Edexcel

Pure Mathematics

Year 2

Trigonometry

Past paper questions from Core Maths 3 and IAL C34

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Past paper questions from

Edexcel Core Maths 3 and IAL C34.

From June 2005 to Nov 2019.

The Section 1 has 42 Questions on

- Identities involving Cosec, Sec and Cot
- The Addition Formulas
- The Double Angle Formulas

The Sections 2 has 37 questions on

- The *R* Addition Formulas
- The Trigonometry Modelling

Please check the Edexcel website for the solutions.

Section_01

1. (*a*) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \tan^2\theta \equiv \sec^2\theta$.

(2)

(*b*) Solve, for $0 \leq \theta < 360°$, the equation

$$2\tan^2\theta + \sec\theta = 1,$$

giving your answers to 1 decimal place.

(6)

[2005 June Q1]

2. (*a*) Show that

(i) $\dfrac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x, \quad x \neq (n - \tfrac{1}{4})\pi, \ n \in \mathbb{Z},$

(2)

(ii) $\tfrac{1}{2}(\cos 2x - \sin 2x) \equiv \cos^2 x - \cos x \sin x - \tfrac{1}{2}$.

(3)

(*b*) Hence, or otherwise, show that the equation

$$\cos\theta\left(\frac{\cos 2\theta}{\cos\theta + \sin\theta}\right) = \frac{1}{2}$$

can be written as

$$\sin 2\theta = \cos 2\theta.$$

(3)

(*c*) Solve, for $0 \leq \theta < 2\pi$,

$$\sin 2\theta = \cos 2\theta,$$

giving your answers in terms of π.

(4)

[2006 January Q7]

3. (*a*) Using $\sin^2\theta + \cos^2\theta \equiv 1$, show that the $\operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$.

(2)

(*b*) Hence, or otherwise, prove that

$$\operatorname{cosec}^4\theta - \cot^4\theta \equiv \operatorname{cosec}^2\theta + \cot^2\theta.$$

(2)

(*c*) Solve, for $90° < \theta < 180°$,

$$\operatorname{cosec}^4\theta - \cot^4\theta = 2 - \cot\theta.$$

(6)

[2006 June Q6]

4. (*a*) Given that $\cos A = \frac{3}{4}$, where $270° < A < 360°$, find the exact value of $\sin 2A$.

(5)

(*b*) (i) Show that $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x$.

(3)

Given that

$$y = 3\sin^2 x + \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right),$$

(ii) show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \sin 2x$.

(4)

[2006 June Q8]

5. (*a*) By writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(5)

(*b*) Given that $\sin\theta = \frac{\sqrt{3}}{4}$, find the exact value of $\sin 3\theta$.

(2)

[2007 January Q1]

6. (*a*) Prove that

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2\operatorname{cosec} 2\theta, \quad \theta \neq 90n°.$$

(4)

(*b*) Sketch the graph of $y = 2\operatorname{cosec} 2\theta$ for $0° < \theta < 360°$.

(2)

(*c*) Solve, for $0° < \theta < 360°$, the equation

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 3$$

giving your answers to 1 decimal place.

(6)

[2007 June Q7]

7. (*a*) Use the double angle formulae and the identity

$$\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.

(4)

(*b*) (i) Prove that

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} \equiv 2\sec x, \qquad x \neq (2n+1)\frac{\pi}{2}.$$

(4)

(ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 4.$$

(3)

[2008 January Q6]

8. (*a*) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$.

(2)

(*b*) Solve, for $0 \le \theta < 180°$, the equation

$$2\cot^2\theta - 9\operatorname{cosec}\theta = 3,$$

giving your answers to 1 decimal place.

(6)

[2008 June Q5]

9. (*a*) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8 \sin^3 \theta - 6 \sin \theta + 1 = 0.$$

Give your answers in terms of π.

(5)

(*b*) Using $\sin (\theta - \alpha) = \sin \theta \cos \alpha - \cos \theta \sin \alpha$, or otherwise, show that

$$\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$ **(4)**

[2009 January Q6]

10. (*a*) Use the identity $\cos^2 \theta + \sin^2 \theta = 1$ to prove that $\tan^2 \theta = \sec^2 \theta - 1$.

(2)

(*b*) Solve, for $0 \leq \theta < 360°$, the equation

$$2 \tan^2 \theta + 4 \sec \theta + \sec^2 \theta = 2.$$

(6)
[2009 June Q2]

11. (*a*) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$.

(1)

(*b*) Find, for $0 < x < \pi$, all the solutions of the equation

$$\text{cosec } x - 8 \cos x = 0.$$

giving your answers to 2 decimal places.

(5)
[2009 June Q8]

12. Solve

$$\text{cosec}^2 2x - \cot 2x = 1$$

for $0 \leq x \leq 180°$.

(7)
[2010 January Q8]

13. (*a*) Show that

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta.$$

(2)

(*b*) Hence find, for $-180° \leq \theta < 180°$, all the solutions of

$$\frac{2\sin 2\theta}{1+\cos 2\theta} = 1.$$

Give your answers to 1 decimal place.

(3)
[2010 June Q1]

14. Find all the solutions of

$$2\cos 2\theta = 1 - 2\sin\theta$$

in the interval $0 \leq \theta < 360°$.

(6)

[2011January Q3]

15. (*a*) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan\theta, \qquad \theta \neq 90n°, \quad n \in \mathbb{Z}.$$

(4)

(*b*) Hence, or otherwise,

(i) show that $\tan 15° = 2 - \sqrt{3}$,

(3)

(ii) solve, for $0 < x < 360°$,

$$\operatorname{cosec} 4x - \cot 4x = 1.$$

(5)
[2011 June Q6]

16. Solve, for $0 \leq \theta \leq 180°$,

$$2\cot^2 3\theta = 7\operatorname{cosec} 3\theta - 5.$$

Give your answers in degrees to 1 decimal place.

