Edexcel

Pure Mathematics

Year 1

Trigonometry

Past paper questions from Core Maths 2 and IAL C12

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Past paper questions from

Edexcel Core Maths 2 and IAL C12.

From Jan 2005 to Oct 2019.

This Section 1 has 44 Questions on

- Solving Trigonometry Equations
- Identities

Please check the Edexcel website for the solutions.

1. (*a*) Show that the equation

$$5 \cos^2 x = 3(1 + \sin x)$$

can be written as

$$5 \sin^2 x + 3 \sin x - 2 = 0.$$

(2)

(*b*) Hence solve, for $0 \leq x < 360°$, the equation

$$5 \cos^2 x = 3(1 + \sin x),$$

giving your answers to 1 decimal place where appropriate.

(5)

Jan 2005, Q4

2. Solve, for $0 \leq x \leq 180°$, the equation

(*a*) $\sin(x + 10°) = \frac{\sqrt{3}}{2}$,

(4)

(*b*) $\cos 2x = -0.9$, giving your answers to 1 decimal place.

(4)

June 2005, Q5

3. (*a*) Find all the values of θ, to 1 decimal place, in the interval $0° \leq \theta < 360°$ for which

$$5 \sin(\theta + 30°) = 3.$$

(4)

(*b*) Find all the values of θ, to 1 decimal place, in the interval $0° \leq \theta < 360°$ for which

$$\tan^2 \theta = 4.$$

(5)

Jan 2006, Q8

4. (*a*) Given that $\sin \theta = 5 \cos \theta$, find the value of $\tan \theta$.

(1)

(*b*) Hence, or otherwise, find the values of θ in the interval $0 \leq \theta < 360°$ for which

$$\sin \theta = 5 \cos \theta,$$

giving your answers to 1 decimal place.

(3)

May 2006, Q6

5. Find all the solutions, in the interval $0 \leq x < 360°$, of the equation

$$2 \cos^2 x + 1 = 5 \sin x,$$

giving each solution in exact form.

(6)

Jan 2007, Q6

6. (*a*) Sketch, for $0 \leq x \leq 360°$, the graph of $y = \sin(x + 30)$.

(2)

(*b*) Write down the exact coordinates of the points where the graph meets the coordinate axes.

(3)

(*c*) Solve, for $0 \leq x \leq 360°$, the equation

$$\sin(x + 30) = 0.65,$$

giving your answers in degrees to 2 decimal places.

(5)

May 2007, Q9

7. (*a*) Show that the equation

$$3 \sin^2 \theta - 2 \cos^2 \theta = 1$$

can be written as

$$5 \sin^2 \theta = 3.$$

(2)

(*b*) Hence solve, for $0° \leq \theta < 360°$, the equation

$$3 \sin^2 \theta - 2 \cos^2 \theta = 1,$$

giving your answer to 1 decimal place.

(7)

Jan 2008, Q4

8. Solve, for $0 \leq x < 360°$,

(*a*) $\sin(x - 20°) = \dfrac{1}{\sqrt{2}}$,

(4)

(*b*) $\cos 3x = -\dfrac{1}{2}$.

(6)

June 2008, Q9

9. (*a*) Show that the equation

$$4 \sin^2 x + 9 \cos x - 6 = 0$$

can be written as

$$4 \cos^2 x - 9 \cos x + 2 = 0.$$

(2)

(*b*) Hence solve, for $0 \leq x < 720°$,

$$4 \sin^2 x + 9 \cos x - 6 = 0,$$

giving your answers to 1 decimal place.

(6)

Jan 2009, Q8

10. (i) Solve, for $-180° \leq \theta < 180°$,

$$(1 + \tan \theta)(5 \sin \theta - 2) = 0.$$

(4)

(ii) Solve, for $0 \leq x < 360°$,

$$4 \sin x = 3 \tan x.$$

(6)

June 2009, Q7

11. (*a*) Show that the equation

$$5 \sin x = 1 + 2 \cos^2 x$$

can be written in the form

$$2 \sin^2 x + 5 \sin x - 3 = 0.$$

(2)

(*b*) Solve, for $0 \leq x < 360°$,

$$2 \sin^2 x + 5 \sin x - 3 = 0.$$

(4)

Jan 2010, Q2

12. (*a*) Given that $5 \sin \theta = 2 \cos \theta$, find the value of $\tan \theta$.

