OCR Core Maths 2

Past paper questions

Surds & Indices

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Surds

- Know and understand the laws

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b} \qquad \text{and} \qquad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}.$$

  In particular know how to deal with $\frac{\sqrt{44}}{2}$; it is *not* $\sqrt{22}$! It is $\frac{\sqrt{44}}{2} = \frac{\sqrt{44}}{\sqrt{4}} = \sqrt{\frac{44}{4}} = \sqrt{11}$. This comes up in solving quadratics by the formula; check that when you solve $x^2 + 4x - 2 = 0$ by the formula you obtain $x = -2 \pm \sqrt{6}$.

- You also need to be able to rationalise the denominator of certain types of surd expressions. For example to rationalise $\frac{9}{\sqrt{3}}$ is easy; just multiply by $\frac{\sqrt{3}}{\sqrt{3}}$ to obtain $\frac{9\sqrt{3}}{3} = 3\sqrt{3}$. In harder examples you must multiply the top and bottom of the fraction by the denominator with the sign 'flipped'. For example

$$\frac{2+2\sqrt{3}}{5-2\sqrt{3}} = \frac{2+2\sqrt{3}}{5-2\sqrt{3}} \times \boxed{\frac{5+2\sqrt{3}}{5+2\sqrt{3}}} = \frac{10+4\sqrt{3}+10\sqrt{3}+12}{25+10\sqrt{3}-10\sqrt{3}-12} = \frac{22+14\sqrt{3}}{13}.$$

Index Notation

- $(ab)^m = a^m \times b^m$. For example $6^5 = 2^5 \times 3^5$.
- When multiplying a number raised to different powers the powers *add*. Therefore $a^m \times a^n = a^{m+n}$. You can think of this as follows $2^2 \times 2^4 = (2 \times 2) \times (2 \times 2 \times 2 \times 2) = 2^6$.
- Know that $a^{-m} = \frac{1}{a^m}$. Remember this by the standard result that $2^{-1} = \frac{1}{2}$. "When moving something from the bottom line of a fraction to the top (or vice versa), the sign changes."
- From the above two results we can obtain the result $\frac{a^m}{a^n} = a^{m-n}$. This is derived thus; $\frac{a^m}{a^n} = a^m \times a^{-n} = a^{m-n}$ as required.
- We can also derive the important result $a^0 = 1$ for any $a \neq 0$. Derived by considering something like this; $a^0 = a^{1-1} = \frac{a^1}{a^1} = \frac{a}{a} = 1$.
- Know that $(a^m)^n = a^{mn}$. Think about it like this; $(a^3)^4 = a^3 \times a^3 \times a^3 \times a^3 = a^{12}$.
- The n^{th} root of a number can be expressed as a power thus; $\sqrt[n]{a} = a^{\frac{1}{n}}$.
- A few examples:

  1. Write 8 as a power of 4; well $8 = 2^3 = \left(4^{\frac{1}{2}}\right)^3 = 4^{\frac{3}{2}}$.

  2. Simplify $\sqrt[4]{16^3} = \left(\left(2^4\right)^3\right)^{\frac{1}{4}} = \left(2^{12}\right)^{\frac{1}{4}} = 2^3 = 8$.

  3. Simplify $\dfrac{12x^8y^{\frac{3}{2}}}{6x^6y^{\frac{5}{2}}} = 2x^{8-6}y^{\frac{3}{2}-\frac{5}{2}} = 2x^2y^{-1}$.

  4. Simplify $\sqrt{x^6y^4} \times \sqrt[3]{x^3y^{-6}} = \left(x^6y^4\right)^{\frac{1}{2}} \times \left(x^3y^{-6}\right)^{\frac{1}{3}} = x^3y^2x^1y^{-2} = x^4$.

1.

(a) Simplify $2x^{\frac{2}{3}} \times 3x^{-1}$. [2]

(b) Express $2^{40} \times 4^{30}$ in the form 2^n. [2]

(c) Express $\dfrac{26}{4-\sqrt{3}}$ in the form $a + b\sqrt{3}$. [3]

Q5 June 2005

2.

Solve the equations

(i) $x^{\frac{1}{3}} = 2$, [1]

(ii) $10^t = 1$, [1]

(iii) $\left(y^{-2}\right)^2 = \frac{1}{81}$. [2]

Q1 Jan 2006

3.

(i) Evaluate $27^{-\frac{2}{3}}$. [2]

(ii) Express $5\sqrt{5}$ in the form 5^n. [1]

(iii) Express $\dfrac{1-\sqrt{5}}{3+\sqrt{5}}$ in the form $a + b\sqrt{5}$. [3]

Q2 June 2006

4.

Express $\dfrac{5}{2-\sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers. [3]

Q1 Jan 2007

5.

Evaluate

(i) 6^0, [1]

(ii) $2^{-1} \times 32^{\frac{4}{5}}$. [3]

Q2 Jan 2007

6.

Simplify the following, expressing each answer in the form $a\sqrt{5}$.

(i) $3\sqrt{10} \times \sqrt{2}$ [2]

(ii) $\sqrt{500} + \sqrt{125}$ [3]

Q3 June 2007

7.