(10)
[2012January Q5]

17. (*a*) Starting from the formulae for $\sin(A+B)$ and $\cos(A+B)$, prove that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

(4)

(*b*) Deduce that

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3}\tan\theta}{\sqrt{3} - \tan\theta}.$$

(3)

(*c*) Hence, or otherwise, solve, for $0 \le \theta \le \pi$,

$$1 + \sqrt{3}\tan\theta = (\sqrt{3} - \tan\theta)\tan(\pi - \theta).$$

Give your answers as multiples of π.

(6)

[2012January Q8]

18. (*a*) Express $4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta$ in terms of $\sin\theta$ and $\cos\theta$.

(2)

(*b*) Hence show that

$$4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta = \sec^2\theta.$$

(4)

(*c*) Hence or otherwise solve, for $0 < \theta < \pi$,

$$4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta = 4$$

giving your answers in terms of π.

(3)

[2012June Q5]

19. (i) Without using a calculator, find the exact value of

$$(\sin 22.5° + \cos 22.5°)^2.$$

You must show each stage of your working.

(5)

(ii) (*a*) Show that $\cos 2\theta + \sin\theta = 1$ may be written in the form

$$k\sin^2\theta - \sin\theta = 0, \text{ stating the value of } k.$$

(2)

(*b*) Hence solve, for $0 \le \theta < 360°$, the equation

$$\cos 2\theta + \sin\theta = 1.$$

(4)

[2013January Q6]

20. Given that

$$2\cos(x + 50)° = \sin(x + 40)°$$

(*a*) Show, without using a calculator, that

$$\tan x° = \frac{1}{3} \tan 40°$$

(4)

(*b*) Hence solve, for $0 \leq \theta < 360$,

$$2\cos(2\theta + 50)° = \sin(2\theta + 40)°$$

giving your answers to 1 decimal place.

(4)

[2013June Q3]

21. (i) Use an appropriate double angle formula to show that

$$\operatorname{cosec} 2x = \lambda \operatorname{cosec} x \sec x,$$

and state the value of the constant λ.

(3)

(ii) Solve, for $0 \leq \theta < 2\pi$, the equation

$$3\sec^2 \theta + 3 \sec \theta = 2 \tan^2 \theta$$

You must show all your working. Give your answers in terms of π.

(6)

[2013_R June Q6]

22. (*a*) Show that

$$\operatorname{cosec} 2x + \cot 2x = \cot x, \qquad x \neq 90n°, \qquad n \in \mathbb{Z}$$

(5)

(*b*) Hence, or otherwise, solve, for $0 \leq \theta < 180°$,

$$\operatorname{cosec}(4\theta + 10°) + \cot(4\theta + 10°) = \sqrt{3}$$

You must show your working.

(5)

[2014June Q7]

23. (i) (*a*) Show that $2 \tan x - \cot x = 5 \operatorname{cosec} x$ may be written in the form

$$a \cos^2 x + b \cos x + c = 0$$

stating the values of the constants a, b and c.

(4)

(*b*) Hence solve, for $0 \le x < 2\pi$, the equation

$$2 \tan x - \cot x = 5 \operatorname{cosec} x$$

giving your answers to 3 significant figures.

(4)

(ii) Show that

$$\tan\theta + \cot\theta \equiv \lambda \operatorname{cosec} 2\theta, \qquad \theta = \frac{n\pi}{2}, \qquad n \in \mathbb{Z}$$

stating the value of the constant λ.

(4)

[2014_R June Q3]

24. Given that

$$\tan\theta° = p, \text{ where } p \text{ is a constant, } p \neq \pm 1,$$

use standard trigonometric identities, to find in terms of p,

(*a*) $\tan 2\theta°$,

(2)

(*b*) $\cos\theta°$,

(2)

(*c*) $\cot(\theta - 45)°$.

(2)

(5)

[2015June Q1]

Write each answer in its simplest form.

25. (*a*) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \qquad A \neq \frac{(2n+1)\pi}{4}, \qquad n \in \mathbb{Z}.$$

(5)

(*b*) Hence solve, for $0 \le \theta < 2\pi$,

$$\sec 2\theta + \tan 2\theta = \frac{1}{2}.$$

Give your answers to 3 decimal places.

(4)

[2015, June Q8]

26. (a) Prove that

$$2 \cot 2x + \tan x \equiv \cot x, \qquad x \neq \frac{n\pi}{2}, \qquad n \in \mathbb{Z}.$$

(4)

(b) Hence, or otherwise, solve, for $-\pi \leq x < \pi$,

$$6 \cot 2x + 3 \tan x = \operatorname{cosec}^2 x - 2.$$

Give your answers to 3 decimal places.

(6)

[2016, June Q8]

27. (a) Prove that

$$\sin 2x - \tan x \equiv \tan x \cos 2x, \qquad x \neq (2n + 1)90°, \quad n \in \mathbb{Z}$$

(4)

(b) Given that $x \neq 90°$ and $x \neq 270°$, solve, for $0 \leqslant x < 360°$,

$$\sin 2x - \tan x = 3 \tan x \sin x$$

Give your answers in degrees to one decimal place where appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(5)

[2017, June Q9]

28. Solve, for $0 \leq \theta < 2\pi$

$$2\cos 2\theta = 5 - 13\sin\theta$$

Give your answers in radians to 3 decimal places.

(5)

[2015, Jan. IAL Q2]

29. (a) Given that

$$2 \cos (x + 30)° = \sin (x - 30)°$$

without using a calculator, show that

$$\tan x° = 3\sqrt{3} - 4$$

(5)

(b) Hence or otherwise solve, for $0 \leq \theta < 180$,

$$2 \cos (2\theta + 40)° = \sin (2\theta - 20)°$$

Give your answers to one decimal place.