(1)

(*b*) Solve, for $0 \leq x < 360°$,

$$5 \sin 2x = 2 \cos 2x ,$$

giving your answers to 1 decimal place.

(5)

June 2010, Q5

13. (*a*) Show that the equation

$$3 \sin^2 x + 7 \sin x = \cos^2 x - 4$$

can be written in the form

$$4 \sin^2 x + 7 \sin x + 3 = 0.$$

(2)

(*b*) Hence solve, for $0 \leq x < 360°$,

$$3 \sin^2 x + 7 \sin x = \cos^2 x - 4$$

giving your answers to 1 decimal place where appropriate.

(5)

Jan 2011, Q7

14. (*a*) Solve for $0 \leq x < 360°$, giving your answers in degrees to one decimal place,

$$3 \sin (x + 45°) = 2.$$

(4)

(*b*) Find, for $0 \leq x < 360°$, all the solutions of

$$2 \sin^2 x + 2 = 7\cos x,$$

giving your answers in degrees to one decimal place.

You must show clearly how you obtained your answers.

(6)

May 2011, Q7

15. (i) Find the solutions of the equation $\sin(3x - 15°) = \frac{1}{2}$, for which $0 \le x \le 180°$.

(6)

(ii)

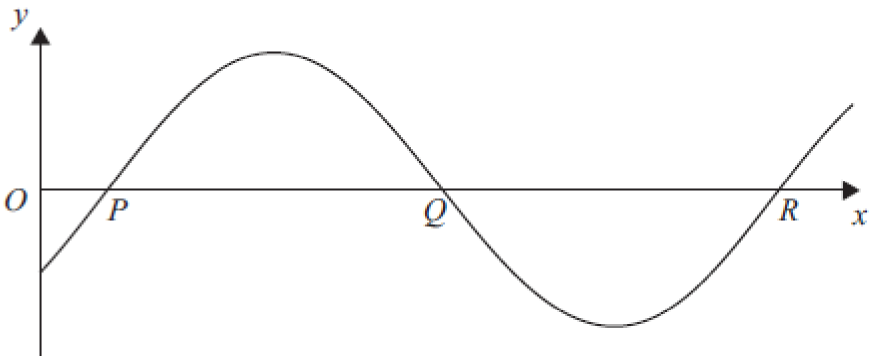

Figure 4

Figure 4 shows part of the curve with equation

$$y = \sin(ax - b), \text{ where } a > 0, \quad 0 < b < 180.$$

The curve cuts the x-axis at the points P, Q and R as shown.

Given that the coordinates of P, Q and R are $(18, 0)$, $(108, 0)$ and $(198, 0)$ respectively, find the values of a and b.

(4)

Jan 2012, Q9

16. (*a*) Show that the equation

$$\tan 2x = 5 \sin 2x$$

can be written in the form

$$(1 - 5 \cos 2x) \sin 2x = 0.$$

(2)

(*b*) Hence solve, for $0 \le x \le 180°$,

$$\tan 2x = 5 \sin 2x,$$

giving your answers to 1 decimal place where appropriate.

You must show clearly how you obtained your answers.

(5)

May 2012, Q6

17. Solve, for $0 \leq x < 180°$,

$$\cos (3x - 10°) = -0.4,$$

giving your answers to 1 decimal place. You should show each step in your working.

(7)

Jan 2013, Q4

18. (i) Solve, for $-180° \leq x < 180°$,

$$\tan(x - 40°) = 1.5,$$

giving your answers to 1 decimal place.

(3)

(ii) (*a*) Show that the equation

$$\sin\theta \tan \theta = 3 \cos\theta + 2$$

can be written in the form

$$4 \cos^2\theta + 2 \cos \theta - 1 = 0.$$

(3)

(*b*) Hence solve, for $0 \leq \theta < 360°$,

$$\sin \theta \tan \theta = 3 \cos \theta + 2,$$

showing each stage of your working.

