OCR Core Maths 2

Past paper questions
Quadratics

Edited by K V Kumaran

Email: kvkumaran@gmail.com

Phone: 07961319548

Quadratics

- Factorising quadratics. To check whether a given quadratic factorises calculate the discriminant $b^2 - 4ac$; if it is a perfect square $(4, 49, 81$ etc.) then it factorises.

- When the x^2 coefficient (the number in front of the x^2) is one this is easy. Just spot two numbers which multiply to the constant and add to the x coefficient. For example with $x^2 + 8x + 15$ we need to find two numbers which multiply to 15 and sum to 8; clearly 3 and 5. So $x^2 + 8x + 15 = (x + 3)(x + 5)$.

- If the x^2 coefficient is not one then more work is required. You need to multiply the x^2 coefficient by the constant term and then find 2 numbers which multiply to this and sum to the x coefficient. For example with $6x^2 + x - 12$ we calculate $6 \times -12 = -72$ so the two numbers are clearly 9 and -8. So

$$\begin{aligned} 6x^2 + x - 12 &= 6x^2 + 9x - 8x - 12 & &= 6x^2 - 8x + 9x - 12 \\ &= 3x(2x + 3) - 4(2x + 3) & &= 2x(3x - 4) + 3(3x - 4) \\ &= (3x - 4)(2x + 3) & &= (2x + 3)(3x - 4). \end{aligned}$$

  Notice that it does not matter which way round we write the $9x$ and $-8x$.

- For quadratics that cannot be factorised we need to use the formula. For $ax^2 + bx + c = 0$ the solution is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

- The $b^2 - 4ac$ part is called the *discriminant*. If it is positive then there are two *distinct* roots. If it is zero then there exists only one root and it is *repeated*. If it is negative then there are no roots. For example: find the values of k such that $x^2 + (k + 3)x + 4k = 0$ has only one root. We need the discriminant to be zero, so

$$\begin{aligned} b^2 - 4ac &= 0 \\ (k + 3)^2 - 16k &= 0 \\ k^2 - 10k + 9 &= 0 \\ k &= 9 \text{ or } k = 1. \end{aligned}$$

- Completing the square. All about halving the x coefficient into the bracket and then correcting the constant term. For example $x^2 - 6x + 10 = (x - 3)^2 - 9 + 10 = (x - 3)^2 + 1$. If the x^2 coefficient isn't one then need to factorise it out. For example

$$\begin{aligned} -2x^2 + 4x - 8 &= -2[x^2 - 2x] - 8 \\ &= -2[(x - 1)^2 - 1] - 8 \\ &= -2(x - 1)^2 - 6. \end{aligned}$$

  From this we can find the maximum or minimum of the quadratic. For $y = -2(x - 1)^2 - 6$ it is when $x = 1$ (to make the bracket 0) and therefore $y = -6$. In this case $(1, -6)$ is a maximum due to negative x^2 coefficient.

  We can also find the vertical line of symmetry by completing the square. For example

$$\begin{aligned} 3x^2 + 5x + 1 &= 3[x^2 + \tfrac{5}{3}x] + 1 \\ &= 3[(x + \tfrac{5}{6})^2 - \tfrac{25}{36}] + 1 \\ &= 3(x + \tfrac{5}{6})^2 - \tfrac{25}{12} + \tfrac{12}{12} \\ &= 3(x + \tfrac{5}{6})^2 - \tfrac{13}{12}. \end{aligned}$$

From this we see that the vertex is at $(-\frac{5}{6}, -\frac{13}{12})$ and consequently the line of symmetry is $x = -\frac{5}{6}$.

- You must be on the lookout for *quadratics in disguise*. You spot these when there are two powers on the variable and one is *twice* the other (or can be manipulated into such an equation[2]). Most students like to solve these by means of a substitution (although some students don't need to do this). For example to solve $x^4 + 2x^2 = 8$ work as follows:

$$\begin{aligned} x^4 + 2x^2 - 8 &= 0 && \text{getting everything to one side} \\ u^2 + 2u - 8 &= 0 && \text{substituting } u = x^2 \\ (u+4)(u-2) &= 0 \\ u = -4 \text{ or } u = 2 \quad &\Rightarrow && x^2 = -4 \text{ or } x^2 = 2 \end{aligned}$$

  But $x^2 = -4$ has no solutions, so $x = \pm\sqrt{2}$.