(4)

[2015, Jan, IAL Q7]

30. (*a*) Use the substitution $t = \tan x$ to show that the equation

$$4\tan 2x - 3\cot x \sec^2 x = 0$$

can be written in the form

$$3t^4 + 8t^2 - 3 = 0$$

(4)

(*b*) Hence solve, for $0 \leq x < 2\pi$,

$$4\tan 2x - 3\cot x \sec^2 x = 0$$

Give each answer in terms of π. You must make your method clear.

(4)

[2015, June, IAL Q7]

31. (*a*) Show that

$$\cot^2 x - \operatorname{cosec} x - 11 = 0$$

may be expressed in the form $\operatorname{cosec}^2 x - \operatorname{cosec} x + k = 0$, where k is a constant.

(1)

(*b*) Hence solve for $0 \leq x < 360°$

$$\cot^2 x - \operatorname{cosec} x - 11 = 0$$

Give each solution in degrees to one decimal place.

(5)

[2016, Jan, IAL Q2]

32. (*a*) Prove that

$$\sin 2x - \tan x \equiv \tan x \cos 2x, \qquad x \neq \frac{(2n+1)\pi}{2}, \qquad n \in \mathbb{Z}$$

(4)

(*b*) Hence solve, for $0 \leq \theta < \frac{\pi}{2}$

(i) $\sin 2\theta - \tan\theta = \sqrt{3}\cos 2\theta$

(ii) $\tan(\theta + 1)\cos(2\theta + 2) - \sin(2\theta + 2) = 2$

Give your answers in radians to 3 significant figures, as appropriate.

(Solutions based entirely on graphical or numerical methods are not acceptable)

(7)

[2016, June, IAL Q8]

33. (*a*) Using the trigonometric identity for tan($A + B$), prove that

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}, \qquad x \neq (2n+1)30°, \qquad n \in \mathbb{Z}$$

(4)

(*b*) Hence solve, for $-30° < x < 30°$,

$$\tan 3x = 11 \tan x$$

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(5)

[2017, Jan, IAL Q8]

34. (*a*) Prove that

$$\frac{1 - \cos 2x}{1 + \cos 2x} \equiv \tan^2 x, \qquad x \neq (2n + 1)90°, n \in \mathbb{Z}$$

(3)

(*b*) Hence, or otherwise, solve, for $-90° < \theta < 90°$,

$$\frac{2 - 2\cos 2\theta}{1 + \cos 2\theta} - 2 = 7\sec\theta$$

Give your answers in degrees to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(6)

[2017, June, IAL Q7]

35.

(i) Using the identity for $\tan(A \pm B)$, solve, for $-90° < x < 90°$,

$$\frac{\tan 2x + \tan 32°}{1 - \tan 2x \tan 32°} = 5$$

Give your answers, in degrees, to 2 decimal places.

(4)

(ii) (a) Using the identity for $\tan(A \pm B)$, show that

$$\tan(3\theta - 45°) \equiv \frac{\tan 3\theta - 1}{1 + \tan 3\theta}, \qquad \theta \neq (60n + 45)°, n \in \mathbb{Z}$$

(2)

(b) Hence solve, for $0 < \theta < 180°$,

$$(1 + \tan 3\theta)\tan(\theta + 28°) = \tan 3\theta - 1$$

(5)

[2018, June, Q6]

36. (*a*) Given $-90^\circ < A < 90^\circ$, prove that

$$2\cos(A - 30^\circ)\ \sec A \equiv \tan A + k$$

where k is a constant to be determined.

(3)

(*b*) Hence or otherwise, solve, for $-90^\circ < x < 90^\circ$, the equation

$$2\cos(x - 30^\circ) = \sec x$$

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(5)

[2019, June, Q7]

37. (*a*) Prove that

$$\frac{1 - \cos 2x}{\sin 2x} \equiv \tan x, \qquad x \neq \frac{n\pi}{2}$$

(3)

(*b*) Hence solve, for $0 \leqslant \theta < 2\pi$,

$$3\sec^2\theta - 7 = \frac{1 - \cos 2\theta}{\sin 2\theta}$$

Give your answers in radians to 3 decimal places, as appropriate.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

[2017, Oct, IAL Q4]

38. (*a*) Show that

$$\frac{\cot^2 x}{1 + \cot^2 x} \equiv \cos^2 x$$

(3)

(*b*) Hence solve, for $0 \leqslant x < 360^\circ$,

$$\frac{\cot^2 x}{1 + \cot^2 x} = 8\cos 2x + 2\cos x$$

Give each solution in degrees to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(6)

[2018, Jan, IAL Q9]

39. (a) Show that

$$\cot x - \tan x \equiv 2 \cot 2x, \qquad x \neq 90n°, n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise, solve, for $0 \leqslant \theta \leqslant 180°$

$$5 + \cot(\theta - 15°) - \tan(\theta - 15°) = 0$$

giving your answers to one decimal place.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(5)

[2018, June, IAL Q12]

40. Given

$$\cos \theta° = p, \text{ where } p \text{ is a constant and } \theta° \text{ is acute}$$

use standard trigonometric identities to find, in terms of p,

(a) $\sec \theta°$

(1)

(b) $\sin (\theta - 90)°$

(2)

(c) $\sin 2\theta°$

(3)

Write each answer in its simplest form.

[2018, Oct, IAL Q3]

41. The angle x and the angle y are such that

$$\tan x = m \qquad \text{and} \qquad 4 \tan y = 8m + 5$$

where m is a constant.

Given that $16 \sec^2 x + 16 \sec^2 y = 537$

(a) find the two possible values of m.

(4)

Given that the angle x and the angle y are acute, find the exact value of

(b) $\sin x$

(2)

(c) $\cot y$

(2)

[2019, Jan, IAL Q5]

42. (*a*) Prove that

$$\frac{\sec x}{1+\sec x} - \frac{\sec x}{1-\sec x} \equiv 2\operatorname{cosec}^2 x \qquad x \neq n\pi, \qquad n \in \mathbf{Z}$$

(3)

(*b*) Hence solve, for $0 < \theta < \pi$

$$\frac{\sec 2\theta}{1+\sec 2\theta} - \frac{\sec 2\theta}{1-\sec 2\theta} = 3 - 2\cot^2 2\theta$$

giving your answers in radians to 3 significant figures.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(6)

[2019, Oct, IAL Q6]

Section_02

1. (*a*) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2\sin^2 A.$$

(2)

(*b*) Show that

$$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv \sin\theta(4\cos\theta + 6\sin\theta - 3).$$

(4)

(*c*) Express $4\cos\theta + 6\sin\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$
.

