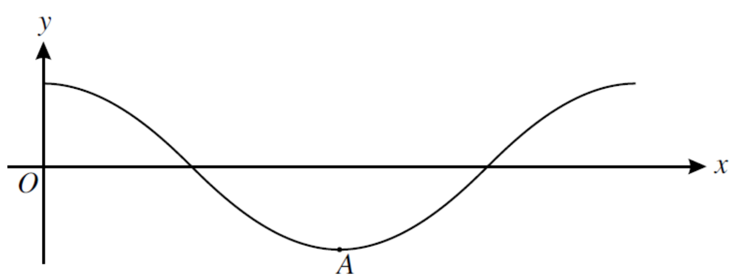
(ii) Find the area of triangle ABC. **[2]**

(iii) D is the point on AC such that angle $BDA = 63°$. Find the length of BD. **[3]**

Q1 June 2011

21.

(a)



The diagram shows part of the curve $y = \cos 2x$, where x is in radians. The point A is the minimum point of this part of the curve.

(i) State the period of $y = \cos 2x$. **[1]**

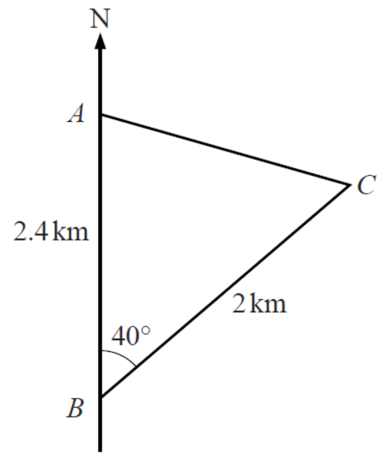
(ii) State the coordinates of A. **[2]**

(iii) Solve the inequality $\cos 2x \leqslant 0.5$ for $0 \leqslant x \leqslant \pi$, giving your answers exactly. **[4]**

(b) Solve the equation $\cos 2x = \sqrt{3} \sin 2x$ for $0 \leqslant x \leqslant \pi$, giving your answers exactly. **[4]**

Q9 June 2011

22.



The diagram shows two points A and B on a straight coastline, with A being 2.4 km due north of B. A stationary ship is at point C, on a bearing of 040° and at a distance of 2 km from B.

(i) Find the distance AC, giving your answer correct to 3 significant figures. **[2]**

(ii) Find the bearing of C from A. **[3]**

(iii) Find the shortest distance from the ship to the coastline. **[2]**

Q4 Jan 2012

23.

(i) Sketch the graph of $y = \tan(\frac{1}{2}x)$ for $-2\pi \leqslant x \leqslant 2\pi$ on the axes provided.

On the same axes, sketch the graph of $y = 3\cos(\frac{1}{2}x)$ for $-2\pi \leqslant x \leqslant 2\pi$, indicating the point of intersection with the y-axis. **[3]**

(ii) Show that the equation $\tan(\frac{1}{2}x) = 3\cos(\frac{1}{2}x)$ can be expressed in the form

$$3\sin^2(\tfrac{1}{2}x) + \sin(\tfrac{1}{2}x) - 3 = 0.$$

Hence solve the equation $\tan(\frac{1}{2}x) = 3\cos(\frac{1}{2}x)$ for $-2\pi \leqslant x \leqslant 2\pi$. **[6]**

Q9 Jan 2012

24.

Solve the equation

$$4\cos^2 x + 7\sin x - 7 = 0,$$

giving all values of x between 0° and 360°. **[6]**

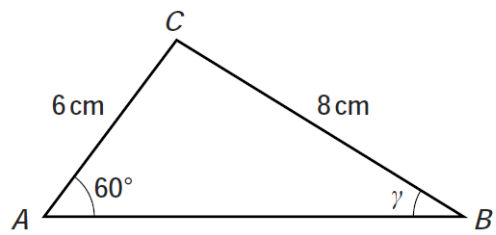
Q4 June 2012

25.

(a) **(i)** Given that α is the acute angle such that $\tan\alpha = \frac{2}{5}$, find the exact value of $\cos\alpha$. **[2]**

(ii) Given that β is the obtuse angle such that $\sin\beta = \frac{3}{7}$, find the exact value of $\cos\beta$. **[3]**

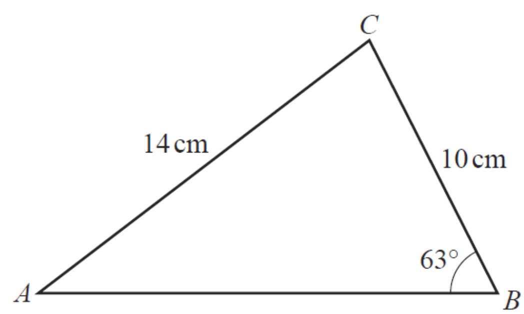
(b)



The diagram shows a triangle ABC with $AC = 6$ cm, $BC = 8$ cm, angle $BAC = 60°$ and angle $ABC = \gamma$. Find the exact value of $\sin\gamma$, simplifying your answer. **[3]**

Q7 June 2012

26.



