OCR Statistics 01

Past paper questions on

- Probability
- Permutations & combinations

Edited by K V Kumaran
Email: kvkumaran@gmail.com
Phone: 07961319548

Probability

- An *independent event* is one which has no effect on subsequent events. The events of spinning a coin and then cutting a pack of cards are independent because the way in which the coin lands has no effect on the cut. For two *independent* events A & B

$$\mathbb{P}(A \text{ and } B) = \mathbb{P}(A) \times \mathbb{P}(B).$$

For example a fair coin is tossed and a card is then drawn from a pack of 52 playing cards. Find the probability that a head and an ace will result.

$$\mathbb{P}(\text{head}) = \tfrac{1}{2}, \qquad \mathbb{P}(\text{ace}) = \tfrac{4}{52} = \tfrac{1}{13}, \qquad \text{so } \mathbb{P}(\text{head and ace}) = \tfrac{1}{2} \times \tfrac{1}{13} = \tfrac{1}{26}.$$

- *Mutually Exclusive Events.* Two events which cannot occur at the same time are called mutually exclusive. The events of throwing a 3 or a 4 in a single roll of a fair die are mutually exclusive. For any two mutually exclusive events

$$\mathbb{P}(A \text{ or } B) = \mathbb{P}(A) + \mathbb{P}(B).$$

For example a fair die with faces of 1 to 6 is rolled once. What is the probability of obtaining either a 5 or a 6?

$$\mathbb{P}(5) = \tfrac{1}{6}, \qquad \mathbb{P}(6) = \tfrac{1}{6}, \qquad \text{so } \mathbb{P}(5 \text{ or } 6) = \tfrac{1}{6} + \tfrac{1}{6} = \tfrac{1}{3}.$$

- *Non-Mutually Exclusive Events.* When two events can both happen they are called non-mutually exclusive events. For example studying English and studying Maths at A Level are non-mutually exclusive. By considering a Venn diagram of two events A & B we find

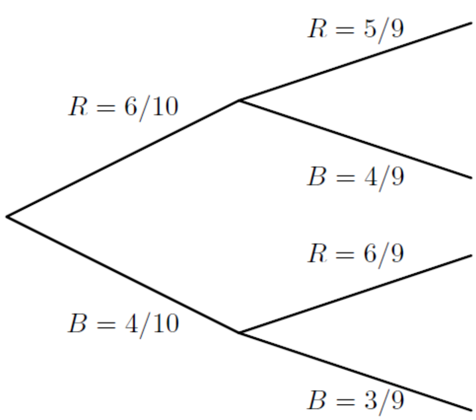
$$\mathbb{P}(A \text{ or } B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \text{ and } B),$$
$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B).$$

- *Tree Diagrams.* These may be used to help solve probability problems when more than one event is being considered. The probabilities on any branch section must sum to one. You multiply along the branches to discover the probability of that branch occurring.

For example a box contains 4 black and 6 red pens. A pen is drawn from the box and it is not replaced. A second pen is then drawn. Find the probability of

(i) two red pens being obtained.
(ii) two black pens being obtained.
(iii) one pen of each colour being obtained.
(iv) two red pens *given* that they are the same colour.

Draw tree diagram to discover:



(i) $\mathbb{P}(\text{two red pens}) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}.$

(ii) $\mathbb{P}(\text{two black pens}) = \frac{4}{10} \times \frac{3}{9} = \frac{12}{90} = \frac{2}{15}.$

(iii) $\mathbb{P}(\text{one of each colour}) = 1 - \frac{30}{90} - \frac{12}{90} = \frac{8}{15}.$

(iv) $\mathbb{P}(\text{two reds} \mid \text{same colour}) = \frac{1/3}{1/3+2/15} = \frac{5}{7}.$

- *Conditional Probability.* In the above example we see that the probability of two red pens is $\frac{1}{3}$, but the probability of two red pens *given that both pens are the same colour* is $\frac{5}{7}$. This is known as conditional probability. $\mathbb{P}(A \mid B)$ mean the probability of A *given* that B has happened. It is governed by