(4)

(*d*) Hence, for $0 \leq \theta < \pi$, solve

$$2\sin 2\theta = 3(\cos 2\theta + \sin\theta - 1),$$

giving your answers in radians to 3 significant figures, where appropriate.

(5)

[2005 June Q5]

2. $$\mathrm{f}(x) = 12\cos x - 4\sin x.$$

Given that $\mathrm{f}(x) = R\cos(x + \alpha)$, where $R \geq 0$ and $0 \leq \alpha \leq 90°$,

(*a*) find the value of R and the value of α.

(4)

(*b*) Hence solve the equation

$$12\cos x - 4\sin x = 7$$

for $0 \leq x < 360°$, giving your answers to one decimal place.

(5)

(*c*) (i) Write down the minimum value of $12\cos x - 4\sin x$.

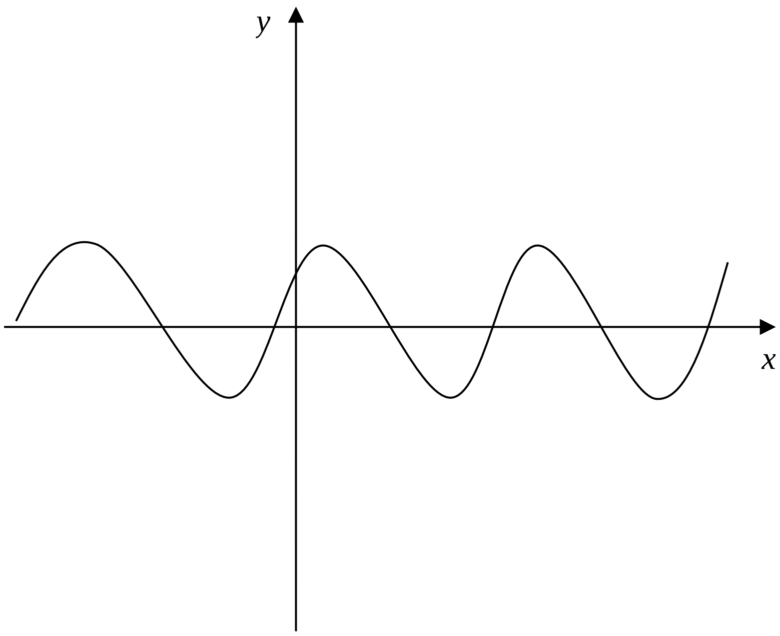
(1)

(ii) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs.

(2)

[2006January Q6]

3. **Figure 1**



The curve on the screen satisfies the equation $y = \sqrt{3} \cos x + \sin x$.

(*a*) Express the equation of the curve in the form $y = R \sin (x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(4)

(*b*) Find the values of x, $0 \leq x < 2\pi$, for which $y = 1$.

(4)

[2007January Q5]

4. (*a*) Express $3 \sin x + 2 \cos x$ in the form $R \sin (x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(4)

(*b*) Hence find the greatest value of $(3 \sin x + 2 \cos x)^4$.

(2)

(*c*) Solve, for $0 < x < 2\pi$, the equation

$$3 \sin x + 2 \cos x = 1,$$

giving your answers to 3 decimal places.

(5)

[2007 June Q7]

5. A curve C has equation

$$y = 3 \sin 2x + 4 \cos 2x, \qquad -\pi \le x \le \pi .$$

The point $A(0, 4)$ lies on C.

(a) Find an equation of the normal to the curve C at A.

(5)

(b) Express y in the form $R \sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 significant figures.

(4)

(c) Find the coordinates of the points of intersection of the curve C with the x-axis. Give your answers to 2 decimal places.

(4)

[2008January Q7]

6. $$\mathrm{f}(x) = 5 \cos x + 12 \sin x.$$

Given that $\mathrm{f}(x) = R \cos (x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

(a) find the value of R and the value of α to 3 decimal places.

(4)

(b) Hence solve the equation

$$5 \cos x + 12 \sin x = 6$$

for $0 \le x < 2\pi$.

(5)

(c) (i) Write down the maximum value of $5 \cos x + 12 \sin x$.

(1)

(ii) Find the smallest positive value of x for which this maximum value occurs.

(2)

[2008 June Q2]

7. (a) Express $3 \cos \theta + 4 \sin \theta$ in the form $R \cos (\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Hence find the maximum value of $3 \cos \theta + 4 \sin \theta$ and the smallest positive value of θ for which this maximum occurs.

(3)

The temperature, f(t), of a warehouse is modelled using the equation

$$f(t) = 10 + 3\cos(15t)° + 4\sin(15t)°,$$

where t is the time in hours from midday and $0 \le t < 24$.

(c) Calculate the minimum temperature of the warehouse as given by this model.

(2)

(d) Find the value of t when this minimum temperature occurs.

(3)

[2009January Q8]

8. (a) Use the identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$, to show that

$$\cos 2A = 1 - 2\sin^2 A$$

(2)

The curves C_1 and C_2 have equations

$$C_1: y = 3\sin 2x$$

$$C_2: y = 4\sin^2 x - 2\cos 2x$$

(b) Show that the x-coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2$$

(3)

(c) Express $4\cos 2x + 3\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

(d) Hence find, for $0 \le x < 180°$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2,$$

giving your answers to 1 decimal place.

(4)

[2009 June Q6]

9. (a) Express $5\cos x - 3\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

(4)

(b) Hence, or otherwise, solve the equation

$$5\cos x - 3\sin x = 4$$

for $0 \le x < 2\pi$, giving your answers to 2 decimal places.

