OCR Core Maths 2

Past paper questions

TheTrapezium Rule

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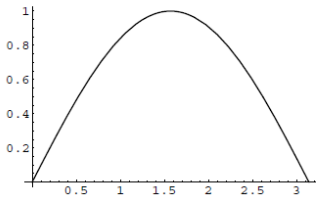
The Trapezium Rule

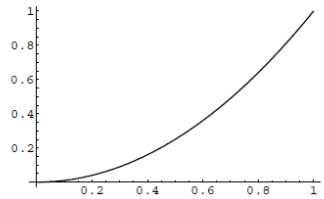
- The area under *any* curve can be *approximated* by the Trapezium Rule. The governing formula is given by (and contained in the formula booklet you will have in the exam)

$$\int_a^b y\,dx \approx \frac{1}{2}h\left[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})\right],$$

where h is the width of each trapezium, y_0 and y_n are the 'end' heights and $y_1 + y_2 + \cdots + y_{n-1}$ are the 'internal' heights.

- By considering the shape of the graph in the interval over which you are approximating it should be clear whether your estimate of the area is an over or under-estimate of the *true* area.





For example if you were to estimate $\int_0^\pi \sin x\,dx$ (above, left) using the trapezium rule, due to the shape of the curve, the trapezia would all fall below the curve, so we would obtain an *under*-estimate. However, with $\int_0^1 x^2\,dx$ (above, right) we would obtain an *over*-estimate.

1.

(b) **(i)** Use the trapezium rule, with 3 strips, to find an approximate value for the area of the region bounded by the curve $y = \tan x$, the x-axis, and the lines $x = 0.1$ and $x = 0.4$. (Values of x are in radians.) [4]

(ii) State with a reason whether this approximation is an underestimate or an overestimate. [1]

Q9 June 2005

2.

(i) Sketch the curve $y = \left(\frac{1}{2}\right)^x$, and state the coordinates of any point where the curve crosses an axis. [3]

(ii) Use the trapezium rule, with 4 strips of width 0.5, to estimate the area of the region bounded by the curve $y = \left(\frac{1}{2}\right)^x$, the axes, and the line $x = 2$. [4]

Q9 June 2006

3.

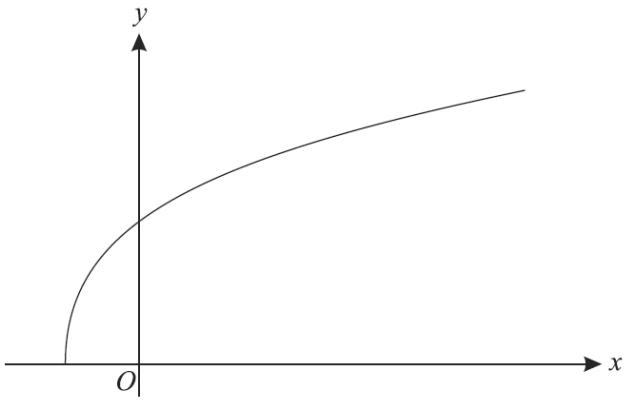
(b) Use the trapezium rule, with two strips of width 3, to find an approximate value for

$$\int_3^9 \log_{10} x \, \mathrm{d}x,$$

giving your answer correct to 3 significant figures. [4]

Q5 Jan 2007

4.



The diagram shows the curve $y = \sqrt{4x + 1}$.

(i) Use the trapezium rule, with strips of width 0.5, to find an approximate value for the area of the region bounded by the curve $y = \sqrt{4x + 1}$, the x-axis, and the lines $x = 1$ and $x = 3$. Give your answer correct to 3 significant figures. [4]

(ii) State with a reason whether this approximation is an under-estimate or an over-estimate. [2]

Q4 June 2007

5.

Use the trapezium rule, with 3 strips each of width 2, to estimate the value of

$$\int_1^7 \sqrt{x^2 + 3}\,\mathrm{d}x. \qquad [4]$$

Q2 Jan 2008

6.