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \text{ and } B)}{\mathbb{P}(B)} = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}.$$

  For example if there are 120 students in a year and 60 study Maths, 40 study English and 10 study both then

$$\mathbb{P}(\text{study English} \mid \text{study Maths}) = \frac{\mathbb{P}(\text{study Maths \& English})}{\mathbb{P}(\text{study Maths})} = \frac{10/120}{60/120} = \frac{1}{6}.$$

- A is independent of B if $\mathbb{P}(A) = \mathbb{P}(A \mid B) = \mathbb{P}(A \mid B')$. (i.e. whatever happens in B the probability of A remains unchanged.) For example flicking a coin and then cutting a deck of cards to try and find an ace are independent because

$$\mathbb{P}(\text{cutting ace}) = \mathbb{P}(\text{cutting ace} \mid \text{flick head}) = \mathbb{P}(\text{cutting ace} \mid \text{flick tail}) = \tfrac{1}{13}.$$

Permutations And Combinations

- Factorials are defined $n! \equiv n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$. Many expressions involving factorials simplify with a bit of thought. For example $\frac{n!}{(n-2)!} = n(n-1)$. Also there is a convention that $0! = 1$.

- The number of ways of arranging n different objects in a line is $n!$ For example how many different arrangements are there if 4 different books are to be placed on a bookshelf? There are 4 ways in which to select the first book, 3 ways in which to choose the second book, 2 ways to pick the third book and 1 way left for the final book. The total number of different ways is $4 \times 3 \times 2 \times 1 = 4!$

- Permutations. The number of ways of selecting r objects from n when *the order of the selection matters* is ${}^nP_r$. It can be calculated by

$${}^nP_r = \frac{n!}{(n-r)!}.$$

  For example in how many ways can the gold, silver and bronze medals be awarded in a race of ten people? The order in which the medals are awarded matters, so the number of ways is given by ${}^{10}P_3 = 720$.

  In another example how many words of four letters can be made from the word CONSIDER? This is an arrangement of four out of eight different objects where the order matters so there are ${}^8P_4 = \frac{8!}{4!} = 1680$ different words.

- Combinations. The number of ways of selecting r objects from n when *the order of the selection does not matter* is ${}^nC_r$. It can be calculated by

$${}^nC_r \equiv \binom{n}{r} = \frac{n!}{r!\,(n-r)!}.$$

  For example in how many ways can a committee of 5 people be chosen from 8 applicants? Solution is given by ${}^8C_5 = \frac{8!}{5! \times 3!} = 56$.

  In another example how many ways are there of selecting your lottery numbers (where one selects 6 numbers from 49)? It does not matter which order you choose your numbers, so there are ${}^{49}C_6 = 13\,983\,816$ possible selections.

- If letters are repeated in a 'word', then you just divide through by the factorials of each repeat. Therefore there are $\frac{11!}{4!\times 4!\times 2!}$ arrangements of the word 'MISSISSIPPI'.[4]

- You must be good at 'choosing committee' questions [be on the lookout, they can be in disguise]. For example how many ways are there of choosing a committee of 3 women and 4 men from a group containing 10 women and 5 men? There are $\binom{10}{3}$ ways of choosing the women (the order doesn't matter) and $\binom{5}{4}$ ways of choosing the men. Therefore overall there are $\binom{5}{4} \times \binom{10}{3}$ ways of choosing the committee.

- Example: If I deal six cards from a standard deck of cards, in how many ways can I get exactly four clubs? Well there are $\binom{13}{4}$ ways of getting the clubs, and $\binom{39}{2}$ ways of getting the non-clubs, so therefore the answer to the original question is $\binom{13}{4} \times \binom{39}{2}$.

- When considering lining things up in a line we start from the principle that there are $n!$ ways of arranging n objects. In the harder examples you need to be a cunning.

  For example three siblings join a queue with 5 other people making 8 in total.