(5)

[2010January Q3]

10. (*a*) Express $2 \sin \theta - 1.5 \cos \theta$ in the form $R \sin (\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

Give the value of α to 4 decimal places.

(3)

(*b*) (i) Find the maximum value of $2 \sin \theta - 1.5 \cos \theta$.

(ii) Find the value of θ, for $0 \leq \theta < \pi$, at which this maximum occurs.

(3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2 \sin \left(\frac{4\pi t}{25}\right) - 1.5 \cos \left(\frac{4\pi t}{25}\right), \quad 0 \leq t < 12,$$

where t hours is the number of hours after midday.

(*c*) Calculate the maximum value of H predicted by this model and the value of t, to 2 decimal places, when this maximum occurs.

(3)

(*d*) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

(6)

[2010 June Q7]

11. (*a*) Express $7 \cos x - 24 \sin x$ in the form $R \cos (x + \alpha)$ where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

Give the value of α to 3 decimal places.

(3)

(*b*) Hence write down the minimum value of $7 \cos x - 24 \sin x$.

(1)

(*c*) Solve, for $0 \leq x < 2\pi$, the equation

$$7 \cos x - 24 \sin x = 10,$$

giving your answers to 2 decimal places.

(5)

[2011January Q1]

12. (*a*) Express $2 \cos 3x - 3 \sin 3x$ in the form $R \cos (3x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your answers to 3 significant figures.

(4)

$$f(x) = e^{2x} \cos 3x.$$

(*b*) Show that f ′(*x*) can be written in the form

$$f'(x) = Re^{2x} \cos (3x + \alpha),$$

where R and α are the constants found in part (*a*).

(5)

(*c*) Hence, or otherwise, find the smallest positive value of x for which the curve with equation $y = f(x)$ has a turning point.

(3)

[2011 June Q8]

13. $$f(x) = 7 \cos 2x - 24 \sin 2x.$$

Given that $f(x) = R \cos (2x + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(*a*) find the value of R and the value of α.

(3)

(*b*) Hence solve the equation

$$7 \cos 2x - 24 \sin 2x = 12.5$$

for $0 \leq x < 180°$, giving your answers to 1 decimal place.

(5)

(*c*) Express $14 \cos^2 x - 48 \sin x \cos x$ in the form $a \cos 2x + b \sin 2x + c$, where a, b, and c are constants to be found.

(2)

(*d*) Hence, using your answers to parts (*a*) and (*c*), deduce the maximum value of

$$14 \cos^2 x - 48 \sin x \cos x.$$

(2)

[2012 June Q8]

14. (*a*) Express 6 cos θ + 8 sin θ in the form R cos (θ – α), where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(4)

(*b*)
$$\mathrm{p}(\theta) = \frac{4}{12 + 6\cos\theta + 8\sin\theta}, \quad 0 \leq \theta \leq 2\pi.$$

Calculate

(i) the maximum value of p(θ),

(ii) the value of θ at which the maximum occurs.

(4)

[2013January Q4]

15.

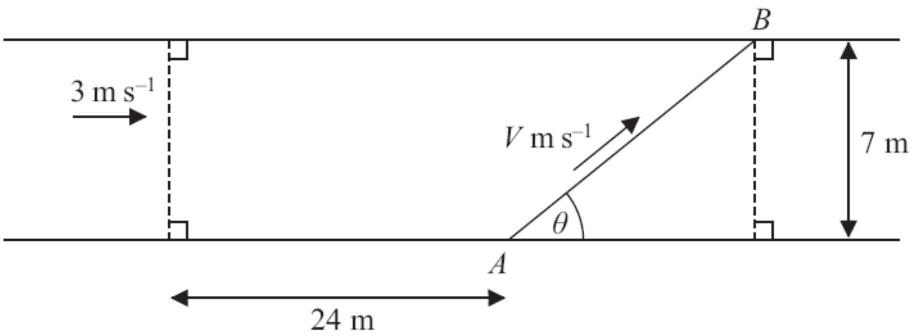


Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1}.
Kate is 24 m ahead of John when she starts to cross the road from the fixed point A.
John passes her as she reaches the other side of the road at a variable point B, as shown in Figure 2.
Kate's speed is V m s^{-1} and she moves in a straight line, which makes an angle θ, $0 < \theta < 150°$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24\sin\theta + 7\cos\theta}, \qquad 0 < \theta < 150°$$

(*a*) Express 24sin θ + 7cos θ in the form Rcos(θ – α), where R and α are constants and where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

Given that θ varies,

(*b*) find the minimum value of V.

(2)

Given that Kate's speed has the value found in part (*b*),

(*c*) find the distance AB.

(3)

Given instead that Kate's speed is 1.68 m s^{-1},

(*d*) find the two possible values of the angle θ, given that $0 < \theta < 150°$.

(6)

[2013 June Q8]

16. $$f(x) = 7\cos x + \sin x$$

Given that $f(x) = R\cos(x - a)$, where R > 0 and $0 < a < 90°$,

(*a*) find the exact value of R and the value of a to one decimal place.

(3)

(*b*) Hence solve the equation

$$7\cos x + \sin x = 5$$

for $0 \leq x < 360°$, giving your answers to one decimal place.

(5)

(*c*) State the values of k for which the equation

$$7\cos x + \sin x = k$$

has only one solution in the interval $0 \leq x < 360°$.

(2)

[2013_R June Q8]

17. (*a*) Express $2\sin\theta - 4\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(3)

$$H(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$$

Find

(*b*) (i) the maximum value of $H(\theta)$,

(ii) the smallest value of θ, for $0 \leq \theta \leq \pi$, at which this maximum value occurs.

(3)

Find

(*c*) (i) the minimum value of $H(\theta)$,

(ii) the largest value of θ, for $0 \le \theta \le \pi$, at which this minimum value occurs. **(3)**

[2014 June Q9]

18.