- For those who don't like substituting, just factorise and solve:

$$\begin{aligned} 2x^{\frac{2}{3}} &= 5x^{\frac{1}{3}} + 3 \\ 2x^{\frac{2}{3}} - 5x^{\frac{1}{3}} - 3 &= 0 \\ (2x^{\frac{1}{3}} + 1)(x^{\frac{1}{3}} - 3) &= 0 \end{aligned}$$

  So $x^{\frac{1}{3}} = -\frac{1}{2}$ or $x^{\frac{1}{3}} = 3$. Therefore cubing we find $x = -\frac{1}{8}$ or $x = 27$.

- Don't be one of the cretins who sees something like $x^4 + 4x^2 = 9$ and then *thinks* that they are square rooting to obtain $x^2 + 2x = 3$. Remember $\sqrt{x^4 + 4x^2} \neq x^2 + 2x$.

  Likewise $x + \sqrt{x} + 3 = 0$ does not square to $x^2 + x + 9 = 0$.

1.

(i) Express $3x^2 + 12x + 7$ in the form $3(x+a)^2 + b$. [4]

(ii) Hence write down the equation of the line of symmetry of the curve $y = 3x^2 + 12x + 7$. [1]

Q2 June 2005

2.

Solve the equation $x^6 + 26x^3 - 27 = 0$. [5]

Q4 June 2005

3.

(i) Calculate the discriminant of each of the following:

(a) $x^2 + 6x + 9$,

(b) $x^2 - 10x + 12$,

(c) $x^2 - 2x + 5$.

[3]

(ii)

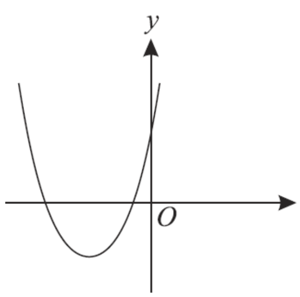

Fig. 1

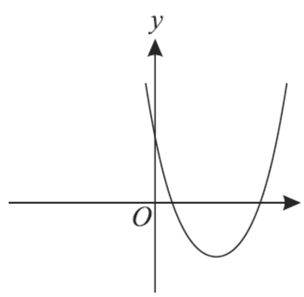

Fig. 2

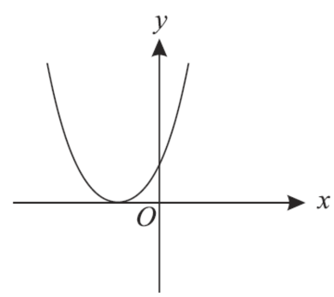

Fig. 3

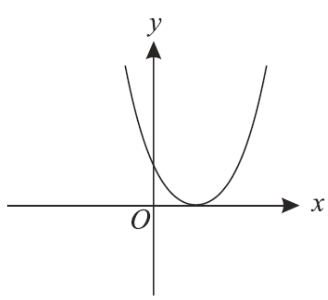

Fig. 4

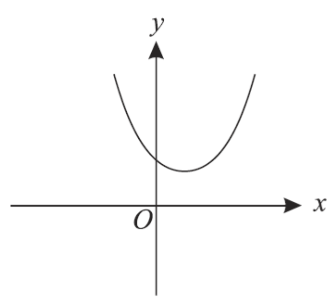

Fig. 5

State with reasons which of the diagrams corresponds to the curve

(a) $y = x^2 + 6x + 9$,

(b) $y = x^2 - 10x + 12$,

(c) $y = x^2 - 2x + 5$.

[4]

Q7 June 2005

4.

(i) Simplify $(3x+1)^2 - 2(2x-3)^2$. [3]

(ii) Find the coefficient of x^3 in the expansion of

$$(2x^3 - 3x^2 + 4x - 3)(x^2 - 2x + 1).$$ [2]

Q2 Jan 2006

5.

(i) Solve the equation $x^2 - 8x + 11 = 0$, giving your answers in simplified surd form. [4]

(ii) Hence sketch the curve $y = x^2 - 8x + 11$, labelling the points where the curve crosses the axes. [3]

(iii) Solve the equation $y - 8y^{\frac{1}{2}} + 11 = 0$, giving your answers in the form $p \pm q\sqrt{5}$. [4]

Q7 Jan 2006

6.

(i) Express $2x^2 + 12x + 13$ in the form $a(x+b)^2 + c$. [4]

(ii) Solve $2x^2 + 12x + 13 = 0$, giving your answers in simplified surd form. [3]

Q3 June 2006

7.