  1. How many way are there of arranging the 8 in a queue? Easy; 8!
  2. How many ways are there of arranging the 8, such that the siblings are together? We, we imaging the three siblings tied together. There are therefore 6! ways of arranging the 5 and the bundle of siblings and then there are 3! ways of arranging the siblings in the bundle. Therefore the answer is $6! \times 3!$
  3. How many ways are there of arranging the siblings so they are not together? There are 5! ways of arranging the five without the siblings. There are then 6 places for the first sibling to go, 5 for the second, and 4 for the third. Therefore $5! \times 6 \times 5 \times 4$.

- To calculate probabilities we go back to first principles and remember that probability is calculated from the number of ways of getting what we want over the total number of possible outcomes. So in the above example, if the 8 are arranged randomly in a line, what is the probability of the siblings being together? $\mathbb{P}(\text{together}) = \frac{6!\times 3!}{8!}$.

  Going back to the four club question if it asked for the probability of getting exactly four clubs if I dealt exactly six cards from the pack, the answer would be $\frac{\binom{13}{4}\times\binom{39}{2}}{\binom{52}{6}}$. The $\binom{52}{6}$ represents the total number of ways I can deal six cards from the 52.

1.

A committee of 7 people is to be chosen at random from 18 volunteers.

(i) In how many different ways can the committee be chosen? [2]

The 18 volunteers consist of 5 people from Gloucester, 6 from Hereford and 7 from Worcester. The committee is to be chosen randomly. Find the probability that the committee will

(ii) consist of 2 people from Gloucester, 2 people from Hereford and 3 people from Worcester, [4]

(iii) include exactly 5 people from Worcester, [4]

(iv) include at least 2 people from each of the three cities. [4]

Q7 June 2005

2.

Jenny and John are each allowed two attempts to pass an examination.

(i) Jenny estimates that her chances of success are as follows.

- The probability that she will pass on her first attempt is $\frac{2}{3}$.
- If she fails on her first attempt, the probability that she will pass on her second attempt is $\frac{3}{4}$.

Calculate the probability that Jenny will pass. [3]

(ii) John estimates that his chances of success are as follows.

- The probability that he will pass on his first attempt is $\frac{2}{3}$.
- Overall, the probability that he will pass is $\frac{5}{6}$.

Calculate the probability that if John fails on his first attempt, he will pass on his second attempt. [3]

Q1 Jan 2006

3.

An examination paper consists of two parts. Section A contains questions A1, A2, A3 and A4. Section B contains questions B1, B2, B3, B4, B5, B6 and B7.

Candidates must choose three questions from section A and four questions from section B. The order in which they choose the questions does not matter.

(i) In how many ways can the seven questions be chosen? [3]

(ii) Assuming that all selections are equally likely, find the probability that a particular candidate chooses question A1 but does **not** choose question B1. [3]

(iii) Following a change of syllabus, the form of the examination remains the same except that candidates who choose question A1 are not allowed to choose question B1. In how many ways can the seven questions now be chosen? [3]

Q6 Jan 2006

4.

Each of the 7 letters in the word DIVIDED is printed on a separate card. The cards are arranged in a row.

(i) How many different arrangements of the letters are possible? [3]

(ii) In how many of these arrangements are all three Ds together? [2]

The 7 cards are now shuffled and 2 cards are selected at random, without replacement.

(iii) Find the probability that at least one of these 2 cards has D printed on it. [3]

Q3 June 2006

5.

The digits 1, 2, 3, 4 and 5 are arranged in random order, to form a five-digit number.

(i) How many different five-digit numbers can be formed? [1]

(ii) Find the probability that the five-digit number is

(a) odd, [2]

(b) less than 23 000. [3]

Q3 Jan 2006

6.

A bag contains 6 white discs and 4 blue discs. Discs are removed at random, one at a time, **without** replacement.