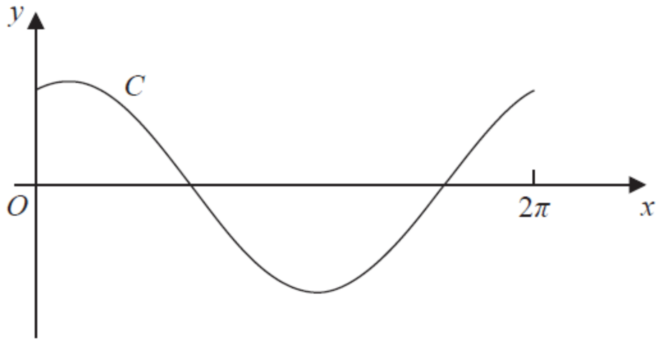


Figure 1

Figure 1 shows the curve *C*, with equation $y = 6\cos x + 2.5\sin x$ for $0 \le x \le 2\pi$.

(*a*) Express $6\cos x + 2.5\sin x$ in the form $R\cos(x - \alpha)$, where R and α are constants with

$R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your value of α to 3 decimal places.

(3)

(*b*) Find the coordinates of the points on the graph where the curve *C* crosses the coordinate axes.

(3)

A student records the number of hours of daylight each Sunday throughout the year. She starts on the last Sunday in May with a recording of 18 hours, and continues until her final recording 52 weeks later.

She models her results with the continuous function given by

$$H = 12 + 6\cos\left(\frac{2\pi t}{52}\right) + 2.5\sin\left(\frac{2\pi t}{52}\right), \qquad 0 \le t \le 52$$

where H is the number of hours of daylight and t is the number of weeks since her first recording.

Use this function to find

(*c*) the maximum and minimum values of H predicted by the model,

(3)

(*d*) the values for t when $H = 16$, giving your answers to the nearest whole number.

(6)

[2014_R June Q7]

19. $$g(\theta) = 4\cos 2\theta + 2\sin 2\theta.$$

Given that $g(\theta) = R\cos(2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(*a*) find the exact value of R and the value of α to 2 decimal places.

(3)

(*b*) Hence solve, for $-90° < \theta < 90°$,

$$4\cos 2\theta + 2\sin 2\theta = 1,$$

giving your answers to one decimal place.

(5)

Given that k is a constant and the equation $g(\theta) = k$ has no solutions,

(*c*) state the range of possible values of k.

(2)

[2015, June Q3]

20. (*a*) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$ Give the exact value of R and give the value of α to 2 decimal places.

(3)

(*b*) Hence solve, for $0 \leq \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15.$$

Give your answers to one decimal place.

(5)

(*c*) Use your solutions to parts (*a*) and (*b*) to deduce the smallest positive value of θ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15.$$

Give your answer to one decimal place.

(2)

[2016, June Q3]

21. (a) Write $5 \cos \theta - 2 \sin \theta$ in the form $R \cos (\theta + \alpha)$, where R and α are constants,

$R > 0$ and $0 \leqslant \alpha < \dfrac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

(b) Show that the equation

$$5 \cot 2x - 3 \operatorname{cosec} 2x = 2$$

can be rewritten in the form

$$5 \cos 2x - 2 \sin 2x = c$$

where c is a positive constant to be determined.

(2)

(c) Hence or otherwise, solve, for $0 \leqslant x < \pi$,

$$5 \cot 2x - 3 \operatorname{cosec} 2x = 2$$

giving your answers to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

[2017, June Q4]

22. (*a*) Express $2\sin\theta + \cos\theta$ in the form $R\sin(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. Give your value of α to 2 decimal places.

(3)

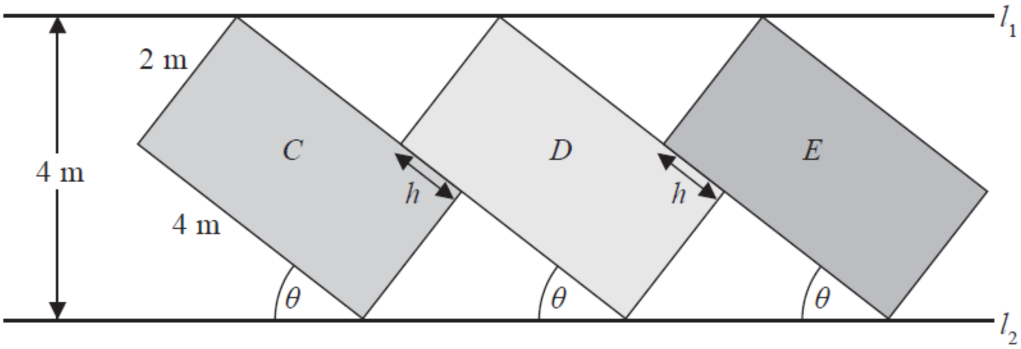


Figure 4

Figure 4 shows the design for a logo that is to be displayed on the side of a large building. The logo consists of three rectangles, C, D and E, each of which is in contact with two horizontal parallel lines l_1 and l_2. Rectangle D touches rectangles C and E as shown in Figure 4.

Rectangles C, D and E each have length 4 m and width 2 m. The acute angle θ between the line l_2 and the longer edge of each rectangle is shown in Figure 4.

Given that l_1 and l_2 are 4 m apart,

(*b*) show that

$$2\sin\theta + \cos\theta = 2$$

(2)

Given also that $0 < \theta < 45°$,

(*c*) solve the equation

$$2\sin\theta + \cos\theta = 2$$

giving the value of θ to 1 decimal place.

(3)

Rectangles C and D and rectangles D and E touch for a distance h m as shown in Figure 4.

Using your answer to part (*c*), or otherwise,

(*d*) find the value of h, giving your answer to 2 significant figures.