(5)

May 2013, Q8

19. (i) Solve, for $0 \leq \theta < 180°$

$$\sin (2\theta - 30°) + 1 = 0.4$$

giving your answers to 1 decimal place.

(5)

(ii) Find all the values of x, in the interval $0 \leq \theta < 360°$, for which

$$9 \cos^2 x - 11 \cos x + 3 \sin^2 x = 0$$

giving your answers to 1 decimal place.

(7)

You must show clearly how you obtained your answers.

May 2013_R, Q9

20. (i) Solve, for $0 \leq \theta < 180°$, the equation

$$\frac{\sin 2\theta}{(4\sin 2\theta - 1)} = 1$$

giving your answers to 1 decimal place.

(3)

(ii) Solve, for $0 \leq x < 360°$, the equation

$$5\sin^2 x - 2\cos x - 5 = 0$$

giving your answers to 2 decimal places.

(5)

May 2014_R, Q7

21. (i) Solve, for $0 \leq \theta < 360°$, the equation $9 \sin (\theta + 60°) = 4$, giving your answers to 1 decimal place. You must show each step of your working.

(4)

(ii) Solve, for $-180° \leq x < 180°$, the equation $2 \tan x - 3 \sin x = 0$, giving your answers to 2 decimal places where appropriate.

(5)

May 2014, Q7

22. (i) Solve, for $0 \leq \theta < 180°$, the equation

$$\sin 3\theta - \sqrt{3} \cos 3\theta = 0,$$

(3)

(ii) Given that

$$4 \sin^2 x + \cos x = 4 - k, \qquad 0 \leq k \leq 3,$$

(*a*) find $\cos x$ in terms of k.

(3)

(*b*) When $k = 3$, find the values of x in the range $0 \leq x < 360°$.

(3)

May 2015, Q8

23. (i) Solve, for $-180° < \theta \leq 180°$,

$$1 - 2\cos(\theta - 36) = 0$$

(3)

(ii) Solve, for $0 \leq x < 360°$,

$$4\cos^2 x + 7\sin x - 2 = 0,$$

giving your answers to one decimal place.

(6)

May 2016, Q6

24. (*a*) Show that the equation

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

can be written in the form

$$(3\sin x - 1)^2 = 2$$

(3)

(*b*) Hence solve, for $0 \leqslant x < 360°$,

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

giving your answers to 2 decimal places.

(5)

May 2017, Q8

25. (i) Solve, for $0 \leq x < 180°$

$$4\cos(x + 70°) = 3$$

giving your answers to 1 decimal place.

(4)

(ii) Find all the values of x, in the interval $0 \leq x < 360°$, for which

$$6\cos^2 x - 5 = 6\sin^2 x + \sin x$$

giving your answers to 1 decimal place.

(5)

May 2018, Q8

26.

(a) Show that the equation

$$6\cos^2 x - \sin x - 4 = 0$$

may be written as

$$6\sin^2 x + \sin x - 2 = 0$$

(2)

(b) Hence solve, for $-90° \leqslant y < 90°$, the equation

$$6\cos^2(2y) - \sin(2y) - 4 = 0$$

giving your answers to one decimal place where appropriate.

(5)

May 2019, Q7

27. (*a*) Show that

$$12\sin^2 x - \cos x - 11 = 0$$

may be expressed in the form

$$12\cos^2 x + \cos x - 1 = 0$$

(1)

(*b*) Hence, using trigonometry, find all the solutions in the interval $0 \leq x \leq 360°$ of

$$12\sin^2 x - \cos x - 11 = 0$$

Give each solution, in degrees, to 1 decimal place.

(4)

IAL, Jan 2014, Q7

28. (*a*) Show that

$$\frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x} \equiv 1 - \tan^2 x,$$

(2)

(*b*) Hence solve, for $0 \leq x < 360°$,

$$\frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x} + 2 = 0$$

(5)

IAL, May 2014, Q6

29.