(b) Use the trapezium rule, with four strips each of width 0.25, to find an approximate value for

$$\int_0^1 \cos x\,\mathrm{d}x,$$

where x is in radians. Give your answer correct to 3 significant figures. [4]

Q9 June 2008

7.

(i) Sketch the graph of $y = 4k^x$, where k is a constant such that $k > 1$. State the coordinates of any points of intersection with the axes. **[2]**

(ii) The point P on the curve $y = 4k^x$ has its y-coordinate equal to $20k^2$. Show that the x-coordinate of P may be written as $2 + \log_k 5$. **[4]**

(iii) **(a)** Use the trapezium rule, with two strips each of width $\frac{1}{2}$, to find an expression for the approximate value of

$$\int_0^1 4k^x\,\mathrm{d}x. \qquad \textbf{[3]}$$

(b) Given that this approximate value is equal to 16, find the value of k. **[3]**

Q9 June 2009

8.

(i) Use the trapezium rule, with 4 strips each of width 0.5, to find an approximate value for

$$\int_3^5 \log_{10}(2 + x)\,\mathrm{d}x,$$

giving your answer correct to 3 significant figures. **[4]**

(ii) Use your answer to part **(i)** to deduce an approximate value for $\int_3^5 \log_{10}\sqrt{2 + x}\,\mathrm{d}x$, showing your method clearly. **[2]**

Q4 Jan 2010

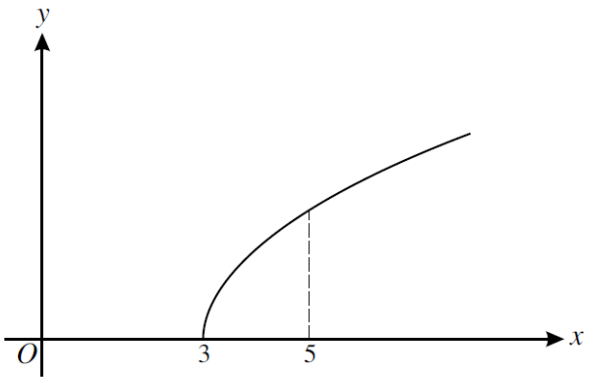
9.

(i) Use the trapezium rule, with 3 strips each of width 3, to estimate the area of the region bounded by the curve $y = \sqrt[3]{7 + x}$, the x-axis, and the lines $x = 1$ and $x = 10$. Give your answer correct to 3 significant figures. **[4]**

(ii) Explain how the trapezium rule could be used to obtain a more accurate estimate of the area. **[1]**

Q2 June 2010

10.



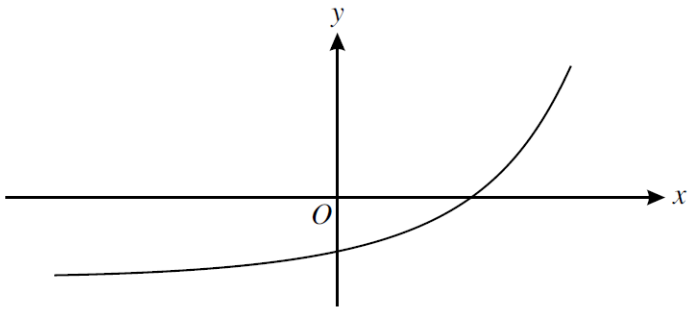
The diagram shows the curve $y = \sqrt{x - 3}$.

(i) Use the trapezium rule, with 4 strips each of width 0.5, to find an approximate value for the area of the region bounded by the curve, the x-axis and the line $x = 5$. Give your answer correct to 3 significant figures. **[4]**

(ii) State, with a reason, whether this approximation is an underestimate or an overestimate. **[2]**

Q2 Jan 2011

11.



The diagram shows the curve $y = 2^x - 3$.