(3)

[2014, June, IAL, Q13]

23. (*a*) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$

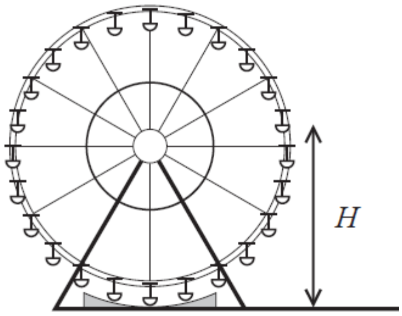
Give the exact value of R and give the value of α to 2 decimal places.

(3)

Alana models the height above the ground of a passenger on a Ferris wheel by the equation

$$H = 12 - 10\cos(30t)° + 3\sin(30t)°$$

where the height of the passenger above the ground is H metres at time t minutes after the wheel starts turning.



(*b*) Calculate

(i) the maximum value of H predicted by this model,

(ii) the value of t when this maximum first occurs.

Give each answer to 2 decimal places.

(4)

(*c*) Calculate the value of t when the passenger is 18m above the ground for the first time.

Give your answer to 2 decimal places.

(4)

(*d*) Determine the time taken for the Ferris wheel to complete two revolutions.

(2)

[2015, Jan, IAL, Q13]

24. (*a*) Express $1.5\sin\theta - 1.2\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of R and the value of α to 3 decimal places.

(3)

The height, H metres, of sea water at the entrance to a harbour on a particular day, is modelled by the equation

$$H = 3 + 1.5\sin\left(\frac{\pi t}{6}\right) - 1.2\cos\left(\frac{\pi t}{6}\right), \qquad 0 \le t < 12$$

where t is the number of hours after midday.

(*b*) Using your answer to part (*a*), calculate the minimum value of H predicted by this
model and the value of t, to 2 decimal places, when this minimum occurs.

(4)

(*c*) Find, to the nearest minute, the times when the height of sea water at the entrance to
the harbour is predicted by this model to be 4 metres.

(6)

[2015, June, IAL, Q11]

25. (*a*) Express $3\sin 2x + 5\cos 2x$ in the form $R\sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α to 3 significant figures.

(3)

(*b*) Solve, for $0 < x < \pi$,

$$3\sin 2x + 5\cos 2x = 4$$

(5)

$$g(x) = 4(3\sin 2x + 5\cos 2x)^2 + 3$$

(*c*) Using your answer to part (*a*) and showing your working,

(i) find the greatest value of $g(x)$,

(ii) find the least value of $g(x)$.

(4)

[2016, Jan, IAL, Q10]

26. (*a*) Express $3\sin 2x + 5\cos 2x$ in the form $R\sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α to 3 significant figures.

(3)

(*b*) Solve, for $0 < x < \pi$,

$$3\sin 2x + 5\cos 2x = 4$$

(5)

$$g(x) = 4(3\sin 2x + 5\cos 2x)^2 + 3$$

(*c*) Using your answer to part (*a*) and showing your working,

(i) find the greatest value of $g(x)$,

(ii) find the least value of $g(x)$.

(4)

[2016, Jan, IAL, Q10]

27. (*a*) Express $3 \cos \theta + 5 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. Give the exact value of R and give the value of α to 2 decimal places.

(3)

(*b*) Hence solve, for $0 \leq \theta < 360°$, the equation

$$3 \cos \theta + 5 \sin \theta = 2$$

Give your answers to one decimal place.

(4)

(*c*) Use your solutions to parts (*a*) and (*b*) to deduce the smallest positive value of θ for which

$$3 \cos \theta - 5 \sin \theta = 2$$

(2)

[2016, June, IAL, Q1]

28. (*a*) Express $35 \sin x \; - 12 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{p}{2}$

Give the exact value of R, and give the value of α, in radians, to 4 significant figures.

(3)

(*b*) Hence solve, for $0 \leq x \leq 2p$,

$$70 \sin x - 24 \cos x = 37$$

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

$$y = \frac{7000}{31 + (35 \sin x - 12 \cos x)^2}, \qquad x > 0$$

(*c*) Use your answer to part (*a*) to calculate

(i) the minimum value of y,

(ii) the smallest value of x, $x > 0$, at which this minimum value occurs.

(4)

[2017, Jan, IAL, Q11]

29. (*a*) Write $2\sin\theta - \cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and

$0 < \alpha \leqslant 90°$. Give the exact value of R and give the value of α to one decimal place.

(3)

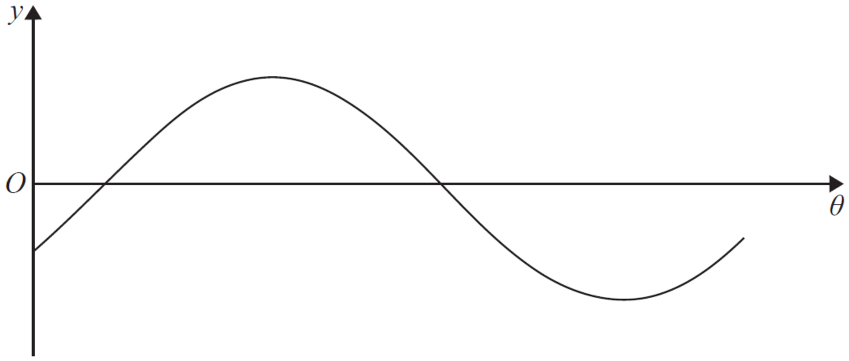


Figure 3

Figure 3 shows a sketch of the graph with equation $y = 2\sin\theta - \cos\theta$, $0 \leqslant \theta < 360°$

(*b*) Sketch the graph with equation

$$y = |\,2\sin\theta - \cos\theta\,|, \quad 0 \leqslant \theta < 360°$$

stating the coordinates of all points at which the graph meets or cuts the coordinate axes.

(3)

The temperature of a warehouse is modelled by the equation

$$f(t) = 5 + |\,2\sin(15t)° - \cos(15t)°\,|, \quad 0 \leqslant t < 24$$

where $f(t)$ is the temperature of the warehouse in degrees Celsius and t is the time measured in hours from midnight.