Solve the equation $x^{\frac{2}{3}} + 3x^{\frac{1}{3}} - 10 = 0$. [5]

Q4 Jan 2007

8.

(i) Express $2x^2 - 24x + 80$ in the form $a(x-b)^2 + c$. [4]

(ii) State the equation of the line of symmetry of the curve $y = 2x^2 - 24x + 80$. [1]

(iii) State the equation of the tangent to the curve $y = 2x^2 - 24x + 80$ at its minimum point. [1]

Q6 Jan 2007

9.

Simplify $(2x+5)^2 - (x-3)^2$, giving your answer in the form $ax^2 + bx + c$. [3]

Q1 June 2007

10.

(i) Find the discriminant of $kx^2 - 4x + k$ in terms of k. [2]

(ii) The quadratic equation $kx^2 - 4x + k = 0$ has equal roots. Find the possible values of k. [3]

Q4 June 2007

11.

By using the substitution $y = (x + 2)^2$, find the real roots of the equation

$$(x + 2)^4 + 5(x + 2)^2 - 6 = 0.$$ [6]

Q6 June 2007

12.

(i) Express $x^2 + 8x + 15$ in the form $(x + a)^2 - b$. [3]

(ii) Hence state the coordinates of the vertex of the curve $y = x^2 + 8x + 15$. [2]

(iii) Solve the inequality $x^2 + 8x + 15 > 0$. [4]

Q8 June 2007

13.

(i) Solve the equation $3x^2 - 14x - 5 = 0$. [3]

A curve has equation $y = 3x^2 - 14x - 5$.

(ii) Sketch the curve, indicating the coordinates of all intercepts with the axes. [3]

(iii) Find the value of c for which the line $y = 4x + c$ is a tangent to the curve. [6]

Q10 June 2007

14.

Given that $3x^2 + bx + 10 = a(x + 3)^2 + c$ for all values of x, find the values of the constants a, b and c. [4]

Q3 Jan 2008

15.

(i) Solve the equation $x^2 + 8x + 10 = 0$, giving your answers in simplified surd form. [3]

(ii) Sketch the curve $y = x^2 + 8x + 10$, giving the coordinates of the point where the curve crosses the y-axis. [3]

(iii) Solve the inequality $x^2 + 8x + 10 \geqslant 0$. [2]

Q6 Jan 2008

16.

Solve the equation $2x - 7x^{\frac{1}{2}} + 3 = 0$. [5]

Q4 June 2008

17.

(i) Express $2x^2 - 6x + 11$ in the form $p(x+q)^2 + r$. [4]

(ii) State the coordinates of the vertex of the curve $y = 2x^2 - 6x + 11$. [2]

(iii) Calculate the discriminant of $2x^2 - 6x + 11$. [2]

(iv) State the number of real roots of the equation $2x^2 - 6x + 11 = 0$. [1]

(v) Find the coordinates of the points of intersection of the curve $y = 2x^2 - 6x + 11$ and the line $7x + y = 14$. [5]

Q10 June 2008

18.

Solve the equation $3x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 = 0$. **[5]**

Q3 Jan 2009

19.

(i) Express $5x^2 + 20x - 8$ in the form $p(x+q)^2 + r$. **[4]**

(ii) State the equation of the line of symmetry of the curve $y = 5x^2 + 20x - 8$. **[1]**

(iii) Calculate the discriminant of $5x^2 + 20x - 8$. **[2]**

(iv) State the number of real roots of the equation $5x^2 + 20x - 8 = 0$. **[1]**

Q6 Jan 2009

20.

(i) Expand and simplify $(2x+1)(x-3)(x+4)$. **[3]**

(ii) Find the coefficient of x^4 in the expansion of

$$x(x^2 + 2x + 3)(x^2 + 7x - 2).$$ **[2]**

Q4 June 2009

21.

Express $x^2 - 12x + 1$ in the form $(x-p)^2 + q$. **[3]**

Q1 Jan 2010

22.

Solve the equation $x - 8\sqrt{x} + 13 = 0$, giving your answers in the form $p \pm q\sqrt{r}$, where p, q and r are integers. **[7]**

Q5 Jan 2010

23.

The quadratic equation $kx^2 - 30x + 25k = 0$ has equal roots. Find the possible values of k. **[4]**

Q10 Jan 2010

24.

Find the real roots of the equation $4x^4 + 3x^2 - 1 = 0$. **[5]**

Q5 June 2010

25.