(i) Find the probability that

(a) the second disc is blue, given that the first disc was blue, [1]

(b) the second disc is blue, [3]

(c) the third disc is blue, given that the first disc was blue. [3]

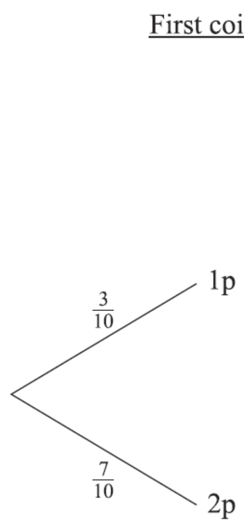
(ii) The random variable X is the number of discs which are removed up to and including the first blue disc. State whether the variable X has a geometric distribution. Explain your answer briefly. [1]

Q4 June 2007

7.

A bag contains three 1p coins and seven 2p coins. Coins are removed at random one at a time, **without** replacement, until the total value of the coins removed is **at least** 3p. Then no more coins are removed.

(i) Copy and complete the probability tree diagram. [5]



Find the probability that

(ii) exactly two coins are removed, [3]

(iii) the total value of the coins removed is 4p. [3]

Q7 Jan 2006

8.

(i) The letters A, B, C, D and E are arranged in a straight line.

(a) How many different arrangements are possible? [2]

(b) In how many of these arrangements are the letters A and B next to each other? [3]

(ii) From the letters A, B, C, D and E, two different letters are selected at random. Find the probability that these two letters are A and B. [2]

Q1 Jan 2008

9.

A supermarket has a large stock of eggs. 40% of the stock are from a firm called Eggzact. 12% of the stock are brown eggs from Eggzact.

An egg is chosen at random from the stock. Calculate the probability that

(i) this egg is brown, given that it is from Eggzact, [2]

(ii) this egg is from Eggzact and is not brown. [2]

Q4 Jan 2008

10.

A class consists of 7 students from Ashville and 8 from Bewton. A committee of 5 students is chosen at random from the class.

(i) Find the probability that 2 students from Ashville and 3 from Bewton are chosen. [3]

(ii) In fact 2 students from Ashville and 3 from Bewton are chosen. In order to watch a video, all 5 committee members sit in a row. In how many different orders can they sit if no two students from Bewton sit next to each other? [2]

Q2 June 2008

11.

(i) A bag contains 12 red discs and 10 black discs. Two discs are removed at random, without replacement. Find the probability that both discs are red. [2]

(ii) Another bag contains 7 green discs and 8 blue discs. Three discs are removed at random, without replacement. Find the probability that exactly two of these discs are green. [3]

(iii) A third bag contains 45 discs, each of which is either yellow or brown. Two discs are removed at random, without replacement. The probability that both discs are yellow is $\frac{1}{15}$. Find the number of yellow discs which were in the bag at first. [4]

Q5 June 2008

12.

A game uses an unbiased die with faces numbered 1 to 6. The die is thrown once. If it shows 4 or 5 or 6 then this number is the final score. If it shows 1 or 2 or 3 then the die is thrown again and the final score is the sum of the numbers shown on the two throws.

(i) Find the probability that the final score is 4. **[3]**

(ii) Given that the die is thrown only once, find the probability that the final score is 4. **[1]**

(iii) Given that the die is thrown twice, find the probability that the final score is 4. **[3]**

Q8 Jan 2009

13.

A test consists of 4 algebra questions, A, B, C and D, and 4 geometry questions, G, H, I and J.

The examiner plans to arrange all 8 questions in a random order, regardless of topic.

(i) (a) How many different arrangements are possible? **[2]**

(b) Find the probability that no two Algebra questions are next to each other and no two Geometry questions are next to each other. **[3]**

Later, the examiner decides that the questions should be arranged in two sections, Algebra followed by Geometry, with the questions in each section arranged in a random order.

(ii) (a) How many different arrangements are possible? **[2]**

(b) Find the probability that questions A and H are next to each other. **[1]**

(c) Find the probability that questions B and J are separated by more than four other questions. **[4]**

Q6 Jan 2009

14.

Three letters are selected at random from the 8 letters of the word COMPUTER, without regard to order.

