Edexcel

Pure Mathematics

Year 2

Recurrence Relations.

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1. The sequence of positive numbers u_1, u_2, u_3, ..., is given by

$$u_{n+1} = (u_n - 3)^2, \qquad u_1 = 1.$$

(*a*) Find u_2, u_3 and u_4.

(3)

(*b*) Write down the value of u_{20}.

(1)

(C1, Q2 Jan 2006)

2. A sequence a_1, a_2, a_3, . . . is defined by

$$a_1 = 3,$$

$$a_{n+1} = 3a_n - 5, \quad n \geq 1.$$

(*a*) Find the value a_2 and the value of a_3.

(2)

(*b*) Calculate the value of $\sum_{r=1}^{5} a_r$.

(3)

(C1, Q4 May 2006)

3. A sequence a_1, a_2, a_3,... is defined by

$$a_1 = k,$$

$$a_{n+1} = 3a_n + 5, \quad n \geq 1,$$

where k is a positive integer.

(*a*) Write down an expression for a_2 in terms of k.

(1)

(*b*) Show that $a_3 = 9k + 20$.

(2)

(*c*) (i) Find $\sum_{r=1}^{4} a_r$ in terms of k.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 10.

(4)

(C1, Q8 May 2007)

4. A sequence is given by

$$x_1 = 1,$$

$$x_{n+1} = x_n(p + x_n),$$

where p is a constant ($p \neq 0$).

(a) Find x_2 in terms of p. **(1)**

(b) Show that $x_3 = 1 + 3p + 2p^2$. **(2)**

Given that $x_3 = 1$,

(c) find the value of p, **(3)**

(d) write down the value of x_{2008}. **(2)**

(C1, Q7 Jan 2008)

5. A sequence x_1, x_2, x_3, … is defined by

$$x_1 = 1,$$

$$x_{n+1} = ax_n - 3, \quad n \geq 1,$$

where a is a constant.

(a) Find an expression for x_2 in terms of a. **(1)**

(b) Show that $x_3 = a^2 - 3a - 3$. **(2)**

Given that $x_3 = 7$,

(c) find the possible values of a. **(3)**

(C1, Q5 June 2008)

6. A sequence a_1, a_2, a_3, ... is defined by

$$a_1 = k,$$

$$a_{n+1} = 2a_n - 7, \quad n \geq 1,$$

where k is a constant.

(a) Write down an expression for a_2 in terms of k. **(1)**

(*b*) Show that $a_3 = 4k - 21$.

(2)

Given that $\sum_{r=1}^{4} a_r = 43$,

(*c*) find the value of *k*.

(4)

(C1, Q7 June 2009)

7. A sequence of positive numbers is defined by

$$a_{n+1} = \sqrt{(a_n^2 + 3)}, \quad n \geq 1,$$

$$a_1 = 2.$$

(*a*) Find a_2 and a_3, leaving your answers in surd form.

(2)

(*b*) Show that $a_5 = 4$.

(2)

(C1, Q5 May 2010)

8. A sequence a_1, a_2, a_3, ... is defined by

$$a_1 = 2,$$

$$a_{n+1} = 3a_n - c$$

where *c* is a constant.

(*a*) Find an expression for a_2 in terms of *c*.

(1)

Given that $\sum_{i=1}^{3} a_i = 0$,

(*b*) find the value of *c*.

(4)

(C1, Q4 Jan 2011)

9. A sequence a_1, a_2, a_3, …, is defined by

$$a_1 = k,$$

$$a_{n+1} = 5a_n + 3, \quad n \geq 1,$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k.

(1)

(b) Show that $a_3 = 25k + 18$.

(2)

(c) (i) Find $\sum_{r=1}^{4} a_r$ in terms of k, in its simplest form.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 6.

(4)

(C1, Q5 May 2011)

10. A sequence x_1, x_2, x_3, . . . is defined by

$$x_1 = 1,$$

$$x_{n+1} = ax_n + 5, \qquad n \geq 1,$$

where a is a constant.

(a) Write down an expression for x_2 in terms of a.

(1)

(b) Show that $x_3 = a^2 + 5a + 5$.

(2)

Given that $x_3 = 41$

(c) find the possible values of a.

(3)

(C1, Q4 Jan 2012)

11. A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3,$$

$$a_{n+1} = 2a_n - c, \qquad (n \geq 1),$$

where c is a constant.

(a) Write down an expression, in terms of c, for a_2.

(1)

(b) Show that $a_3 = 12 - 3c$.

(2)

Given that $\sum_{i=1}^{4} a_i \geq 23$,

(c) find the range of values of c.