(i) Express $2x^2 + 5x$ in the form $2(x + p)^2 + q$. **[3]**

(ii) State the coordinates of the minimum point of the curve $y = 2x^2 + 5x$. **[2]**

(iii) State the equation of the normal to the curve at its minimum point. **[1]**

(iv) Solve the inequality $2x^2 + 5x > 0$. **[4]**

Q8 June 2010

26.

Given that

$$(x - p)(2x^2 + 9x + 10) = (x^2 - 4)(2x + q)$$

for all values of x, find the constants p and q. **[3]**

Q2 Jan 2011

27.

By using the substitution $u = (3x - 2)^2$, find the roots of the equation

$$(3x - 2)^4 - 5(3x - 2)^2 + 4 = 0.$$ **[6]**

Q4 Jan 2011

28.

(i) Express $4x^2 + 12x - 3$ in the form $p(x + q)^2 + r$. **[4]**

(ii) Solve the equation $4x^2 + 12x - 3 = 0$, giving your answers in simplified surd form. **[4]**

(iii) The quadratic equation $4x^2 + 12x - k = 0$ has equal roots. Find the value of k. **[3]**

Q7 Jan 2011

29.

Express $3x^2 - 18x + 4$ in the form $p(x+q)^2 + r$. **[4]**

Q1 June 2011

30.

Solve the equation $3x^{\frac{1}{2}} - 8x^{\frac{1}{4}} + 4 = 0$. **[5]**

Q6 June 2011

31.

Given that

$$5x^2 + px - 8 = q(x-1)^2 + r$$

for all values of x, find the values of the constants p, q and r. **[4]**

Q3 Jan 2012

32.

Find the real roots of the equation $\frac{3}{y^4} - \frac{10}{y^2} - 8 = 0$. **[5]**

Q5 Jan 2012

33.

Simplify $(x-5)(x^2+3) - (x+4)(x-1)$. **[3]**

Q1 June 2012

34.

(i) Express $2x^2 - 20x + 49$ in the form $p(x-q)^2 + r$. **[4]**

(ii) State the coordinates of the vertex of the curve $y = 2x^2 - 20x + 49$. **[2]**

Q4 June 2012

35.

Solve the equation $x - 6x^{\frac{1}{2}} + 2 = 0$, giving your answers in the form $p \pm q\sqrt{r}$, where p, q and r are integers. **[6]**

Q7 June 2012

36.

(i) Simplify $(x+4)(5x-3)-3(x-2)^2$. **[3]**

(ii) The coefficient of x^2 in the expansion of

$$(x+3)(x+k)(2x-5)$$

is –3. Find the value of the constant k. **[3]**

Q5 Jan 2013

37.

The quadratic equation $kx^2+(3k-1)x-4=0$ has no real roots. Find the set of possible values of k. **[7]**

Q8 Jan 2013

38.

Solve the equation $8x^6+7x^3-1=0$. **[5]**

Q2 June 2013

39.

(i) Express $3x^2+9x+10$ in the form $3(x+p)^2+q$. **[3]**

(ii) State the coordinates of the minimum point of the curve $y=3x^2+9x+10$. **[2]**

(iii) Calculate the discriminant of $3x^2+9x+10$. **[2]**

Q4 June 2013

40.

(i) Sketch the curve $y=2x^2-x-6$, giving the coordinates of all points of intersection with the axes. **[5]**

(ii) Find the set of values of x for which $2x^2-x-6$ is a decreasing function. **[3]**

(iii) The line $y=4$ meets the curve $y=2x^2-x-6$ at the points P and Q. Calculate the distance PQ. **[4]**

Q9 June 2013

41.

Express $5x^2+10x+2$ in the form $p(x+q)^2+r$, where p, q and r are integers. **[4]**

Q1 June 2014

42.

Find the real roots of the equation $4x^4+3x^2-1=0$. **[5]**

Q3 June 2014

43.

Solve the equation $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 6 = 0$. **[5]**

Q4 June 2015

44.

(i) Sketch the curve $y = 2x^2 - x - 3$, giving the coordinates of all points of intersection with the axes. **[4]**

(ii) Hence, or otherwise, solve the inequality $2x^2 - x - 3 > 0$. **[2]**

(iii) Given that the equation $2x^2 - x - 3 = k$ has no real roots, find the set of possible values of the constant k. **[3]**

Q8 June 2015