OCR Core Maths 2

Past paper questions

Arithmetic & Geometric Progression

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Sequences

- You must be comfortable with $\sum$-notation. It works as follows; you put in the number at the bottom of the $\sum$ and then keep summing until you reach the top number[1]. For example:

$$\sum_{i=4}^{8}(2i+3) = 11+13+15+17+19 = 75,$$
$$\sum_{i=1}^{n}(i^2+i) = (1+1)+(4+2)+(9+3)+\cdots+(n^2+n).$$

- A 'sequence' is a list of numbers in a specific order. A 'series' is a sum of the terms of a sequence.

- Sequences are sometimes defined *recursively*. For example the sequence $a_{n+1} = a_n + 3$ with $a_1 = 10$ defines the sequence $10, 13, 16, 19\ldots$ We know that this is an arithmetic sequence which can also be defined *deductively* by $a_n = 10 + 3(n-1)$.

- An arithmetic sequence increases or decreases by a constant amount. The letter a always denotes the first term and d is the difference between the terms (negative for a decreasing sequence!). The nth term is denoted a_n and satisfies the important relationship

$$a_n = a + (n-1)d.$$

  For example if told the third term of a sequence is 10 and the seventh term is 34 then we can use the above equation to find the a and d.

$$\begin{matrix} 10 = a + (3-1)d \\ 34 = a + (7-1)d \end{matrix} \quad \Rightarrow \quad 4d = 24 \quad \Rightarrow \quad d = 6 \quad \Rightarrow \quad a = -2.$$

- The sum of the n terms of an arithmetic sequence is given by

$$S = \frac{n}{2}(\text{First} + \text{Last}) = \frac{n}{2}(2a + (n-1)d).$$

  For example the sum of the first 10 terms of a sequence is 130 and the first term is 4. What is the difference?

$$S = \frac{n}{2}(2a + (n-1)d) \quad \Rightarrow \quad 130 = \frac{10}{2}(8 + (10-1)d) \quad \Rightarrow \quad d = 2.$$

Geometric Sequences

- A geometric sequence is one where the terms are multiplied by a constant amount. For example $1, 2, 4, 8, 16, \ldots, [2^{n-1}]$ is a geometric sequence with $a = 1$ and $r = 2$. The n^{th} term is given by

$$a_n = ar^{n-1}.$$

  So for the above example the 20th term is $a_{20} = 1 \times 2^{19} = 524288$.

- The sum of n terms of a geometric sequence is given by

$$S = a\left(\frac{r^n - 1}{r - 1}\right) \qquad \text{or (equivalently) by} \qquad S = a\left(\frac{1 - r^n}{1 - r}\right).$$

  For example sum the first 20 terms of $4, 2, 1, \frac{1}{2}, \ldots, [4 \times \left(\frac{1}{2}\right)^{n-1}]$. This is given by

$$S = 4\left(\frac{\left(\frac{1}{2}\right)^{20} - 1}{\frac{1}{2} - 1}\right) = 7.999992371\ldots$$

- If the ratio (r) lies between -1 and 1 (i.e. $-1 < r < 1$) then there exists a 'sum to infinity' given by

$$S_\infty = \frac{a}{1 - r}.$$

  Therefore S_∞ for the above example is $S_\infty = \frac{4}{1-\frac{1}{2}} = 8$. We can see that the sum to 20 terms is very close to S_∞.

1.

A sequence S has terms $u_1, u_2, u_3, \ldots$ defined by

$$u_n = 3n - 1,$$

for $n \geqslant 1$.

(i) Write down the values of u_1, u_2 and u_3, and state what type of sequence S is. [3]

(ii) Evaluate $\sum_{n=1}^{100} u_n$. [3]

Q1 June 2005

2.

The amounts of oil pumped from an oil well in each of the years 2001 to 2004 formed a geometric progression with common ratio 0.9. The amount pumped in 2001 was 100 000 barrels.