(4)

(C1, Q5 May 2012)

12. A sequence $u_1, u_2, u_3, \ldots$, satisfies

$$u_{n+1} = 2u_n - 1, \quad n \geq 1.$$

Given that $u_2 = 9$,

(a) find the value of u_3 and the value of u_4 ,

(2)

(b) evaluate $\sum_{r=1}^{4} u_r$.

(3)

(C1, Q4 Jan 2013)

13. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4,$$

$$a_{n+1} = k(a_n + 2), \qquad \text{for } n \geq 1$$

where k is a constant.

(a) Find an expression for a_2 in terms of k.

(1)

Given that $\sum_{i=1}^{3} a_i = 2$,

(*b*) find the two possible values of *k*.

(6)

(C1, Q4 May 2013)

14. A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1,$$
$$x_{n+1} = (x_n)^2 - kx_n, \qquad n \geq 1,$$

where *k* is a constant.

(*a*) Find an expression for x_2 in terms of *k*.

(1)

(*b*) Show that $x_3 = 1 - 3k + 2k^2$.

(2)

Given also that $x_3 = 1$,

(*c*) calculate the value of *k*.

(3)

(*d*) Hence find the value of $\sum_{n=1}^{100} x_n$.

(3)

(C1, Q6 May 2013_R)

15. A sequence of numbers $a_1, a_2, a_3\ldots$ is defined by

$$a_{n+1} = 5a_n - 3, \qquad n \geq 1.$$

Given that $a_2 = 7$,

(*a*) find the value of a_1.

(2)

(*b*) Find the value of $\sum_{r=1}^{4} a_r$.

(3)

(C1, Q5 May 2014)

16. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = 4a_n - 3, \qquad n \geq 1$$

$$a_1 = k, \qquad \text{where } k \text{ is a positive integer.}$$

(*a*) Write down an expression for a_2 in terms of k.

(1)

Given that $\sum_{r=1}^{3} a_r = 66$

(*b*) find the value of k.

(4)

(C1, Q3 May 2014_R)

17. (i) A sequence $U_1, U_2, U_3, \ldots$ is defined by

$$U_{n+2} = 2U_{n+1} - U_n, \quad n \geq 1,$$

$$U_1 = 4 \text{ and } U_2 = 4.$$

Find the value of

(*a*) U_3,

(1)

(*b*) $\sum_{n=1}^{20} U_n$.

(2)

(ii) Another sequence $V_1, V_2, V_3, \ldots$ is defined by

$$V_{n+2} = 2V_{n+1} - V_n, \quad n \geq 1,$$

$$V_1 = k \text{ and } V_2 = 2k, \text{ where } k \text{ is a constant.}$$

(*a*) Find V_3 and V_4 in terms of k.

(2)

Given that $\sum_{n=1}^{5} V_n = 165$,

(*b*) find the value of k.

(3)

(C1, Q4 May 2015)

18. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4,$$

$$a_{n+1} = 5 - ka_n, \quad n \geq 1,$$

where k is a constant.

(a) Write down expressions for a_2 and a_3 in terms of k.

(2)

Find

(b) $\sum_{r=1}^{3}(1+a_r)$ in terms of k, giving your answer in its simplest form,

(3)

(c) $\sum_{r=1}^{100}(a_{r+1}+ka_r)$.

(1)

(C1, Q6 May 2016)

19. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 1$$

$$a_{n+1} = \frac{k(a_n+1)}{a_n}, \qquad n \geqslant 1$$

where k is a positive constant.

(a) Write down expressions for a_2 and a_3 in terms of k, giving your answers in their simplest form.

(3)

Given that $\sum_{r=1}^{3} a_r = 10$

(b) find an exact value for k.

(3)

(C1, Q3 May 2017)

20. Given that for all positive integers n,

$$\sum_{r=1}^{n} a_r = 12 + 4n^2$$

(a) find the value of $\sum_{r=1}^{5} a_r$

(2)

(b) find the value of a_6

(3)

(IAL C1, Q5, Jan 2014)

21. A sequence is defined by

$$u_1 = 3$$
$$u_{n+1} = 2 - \frac{4}{u_n}, \quad n \geq 1$$

Find the exact values of

(*a*) u_2, u_3 and u_4

(3)

(*b*) u_{61}

(1)

(*c*) $\sum_{i=1}^{99} u_i$ **(3)**

(IAL C12, Q5, Jan 2014)

22. A sequence is defined by

$$u_1 = k$$
$$u_{n+1} = 3u_n - 12, \quad n \geq 1,$$

(*a*) Write down fully simplified expressions for u_2, u_3 and u_4 in terms of k.

(4)

Given that $u_4 = 15$

(*b*) find the value of k,

(2)

(*c*) find $\sum_{i=1}^{4} u_i$, giving an exact numerical answer.