(i) Find the number of possible selections of 3 letters. **[2]**

(ii) Find the probability that the letter P is included in the selection. **[3]**

Three letters are now selected at random, one at a time, from the 8 letters of the word COMPUTER, and are placed in order in a line.

(iii) Find the probability that the 3 letters form the word TOP. **[3]**

Q7 June 2009

15.

A washing-up bowl contains 6 spoons, 5 forks and 3 knives. Three of these 14 items are removed at random, without replacement. Find the probability that

(i) all three items are of different kinds, **[3]**

(ii) all three items are of the same kind. **[3]**

Q5 Jan 2010

16.

The table shows the numbers of male and female members of a vintage car club who own either a Jaguar or a Bentley. No member owns both makes of car.

	Male	Female
Jaguar	25	15
Bentley	12	8

One member is chosen at random from these 60 members.

(i) Given that this member is male, find the probability that he owns a Jaguar. **[2]**

Now two members are chosen at random from the 60 members. They are chosen one at a time, without replacement.

(ii) Given that the first one of these members is female, find the probability that both own Jaguars. **[4]**

Q7 Jan 2010

17.

The five letters of the word NEVER are arranged in random order in a straight line.

(i) How many different orders of the letters are possible? **[2]**

(ii) In how many of the possible orders are the two Es next to each other? **[2]**

(iii) Find the probability that the first two letters in the order include exactly one letter E. **[3]**

Q8 Jan 2010

18.

The menu below shows all the dishes available at a certain restaurant.

Rice dishes	**Main dishes**	**Vegetable dishes**
Boiled rice	Chicken	Mushrooms
Fried rice	Beef	Cauliflower
Pilau rice	Lamb	Spinach
Keema rice	Mixed grill	Lentils
	Prawn	Potatoes
	Vegetarian	

A group of friends decide that they will share a total of 2 different rice dishes, 3 different main dishes and 4 different vegetable dishes from this menu. Given these restrictions,

(i) find the number of possible combinations of dishes that they can choose to share, **[3]**

(ii) assuming that all choices are equally likely, find the probability that they choose boiled rice. **[2]**

The friends decide to add a further restriction as follows. If they choose boiled rice, they will not choose potatoes.

(iii) Find the number of possible combinations of dishes that they can now choose. **[3]**

Q7 June 2007

19.

Jenny and Omar are each allowed two attempts at a high jump.

(i) The probability that Jenny will succeed on her first attempt is 0.6. If she fails on her first attempt, the probability that she will succeed on her second attempt is 0.7. Calculate the probability that Jenny will succeed. **[3]**

(ii) The probability that Omar will succeed on his first attempt is p. If he fails on his first attempt, the probability that he will succeed on his second attempt is also p. The probability that he succeeds is 0.51. Find p. **[4]**

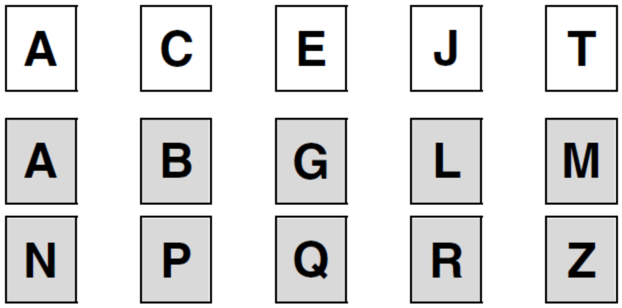
Q4 Jan 2011

20.

(i) The diagram shows 7 cards, each with a digit printed on it. The digits form a 7-digit number.

How many different 7-digit numbers can be formed using these cards? **[3]**

(ii) The diagram below shows 5 white cards and 10 grey cards, each with a letter printed on it.



From these cards, 3 white cards and 4 grey cards are selected at random **without** regard to order.

(a) How many selections of seven cards are possible? **[3]**

(b) Find the probability that the seven cards include exactly one card showing the letter A. **[4]**

Q6 Jan 2011

21.