State

(*c*) (i) the maximum value of $f(t)$,

(ii) the largest value of t, for $0 \leqslant t < 24$, at which this maximum value occurs. Give your answer to one decimal place.

(3)

[2017, June, IAL, Q10]

30.

(a) Express $\sin\theta - 2\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$

Give the exact value of R and the value of α, in radians, to 3 decimal places.

(3)

$$\mathrm{M}(\theta) = 40 + (3\sin\theta - 6\cos\theta)^2$$

(b) Find

(i) the maximum value of $\mathrm{M}(\theta)$,

(ii) the smallest value of θ, in the range $0 < \theta \leqslant 2\pi$, at which the maximum value of $\mathrm{M}(\theta)$ occurs.

(3)

$$\mathrm{N}(\theta) = \frac{30}{5 + 2(\sin 2\theta - 2\cos 2\theta)^2}$$

(c) Find

(i) the maximum value of $\mathrm{N}(\theta)$,

(ii) the largest value of θ, in the range $0 < \theta \leqslant 2\pi$, at which the maximum value of $\mathrm{N}(\theta)$ occurs.

(3)

[2018, June, Q9]

31. (*a*) Write $\cos\theta - 8\sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha \leq 90°$. Give the exact value of R and give the value of α to 2 decimal places.

(3)

The temperature of a cellar is modelled by the equation

$$\mathrm{f}(t) = 13 + \frac{\cos(15t)° - 8\sin(15t)°}{10} \qquad 0 \leq t < 24$$

where $\mathrm{f}(t)$ is the temperature of the cellar in degrees Celsius and t is the time measured in
hours after midnight.

Find, according to the model,

(*b*) the maximum temperature of the cellar, giving your answer to 2 decimal places

(2)

(*c*) the times, after midnight, when then temperature of the cellar is 12.5 °C

(4)

[2019, June, Q3]

32. (a) Express $2\sin x - 4\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{p}{2}$

Give the exact value of R and give the value of α, in radians, to 3 significant figures.

(3)

In a town in Norway, a student records the number of hours of daylight every day for a year. He models the number of hours of daylight, H, by the continuous function given by
the formula

$$H = 12 + 4\sin\left(\frac{2\pi t}{365}\right) - 8\cos\left(\frac{2\pi t}{365}\right), \qquad 0 \leqslant t \leqslant 365$$

where t is the number of days since he began recording.

(b) Using your answer to part (a), or otherwise, find the maximum and minimum number
of hours of daylight given by this formula. Give your answers to 3 significant figures.

(3)

(c) Use the formula to find the values of t when $H = 17$, giving your answers to the nearest integer.

(6)

[2018, Jan, IAL Q12]

33. (a) Express $5\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{p}{2}$

State the value of R and give the value of α to 4 significant figures.

(3)

(b) Solve, for $-\pi < \theta < \pi$,

$$\sqrt{5}\cos\theta - 2\sin\theta = 0.5$$

giving your answers to 3 significant figures.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(4)

$$f(x) = A(\sqrt{5}\cos\theta - 2\sin\theta) + B \qquad \theta \in \mathbb{R}$$

where A and B are constants.

Given that the range of f is

$$-15 \leqslant f(x) \leqslant 33$$

(c) find the value of B and the possible values of A.

(4)

[2018, June, IAL Q6]

34. (*a*) Write $\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants,

$R > 0$ and $0 < \alpha < \frac{p}{2}$. Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

(*b*) Hence solve, for $0 \leqslant \theta < \pi$, the equation

$$\cos 2\theta + 4\sin 2\theta = 1.2$$

giving your answers to 2 decimal places.

(5)

[2018, Oct, IAL Q1]

35. (*a*) Express $7\sin 2\theta - 2\cos 2\theta$ in the form $R\sin(2\theta - \alpha)$, where R and α are constants,

$R > 0$ and $0 < \alpha < 90°$. Give the exact value of R and give the value of α to 2 decimal places.

(3)

(*b*) Hence solve, for $0 \leqslant \theta < 90°$, the equation

$$7\sin 2\theta - 2\cos 2\theta = 4$$

giving your answers in degrees to one decimal place.

(4)

(*c*) Express $28\sin\theta\cos\theta + 8\sin^2\theta$ in the form $a\sin 2\theta + b\cos 2\theta + c$, where a, b and c

are constants to be found.

(3)

(*d*) Use your answers to part (*a*) and part (*c*) to deduce the exact maximum value of $28\sin\theta\cos\theta + 8\sin^2\theta$

(2)

[2019, Jan, IAL Q1]

36.

(a) Express $5\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α, in radians, to 4 decimal places.

(3)

The height of sea water, H metres, on a harbour wall is modelled by the equation

$$H = 6 + 2.5\cos\left(\frac{4\pi t}{25}\right) - 1.5\sin\left(\frac{4\pi t}{25}\right), \qquad 0 \leqslant t < 12$$

where t is the number of hours after midday.

(b) Calculate the times at which the model predicts that the height of sea water on the harbour wall will be 4.6 metres. Give your answers to the nearest minute.

(6)

[2019, June, IAL Q7]

37. (*a*)Express 3 sin x – cos x in the form R sin(x – a), where R and a are constants, $R > 0$ and $0 < a < \frac{\pi}{2}$ Give the exact value of R and give the value of R, in radians, to3 decimal places.

(3)

The temperature, $\theta°$ C, inside a building on a particular day, is modelled by the equation

$$\theta = 19 + 3\sin\left(\frac{\pi t}{12} + 4\right) - \cos\left(\frac{\pi t}{12} + 4\right), \qquad 0 \le t < 24$$

where t is the number of hours after midnight.

(*b*) Using the answer to part (*a*),

(i) state the minimum value of θ predicted by this model,

(ii) find the value of t, to 2 decimal places, when this minimum occurs.

(3)

[2019, Oct, IAL Q1]