(i) Describe the geometrical transformation that transforms the curve $y = 2^x$ to the curve $y = 2^x - 3$. **[2]**

(ii) State the y-coordinate of the point where the curve $y = 2^x - 3$ crosses the y-axis. **[1]**

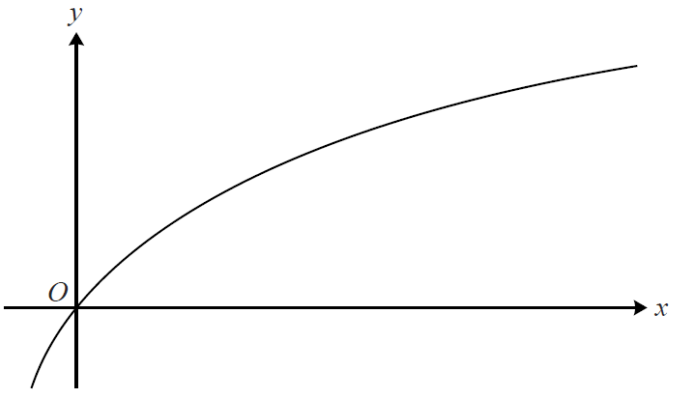
(iii) Find the x-coordinate of the point where the curve $y = 2^x - 3$ crosses the x-axis, giving your answer in the form $\log_a b$. **[2]**

(iv) The curve $y = 2^x - 3$ passes through the point $(p, 62)$. Use logarithms to find the value of p, correct to 3 significant figures. **[3]**

(v) Use the trapezium rule, with 2 strips each of width 0.5, to find an estimate for $\int_3^4 (2^x - 3)\,dx$. Give your answer correct to 3 significant figures. **[3]**

Q8 June 2011

12.



The diagram shows the curve $y = \log_{10}(2x + 1)$.

(i) Use the trapezium rule with 4 strips each of width 1.5 to find an approximation to the area of the region bounded by the curve, the x-axis and the lines $x = 4$ and $x = 10$. Give your answer correct to 3 significant figures. **[4]**

(ii) Explain why this approximation is an under-estimate. **[1]**

Q2 Jan 2012

13.

(i) Use the trapezium rule, with 2 strips each of width 4, to show that an approximate value of $\int_1^9 4\sqrt{x}\,\mathrm{d}x$ is $32 + 16\sqrt{5}$. **[3]**

(ii) Use a sketch graph to explain why the actual value of $\int_1^9 4\sqrt{x}\,\mathrm{d}x$ is greater than $32 + 16\sqrt{5}$. **[2]**

(iii) Use integration to find the exact value of $\int_1^9 4\sqrt{x}\,\mathrm{d}x$. **[4]**

Q6 June 2012

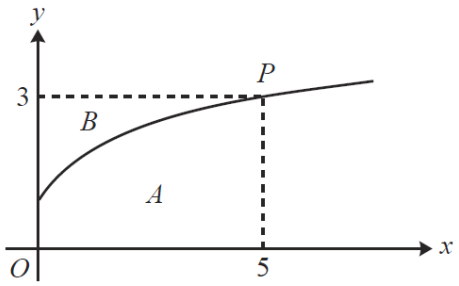
14.

Use the trapezium rule, with 3 strips each of width 2, to estimate the value of

$$\int_5^{11} \frac{8}{x}\,\mathrm{d}x. \quad \textbf{[4]}$$

Q1 June 2013

15.



The diagram shows part of the curve $y = -3 + 2\sqrt{x+4}$. The point $P(5, 3)$ lies on the curve. Region A is bounded by the curve, the x-axis, the y-axis and the line $x = 5$. Region B is bounded by the curve, the y-axis and the line $y = 3$.

(i) Use the trapezium rule, with 2 strips each of width 2.5, to find an approximate value for the area of region A, giving your answer correct to 3 significant figures. **[3]**

(ii) Use your answer to part **(i)** to deduce an approximate value for the area of region B. **[2]**

(iii) By first writing the equation of the curve in the form $x = \mathrm{f}(y)$, use integration to show that the exact area of region B is $\frac{14}{3}$. **[7]**

Q9 June 2014

16.

(i) Use the trapezium rule, with 4 strips each of width 1.5, to estimate the value of

$$\int_4^{10} \sqrt{2x-1}\,\mathrm{d}x,$$

giving your answer correct to 3 significant figures. **[4]**

(ii) Explain how the trapezium rule could be used to obtain a more accurate estimate. **[1]**

Q2 June 2015