(i) Calculate the amount pumped in 2004. [2]

It is assumed that the amounts of oil pumped in future years will continue to follow the same geometric progression. Production from the well will stop at the end of the first year in which the amount pumped is less than 5000 barrels.

(ii) Calculate in which year the amount pumped will fall below 5000 barrels. [4]

(iii) Calculate the total amount of oil pumped from the well from the year 2001 up to and including the final year of production. [3]

Q8 June 2005

3.

The 20th term of an arithmetic progression is 10 and the 50th term is 70.

(i) Find the first term and the common difference. [4]

(ii) Show that the sum of the first 29 terms is zero. [2]

Q1 Jan 2006

4.

In a geometric progression, the first term is 5 and the second term is 4.8.

(i) Show that the sum to infinity is 125. [2]

(ii) The sum of the first n terms is greater than 124. Show that

$$0.96^n < 0.008,$$

and use logarithms to calculate the smallest possible value of n. [6]

Q5 Jan 2006

5.

A sequence of terms $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 2 \qquad \text{and} \qquad u_{n+1} = 1 - u_n \text{ for } n \geqslant 1.$$

(i) Write down the values of u_2, u_3 and u_4. [2]

(ii) Find $\sum_{n=1}^{100} u_n$. [3]

Q2 June 2006

6.

(i) John aims to pay a certain amount of money each month into a pension fund. He plans to pay £100 in the first month, and then to increase the amount paid by £5 each month, i.e. paying £105 in the second month, £110 in the third month, etc.

If John continues making payments according to this plan for 240 months, calculate

(a) how much he will pay in the final month, [2]

(b) how much he will pay altogether over the whole period. [2]

(ii) Rachel also plans to pay money monthly into a pension fund over a period of 240 months, starting with £100 in the first month. Her monthly payments will form a geometric progression, and she will pay £1500 in the final month.

Calculate how much Rachel will pay altogether over the whole period. [5]

Q6 June 2006

7.

In an arithmetic progression the first term is 15 and the twentieth term is 72. Find the sum of the first 100 terms. [4]

Q1 Jan 2007

8.

On its first trip between Malby and Grenlish, a steam train uses 1.5 tonnes of coal. As the train does more trips, it becomes less efficient so that each subsequent trip uses 2% more coal than the previous trip.

(i) Show that the amount of coal used on the fifth trip is 1.624 tonnes, correct to 4 significant figures. [2]

(ii) There are 39 tonnes of coal available. An engineer wishes to calculate N, the total number of trips possible. Show that N satisfies the inequality

$$1.02^N \leqslant 1.52. \qquad [4]$$

(iii) Hence, by using logarithms, find the greatest number of trips possible. [4]

Q9 Jan 2007

9.

A geometric progression $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 15 \qquad \text{and} \qquad u_{n+1} = 0.8u_n \text{ for } n \geqslant 1.$$

(i) Write down the values of u_2, u_3 and u_4. [2]

(ii) Find $\sum_{n=1}^{20} u_n$. [3]

Q1 June 2007

10.

(a) In an arithmetic progression, the first term is 12 and the sum of the first 70 terms is 12 915. Find the common difference. [4]

(b) In a geometric progression, the second term is −4 and the sum to infinity is 9. Find the common ratio. [7]

Q7 June 2007

11.

A sequence of terms $u_1, u_2, u_3, \ldots$ is defined by

$$u_n = 2n + 5, \quad \text{for } n \geqslant 1.$$

(i) Write down the values of u_1, u_2 and u_3. [2]

(ii) State what type of sequence it is. [1]

(iii) Given that $\sum_{n=1}^{N} u_n = 2200$, find the value of N. [5]

Q6 Jan 2008

12.

The first term of a geometric progression is 10 and the common ratio is 0.8.