Express $\dfrac{4}{3-\sqrt{7}}$ in the form $a + b\sqrt{7}$, where a and b are integers. [3]

Q1 Jan 2008

8.

Solve the equations

(i) $10^p = 0.1$, [1]

(ii) $\left(25k^2\right)^{\frac{1}{2}} = 15$, [3]

(iii) $t^{-\frac{1}{3}} = \frac{1}{2}$. [2]

Q4 Jan 2008

9.

Express each of the following in the form 4^n:

(i) $\frac{1}{16}$, [1]

(ii) 64, [1]

(iii) 8. [2]

Q1 June 2008

10.

Express each of the following in the form $k\sqrt{2}$, where k is an integer:

(i) $\sqrt{200}$, [1]

(ii) $\dfrac{12}{\sqrt{2}}$, [1]

(iii) $5\sqrt{8} - 3\sqrt{2}$. [2]

Q3 June 2008

11.

Express $\sqrt{45} + \dfrac{20}{\sqrt{5}}$ in the form $k\sqrt{5}$, where k is an integer. **[3]**

Q1 Jan 2009

12.

Simplify

(i) $\left(\sqrt[3]{x}\right)^6$, **[1]**

(ii) $\dfrac{3y^4 \times (10y)^3}{2y^5}$. **[3]**

Q2 Jan 2009

13.

Express $\dfrac{8+\sqrt{7}}{2+\sqrt{7}}$ in the form $a + b\sqrt{7}$, where a and b are integers. **[4]**

Q2 June 2009

14.

Express each of the following in the form 3^n:

(i) $\frac{1}{9}$, **[1]**

(ii) $\sqrt[3]{3}$, **[1]**

(iii) $3^{10} \times 9^{15}$. **[2]**

Q3 June 2009

15.

Solve the equations

(i) $3^m = 81$, **[1]**

(ii) $\left(36p^4\right)^{\frac{1}{2}} = 24$, **[3]**

(iii) $5^n \times 5^{n+4} = 25$. **[3]**

Q4 Jan 2010

16.

(i) Evaluate 9^0. **[1]**

(ii) Express $9^{-\frac{1}{2}}$ as a fraction. **[2]**

Q1 June 2010

17.

(i) Express $\frac{12}{3+\sqrt{5}}$ in the form $a - b\sqrt{5}$, where a and b are positive integers. **[3]**

(ii) Express $\sqrt{18} - \sqrt{2}$ in simplified surd form. **[2]**

Q3 June 2010

18.

Express each of the following in the form 8^p:

(i) $\sqrt{8}$, **[1]**

(ii) $\frac{1}{64}$, **[1]**

(iii) $2^6 \times 2^2$. **[3]**

Q3 Jan 2011

19.

Simplify

(i) $\frac{(4x)^2 \times 2x^3}{x}$, **[2]**

(ii) $\left(36x^{-2}\right)^{-\frac{1}{2}}$. **[3]**

Q3 June 2011

20.

(i) Express $\sqrt{300} - \sqrt{48}$ in the form $k\sqrt{3}$, where k is an integer. **[3]**

(ii) Express $\frac{15+\sqrt{40}}{\sqrt{5}}$ in the form $a\sqrt{5} + b\sqrt{2}$, where a and b are integers. **[3]**

Q5 June 2011

21.

Express $\frac{15+\sqrt{3}}{3-\sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers. **[4]**

Q1 Jan 2012

22.

Evaluate

(i) 3^{-2}, **[1]**

(ii) $16^{\frac{3}{4}}$, **[2]**

(iii) $\dfrac{\sqrt{200}}{\sqrt{8}}$. **[2]**

Q4 Jan 2012

23.

Express each of the following in the form 7^k:

(i) $\sqrt[4]{7}$, **[1]**

(ii) $\dfrac{1}{7\sqrt{7}}$, **[2]**

(iii) $7^4 \times 49^{10}$. **[2]**

Q2 June 2012

24.

Solve the equations

(i) $3^n = 1$, **[1]**

(ii) $t^{-3} = 64$, **[2]**

(iii) $(8p^6)^{\frac{1}{3}} = 8$. **[3]**

Q2 Jan 2013

25.

Express each of the following in the form $a\sqrt{5}$, where a is an integer.

(i) $4\sqrt{15} \times \sqrt{3}$ **[2]**

(ii) $\dfrac{20}{\sqrt{5}}$ **[1]**

(iii) $5^{\frac{3}{2}}$ **[1]**

Q1 June 2013

26.

Express each of the following in the form $k\sqrt{3}$, where k is an integer.

(i) $\frac{6}{\sqrt{3}}$ **[1]**

(ii) $10\sqrt{3}-6\sqrt{27}$ **[2]**

(iii) $3^{\frac{5}{2}}$ **[2]**

Q2 June 2014

27.

Express $\frac{8}{\sqrt{3}-1}$ in the form $a\sqrt{3}+b$, where a and b are integers. **[3]**

Q1 June 2015

28.

Express each of the following in the form 5^k.

(i) 25^4 **[1]**

(ii) $\frac{1}{\sqrt[4]{5}}$ **[2]**

(iii) $(5\sqrt{5})^3$ **[2]**

Q3 June 2015