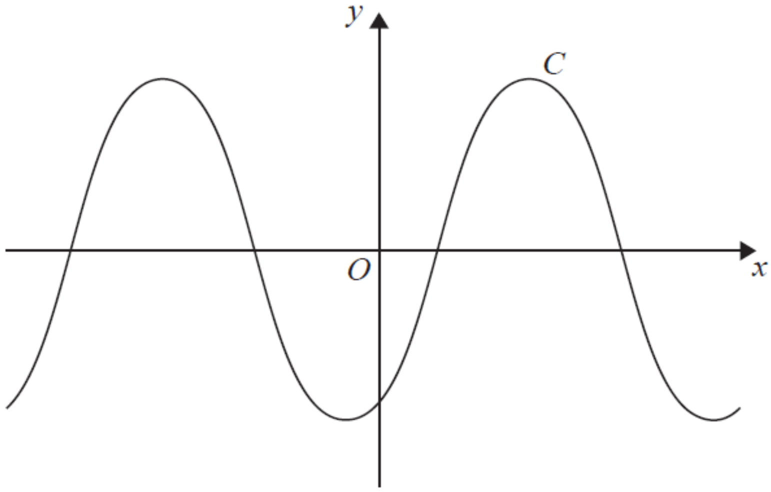

Figure 4

Figure 4 shows a sketch of the curve C with equation $y = \sin(x - 60°)$, $-360° \leq x \leq 360°$.

(a) Write down the exact coordinates of the points at which C meets the two coordinate axes.

(3)

(b) Solve, for $-360° \leq x \leq 360°$,

$$4 \sin(x - 60°) = \sqrt{6} - \sqrt{2}$$

showing each stage of your working.

(5)

IAL, Jan 2015, Q11

30. *In this question, solutions based entirely on graphical or numerical methods are not acceptable.*

(i) Solve, for $0 \leq x < 360°$,

$$3\sin x + 7\cos x = 0$$

Give each solution, in degrees, to one decimal place.

(4)

(ii) Solve, for $0 \leq \theta < 360°$

$$10 \cos^2\theta + \cos\theta = 11\sin^2\theta - 9$$

(6)

IAL, Jan 2015, Q14

31. (i) Showing each step in your reasoning, prove that

$$(\sin x + \cos x)(1 - \sin x\cos x) \equiv \sin^3 x + \cos^3 x$$

(3)

(ii) Solve, for $0 \leq \theta < 360°$,

$$3\sin\theta = \tan\theta$$

giving your answers in degrees to 1 decimal place, as appropriate.

(6)

IAL, May 2015, Q13

32. (*a*) Given that $7\sin x = 3 \cos x$, find the exact value of $\tan x$.

(1)

(*b*) Hence solve for $0 \leq \theta < 360°$

$$7 \sin(2\theta + 30°) = 3 \cos(2\theta + 30°)$$

giving your answers to one decimal place.

(5)

IAL, Jan 2016, Q8

33.

(*a*) Show that the equation

$$\frac{5+\sin\theta}{3\cos\theta} = 2\cos\theta, \qquad \theta \neq (2n+1)90°, \quad n \in \mathbb{Z}$$

may be rewritten as

$$6\sin^2\theta + \sin\theta - 1 = 0$$

(3)

(*b*) Hence solve, for $-90° < \theta < 90°$, the equation

$$\frac{5+\sin\theta}{3\cos\theta} = 2\cos\theta$$

Give your answers to one decimal place, where appropriate.

(4)

IAL, May 2016, Q8

34. (*a*) Given that

$$8\tan x = -3\cos x$$

show that

$$3\sin^2 x - 8\sin x - 3 = 0$$

(3)

(*b*) Hence solve, for $0 \leq \theta < 360°$,

$$8\tan 2\theta = -3\cos 2\theta$$

giving your answers to one decimal place.

(5)

IAL, Oct 2016, Q10

35.

(i) Solve, for $0 \leq x < 360°$,

$$3\cos^2 x + 1 = 4\sin^2 x$$

giving your answers in degrees to 2 decimal places.

(5)

(ii) Solve, for $0 \leq \theta < 360°$

$$5\sin(q + 10°) = \cos(q + 10°)$$

giving your answers in degrees to one decimal place.