(i) Find the fourth term. [2]

(ii) Find the sum of the first 20 terms, giving your answer correct to 3 significant figures. [2]

(iii) The sum of the first N terms is denoted by S_N, and the sum to infinity is denoted by S_∞.

Show that the inequality $S_\infty - S_N < 0.01$ can be written as

$$0.8^N < 0.0002,$$

and use logarithms to find the smallest possible value of N. [7]

Q8 Jan 2008

13.

A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 3 \quad \text{and} \quad u_{n+1} = 1 - \frac{1}{u_n} \text{ for } n \geqslant 1.$$

(i) Write down the values of u_2, u_3 and u_4. [3]

(ii) Describe the behaviour of the sequence. [1]

Q2 June 2008

14.

Jamie is training for a triathlon, which involves swimming, running and cycling.

- On Day 1, he swims 2 km and then swims the same distance on each subsequent day.
- On Day 1, he runs 2 km and, on each subsequent day, he runs 0.5 km further than on the previous day. (Thus he runs 2.5 km on Day 2, 3 km on Day 3, and so on.)
- On Day 1 he cycles 2 km and, on each subsequent day, he cycles a distance 10% further than on the previous day.

(i) Find how far Jamie runs on Day 15. [2]

(ii) Verify that the distance cycled in a day first exceeds 12 km on Day 20. [3]

(iii) Find the day on which the total distance cycled, up to and including that day, first exceeds 200 km. [4]

(iv) Find the total distance travelled, by swimming, running and cycling, up to and including Day 30. [4]

Q10 June 2008

15.

A sequence of terms $u_1, u_2, u_3, \ldots$ is defined by

$$u_n = 24 - \tfrac{2}{3}n.$$

(i) Write down the exact values of u_1, u_2 and u_3. **[2]**

(ii) Find the value of k such that $u_k = 0$. **[2]**

(iii) Find $\sum_{n=1}^{20} u_n$. **[3]**

Q3 Jan 2009

16.

A geometric progression has first term 20 and common ratio 0.9.

(i) Find the sum to infinity. **[2]**

(ii) Find the sum of the first 30 terms. **[2]**

(iii) Use logarithms to find the smallest value of p such that the pth term is less than 0.4. **[4]**

Q6 Jan 2009

17.

The tenth term of an arithmetic progression is equal to twice the fourth term. The twentieth term of the progression is 44.

(i) Find the first term and the common difference. **[4]**

(ii) Find the sum of the first 50 terms. **[2]**

Q2 June 2009

18.

A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 8 \qquad \text{and} \qquad u_{n+1} = u_n + 3.$$

(i) Show that $u_5 = 20$. **[2]**

(ii) The nth term of the sequence can be written in the form $u_n = pn + q$. State the values of p and q. **[2]**

(iii) State what type of sequence it is. **[1]**

(iv) Find the value of N such that $\sum_{n=1}^{2N} u_n - \sum_{n=1}^{N} u_n = 1256$. **[5]**

Q8 Jan 2010

19.

A sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = 5n + 1$.

(i) State the values of u_1, u_2 and u_3. **[1]**

(ii) Evaluate $\sum_{n=1}^{40} u_n$. **[3]**

Another sequence $w_1, w_2, w_3, \ldots$ is defined by $w_1 = 2$ and $w_{n+1} = 5w_n + 1$.

(iii) Find the value of p such that $u_p = w_3$. **[3]**

Q4 June 2010

20.

A geometric progression has first term a and common ratio r, and the terms are all different. The first, second and fourth terms of the geometric progression form the first three terms of an arithmetic progression.

(i) Show that $r^3 - 2r + 1 = 0$. **[3]**

(ii) Given that the geometric progression converges, find the exact value of r. **[5]**

(iii) Given also that the sum to infinity of this geometric progression is $3 + \sqrt{5}$, find the value of the integer a. **[4]**

Q9 June 2010

21.