(3)

(IAL C12, Q8, Jan 2015)

23. A sequence is defined by

$$u_1 = 4$$
$$u_{n+1} = \frac{2u_n}{3}, \quad n \geq 1$$

(*a*) Find the exact values of u_2, u_3 and u_4.

(2)

(*b*) Find the value of u_{20}, giving your answer to 3 significant figures.

(2)

(*c*) Evaluate

$$12 - \sum_{i=1}^{16} u_1$$

giving your answer to 3 significant figures.

(3)

(*d*) Explain why $\sum_{i=1}^{N} u_1$ < 12 for all positive integer values of N. **(1)**

(IAL C12, Q10, May 2015)

24. A sequence of numbers $u_1, u_2, u_3, \ldots$ satisfies

$$u_{n+1} = 2u_n - 6, \qquad n \geq 1$$

Given that $u_1 = 2$

(*a*) find the value of u_3

(2)

(*b*) evaluate $\sum_{i=1}^{4} u_i$

(3)

(IAL C12, Q1, Jan 2016)

25. A sequence is defined by

$$u_1 = 36$$

$$u_{n+1} = \frac{2}{3}u_n, \qquad n \geq 1$$

(*a*) Find the exact simplified values of u_2, u_3 and u_4

(2)

(*b*) Write down the common ratio of the sequence.

(1)

(*c*) Find, giving your answer to 4 significant figures, the value of u_{11}

(2)

(*d*) Find the exact value of $\sum_{i=1}^{6} u_i$

(2)

(*e*) Find the value of $\sum_{i=1}^{\infty} u_i$

(2)

(IAL C12, Q6, Oct 2016)

26. A sequence is defined by

$$u_1 = 1$$

$$u_{n+1} = 2 - 3u_n \qquad n \geqslant 1$$

(*a*) Find the value of u_2 and the value of u_3

(2)

(*b*) Calculate the value of

$$\sum_{r=1}^{4} r - u_n$$

(IAL C12, Q2, Jan 2018)

27. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4$$
$$a_{n+1} = \frac{a_n}{a_n + 1}, \qquad n \geqslant 1, n \in \mathbb{N}$$

(*a*) Find the values of a_2, a_3 and a_4

Write your answers as simplified fractions.

(3)

Given that

$$a_n = \frac{4}{pn+q}, \text{ where } p \text{ and } q \text{ are constants}$$

(*b*) state the value of p and the value of q.

(2)

(*c*) Hence calculate the value of N such that $a_N = \frac{4}{321}$

(2)

(C1, Q6 May 2018)

28. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 8$$
$$a_{n+1} = 4(a_n - c) \qquad n \geq 1$$

where c is a constant.

(*a*) Find an expression for a_2, in terms of c. **(1)**

Given that $a_3 = 28$

(*b*) find the numerical value of $\sum_{i=1}^{4} a_i$ **(6)**

(C1, Q4 May 2019)

29. A sequence is defined by

$$u_1 = 3$$
$$u_{n+1} = u_n - 5, \qquad n \geqslant 1$$

Find the values of

(*a*) u_2, u_3 and u_4

(2)

(*b*) u_{100}

(3)

(c)

$$\sum_{i=1}^{100} u_i$$

(3)

(IAL C12, Q7, May 2018)

30. A sequence of numbers $u_1, u_2, u_3, \ldots$ satisfies

$$u_n = kn - 3^n$$

where k is a constant.
Given that $u_2 = u_4$

(a) find the value of k

(3)

(b) evaluate

$$\sum_{n=1}^{4} u_r$$

(3)
(IAL C12, Q7, Oct 2018)

31. A sequence is defined by

$$u_1 = k, \text{ where } k \text{ is a constant}$$
$$u_{n+1} = 4u_n - 3, \qquad n \geqslant 1$$

(a) Find u_2 and u_3 in terms of k, simplifying your answers as appropriate.

(3)

Given

$$\sum_{n=1}^{3} u_n = 18$$

(b) find k.

(3)
(IAL C12, Q7, Jan 2019)

32. A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 1$$
$$u_{n+1} = k - \frac{8}{u_n} \qquad n \geq 1$$

Where k is a constant

(a) Write down expressions for u_2 and u_3 in terms of k.

(2)

Given that $u_3 = 6$

(b) find the possible values of k.

(4)
(IAL C12, Q5, May 2019)

33. A sequence of numbers $u_1, u_2, u_3, \ldots$ satisfies

$$u_n = p - qn, \qquad n \in \mathbb{N},\ n \geq 1$$

where p and q are positive constants.

Given that $u_2 = 21$ and $u_8 = -9$

(*a*) find the value of p and the value of q.

(4)

Hence find

(*b*) the value of u_{100}

(2)

(*c*) the value of $\sum_{n=6}^{30} u_n$

(3)

(IAL C12, Q9, Oct 2019)