(5)

IAL, Jan 2017, Q11

36. (*a*) Show that the equation

$$5\cos x + 1 = \sin x \tan x$$

can be written in the form

$$6\cos^2 x + \cos x - 1 = 0$$

(4)

(*b*) Hence solve, for $0 \leqslant \theta < 180°$

$$5\cos 2\theta + 1 = \sin 2\theta \tan 2\theta$$

giving your answers, where appropriate, to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(6)

IAL, May 2017, Q13

37. The height of water, H metres, in a harbour on a particular day is given by the equation

$$H = 4 + 1.5\sin(30t), \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight, and $30t$ is measured in degrees.

(*a*) Show that the height of the water at 1 a.m. is 4.75 metres.

(1)

(*b*) Find the height of the water at 2 p.m.

(2)

(*c*) Find, to the nearest minute, the first two times when the height of the water is 3 metres.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(6)

IAL, May 2017, Q15

38. (i) Solve, for $0 < \theta \leqslant 360°$,

$$3\sin(\theta + 30°) = 2\cos(\theta + 30°)$$

giving your answers, in degrees, to 2 decimal places.

(4)

(ii) (*a*) Given that

$$\frac{\cos^2 x + 2\sin^2 x}{1 - \sin^2 x} = 5$$

show that

$$\tan^2 x = k, \qquad \text{where } k \text{ is a constant.}$$

(*b*) Hence solve, for $0 < x \leqslant 360°$,

$$\frac{\cos^2 x + 2\sin^2 x}{1 - \sin^2 x} = 5$$

giving your answers, in in degrees, to one decimal place.

(7)

IAL, Oct 2017, Q12

39.

(i) Solve, for $0 < \theta < 180°$

$$5\sin 3\theta - 7\cos 3\theta = 0$$

Give each solution, in degrees, to 3 significant figures.

(5)

(ii) Solve, for $0 < x < 360°$

$$9\cos^2 x + 5\cos x = 3\sin^2 x$$

Give each solution, in degrees, to one decimal place.

(6)

IAL, Jan 2018, Q5

40.

(i) Solve for $0 \leqslant x < 360°$,

$$5\sin(x + 65°) + 2 = 0$$

giving your answers in degrees to one decimal place.

(4)

(ii) Find, for $0 \leqslant \theta < 360°$, all the solutions of

$$12\sin^2 \theta + \cos \theta = 6$$

giving your answers in degrees to one decimal place.

(6)

IAL, May 2018, Q12

41. (*a*) Show that the equation

$$6\cos x - 5\tan x = 0$$

may be expressed in the form

$$6\sin^2 x + 5\sin x - 6 = 0$$

(3)

(*b*) Hence solve for $0 \leqslant \theta < 360°$

$$6\cos(2\theta - 10°) - 5\tan(2\theta - 10°) = 0$$

giving your answers to one decimal place.

(5)

IAL, Oct 2018, Q12

42.

(i) Solve, for $-180° \leqslant x < 180°$, the equation

$$\sin(x + 60°) = -0.4$$

giving your answers, in degrees, to one decimal place.

(4)

(ii) (*a*) Show that the equation

$$2\sin\theta\tan\theta - 3 = \cos\theta$$

can be written in the form

$$3\cos^2\theta + 3\cos\theta - 2 = 0$$

(3)

(*b*) Hence solve, for $0 \leqslant \theta < 360°$, the equation

$$2\sin\theta\tan\theta - 3 = \cos\theta$$

showing each stage of your working and giving your answers, in degrees, to one decimal place.

(4)

IAL, Jan 2019, Q14

43. (i) Solve, for $0 \leq \theta < 180°$, the equation

$$7\sin 2\theta = 5\cos 2\theta$$

giving your answers, in degrees, to one decimal place.

(4)

(ii) Solve, for $0 \leq x < 360°$, the equation

$$24\tan x = 5\cos x$$

giving your answers, in degrees, to one decimal place.

(5)

IAL, Oct 2019, Q13

44.

(a) Show that

$$\frac{2 + \cos x}{3 + \sin^2 x} = \frac{4}{7}$$

may be expressed in the form

$$a \cos^2 x + b \cos x + c = 0$$

where a, b and c are constants to be found.

(3)

(b) Hence solve, for $0 \leq x \leq 360^0$, the equation

$$\frac{2 + cosx}{3 + sin^2x} = \frac{4}{7}$$

giving your answers, in degrees, to one decimal place.

(5)

IAL, May 2019, Q12