A sequence S has terms $u_1, u_2, u_3, \ldots$ defined by $u_n = 3n + 2$ for $n \geqslant 1$.

(i) Write down the values of u_1, u_2 and u_3. **[2]**

(ii) State what type of sequence S is. **[1]**

(iii) Find $\sum_{n=101}^{200} u_n$. **[3]**

Q2 Jan 2011

22.

In a geometric progression, the sum to infinity is four times the first term.

(i) Show that the common ratio is $\frac{3}{4}$. **[3]**

(ii) Given that the third term is 9, find the first term. **[3]**

(iii) Find the sum of the first twenty terms. **[2]**

Q5 Jan 2011

23.

(a) The first term of a geometric progression is 7 and the common ratio is -2.

(i) Find the ninth term. **[2]**

(ii) Find the sum of the first 15 terms. **[2]**

(b) The first term of an arithmetic progression is 7 and the common difference is -2. The sum of the first N terms is -2900. Find the value of N. **[5]**

Q7 June 2011

24.

A sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = 85 - 5n$ for $n \geqslant 1$.

(i) Write down the values of u_1, u_2 and u_3. **[2]**

(ii) Find $\sum_{n=1}^{20} u_n$. **[3]**

(iii) Given that u_1, u_5 and u_p are, respectively, the first, second and third terms of a geometric progression, find the value of p. **[4]**

(iv) Find the sum to infinity of the geometric progression in part **(iii)**. **[2]**

Q6 Jan 2012

25.

(a) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 4 \quad \text{and} \quad u_{n+1} = \frac{2}{u_n} \quad \text{for } n \geqslant 1.$$

(i) Write down the values of u_2 and u_3. **[2]**

(ii) Describe the behaviour of the sequence. **[1]**

(b) In an arithmetic progression the ninth term is 18 and the sum of the first nine terms is 72. Find the first term and the common difference. **[5]**

Q5 June 2012

26.

A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 7 \quad \text{and} \quad u_{n+1} = u_n + 4 \quad \text{for } n \geqslant 1.$$

(i) Show that $u_{17} = 71$. **[2]**

(ii) Show that $\sum_{n=1}^{35} u_n = \sum_{n=36}^{50} u_n$. **[4]**

Q2 Jan 2013

_27._

(i) The first three terms of an arithmetic progression are $2x$, $x + 4$ and $2x - 7$ respectively. Find the value of x. **[3]**

(ii) The first three terms of another sequence are also $2x$, $x + 4$ and $2x - 7$ respectively.

(a) Verify that when $x = 8$ the terms form a geometric progression and find the sum to infinity in this case. **[4]**

(b) Find the other possible value of x that also gives a geometric progression. **[4]**

Q6 Jan 2013

_28._

A sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = 3n - 1$, for $n \geqslant 1$.

(i) Find the values of u_1, u_2 and u_3. **[2]**

(ii) Find $\sum_{n=1}^{40} u_n$. **[3]**

Q2 June 2014

_29._

(a) The first term of a geometric progression is 50 and the common ratio is 0.8. Use logarithms to find the smallest value of k such that the value of the kth term is less than 0.15. **[4]**

(b) In a different geometric progression, the second term is –3 and the sum to infinity is 4. Show that there is only one possible value of the common ratio and hence find the first term. **[8]**

Q8 June 2014

_30._

A geometric progression has first term 3 and second term –6.

(i) State the value of the common ratio. **[1]**

(ii) Find the value of the eleventh term. **[2]**

(iii) Find the sum of the first twenty terms. **[2]**

Q1 June 2015

31.

In an arithmetic progression the first term is 5 and the common difference is 3. The nth term of the progression is denoted by u_n.

(i) Find the value of u_{20}. **[2]**

(ii) Show that $\sum_{n=10}^{20} u_n = 517$. **[3]**

(iii) Find the value of N such that $\sum_{n=N}^{2N} u_n = 2750$. **[6]**

Q7 June 2015