The diagram shows triangle ABC, with $AC = 14$ cm, $BC = 10$ cm and angle $ABC = 63°$.

(i) Find angle CAB. **[2]**

(ii) Find the length of AB. **[2]**

Q1 Jan 2013

27.

(i) Show that the equation $2\sin x = \dfrac{4\cos x - 1}{\tan x}$ can be expressed in the form

$$6\cos^2 x - \cos x - 2 = 0.$$ **[3]**

(ii) Hence solve the equation $2\sin x = \dfrac{4\cos x - 1}{\tan x}$, giving all values of x between 0° and 360°. **[4]**

Q5 Jan 2013

28.

Solve each of the following equations, for $0° \leqslant x \leqslant 360°$.

(i) $\sin\frac{1}{2}x = 0.8$ **[3]**

(ii) $\sin x = 3\cos x$ **[3]**

Q2 June 2013

29.

The cubic polynomial $f(x)$ is defined by $f(x) = 4x^3 - 7x - 3$.

(i) Find the remainder when $f(x)$ is divided by $(x - 2)$. **[2]**

(ii) Show that $(2x + 1)$ is a factor of $f(x)$ and hence factorise $f(x)$ completely. **[6]**

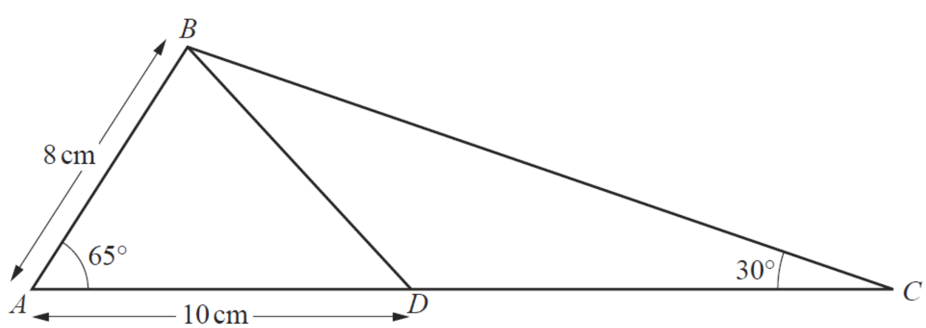
(iii) Solve the equation

$$4\cos^3\theta - 7\cos\theta - 3 = 0$$

for $0 \leqslant \theta \leqslant 2\pi$. Give each solution for θ in an exact form. **[4]**

Q9 June 2013

30.



The diagram shows triangle ABC, with $AB = 8$ cm, angle $BAC = 65°$ and angle $BCA = 30°$. The point D is on AC such that $AD = 10$ cm.

(i) Find the area of triangle ABD. **[2]**

(ii) Find the length of BD. **[2]**

(iii) Find the length of BC. **[2]**

Q1 June 2014

31.

(i) Show that the equation

$$\sin x - \cos x = \frac{6\cos x}{\tan x}$$

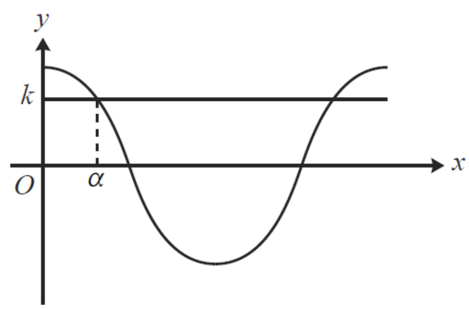
can be expressed in the form

$$\tan^2 x - \tan x - 6 = 0. \qquad \textbf{[2]}$$

(ii) Hence solve the equation $\sin x - \cos x = \frac{6\cos x}{\tan x}$ for $0° \leqslant x \leqslant 360°$. **[4]**

Q4 June 2014

32.



The diagram shows part of the curve $y = 2\cos\frac{1}{3}x$, where x is in radians, and the line $y = k$.

(i) The smallest positive solution of the equation $2\cos\frac{1}{3}x = k$ is denoted by α. State, in terms of α,

(a) the next smallest positive solution of the equation $2\cos\frac{1}{3}x = k$, **[1]**

(b) the smallest positive solution of the equation $2\cos\frac{1}{3}x = -k$. **[2]**

(ii) The curve $y = 2\cos\frac{1}{3}x$ is shown in the Printed Answer Book. On the diagram, and for the same values of x, sketch the curve of $y = \sin\frac{1}{3}x$. **[2]**

(iii) Calculate the x-coordinates of the points of intersection of the curves in part **(ii)**. Give your answers in radians correct to 3 significant figures. **[4]**

Q9 June 2015