A group of 7 students sit in random order on a bench.

(i) **(a)** Find the number of orders in which they can sit. **[1]**

(b) The 7 students include Tom and Jerry. Find the probability that Tom and Jerry sit next to each other. **[3]**

(ii) The students consist of 3 girls and 4 boys. Find the probability that

(a) no two boys sit next to each other, **[2]**

(b) all three girls sit next to each other. **[3]**

Q6 June 2011

22.

A bag contains 9 discs numbered 1, 2, 3, 4, 5, 6, 7, 8, 9.

(i) Andrea chooses 4 discs at random, without replacement, and places them in a row.

(a) How many different 4-digit numbers can be made? **[2]**

(b) How many different **odd** 4-digit numbers can be made? **[3]**

(ii) Andrea's 4 discs are put back in the bag. Martin then chooses 4 discs at random, without replacement. Find the probability that

(a) the 4 digits include at least 3 odd digits, **[4]**

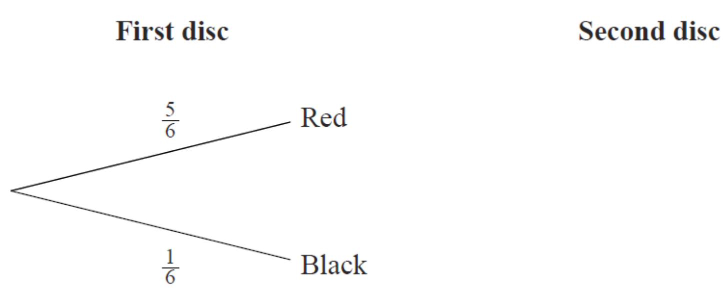
(b) the 4 digits add up to 28. **[3]**

Q9 June 2012

23.

A bag contains 5 red discs and 1 black disc. Tina takes two discs from the bag at random without replacement.

(i) The diagram shows part of a tree diagram to illustrate this situation.



Complete the tree diagram in your Answer Book showing all the probabilities. **[2]**

(ii) Find the probability that exactly one of the two discs is red. **[3]**

All the discs are replaced in the bag. Tony now takes three discs from the bag at random without replacement.

(iii) Given that the first disc Tony takes is red, find the probability that the third disc Tony takes is also red. **[2]**

Q4 June 2012

24.

(i) 5 of the 7 letters A, B, C, D, E, F, G are arranged in a random order in a straight line.

(a) How many different arrangements of 5 letters are possible? **[2]**

(b) How many of these arrangements end with a vowel (A or E)? **[3]**

(ii) A group of 5 people is to be chosen from a list of 7 people.

(a) How many different groups of 5 people can be chosen? **[1]**

(b) The list of 7 people includes Jill and Jo. A group of 5 people is chosen at random from the list. Given that either Jill and Jo are both chosen or neither of them is chosen, find the probability that both of them are chosen. **[3]**

Q7 June 2012

25.

(i) Kathryn is allowed three attempts at a high jump. If she succeeds on any attempt, she does not jump again. The probability that she succeeds on her first attempt is $\frac{3}{4}$. If she fails on her first attempt, the probability that she succeeds on her second attempt is $\frac{3}{8}$. If she fails on her first two attempts, the probability that she succeeds on her third attempt is $\frac{3}{16}$. Find the probability that she succeeds. **[3]**

(ii) Khaled is allowed two attempts to pass an examination. If he succeeds on his first attempt, he does not make a second attempt. The probability that he passes at the first attempt is 0.4 and the probability that he passes on either the first or second attempt is 0.58. Find the probability that he passes on the second attempt, given that he failed on the first attempt. **[3]**

Q2 Jan 2013

26.

(i) How many different 3-digit numbers can be formed using the digits 1, 2 and 3 when

(a) no repetitions are allowed, **[1]**

(b) any repetitions are allowed, **[2]**

(c) each digit may be included at most twice? **[2]**

(ii) How many different **4-digit** numbers can be formed using the digits 1, 2 and 3 when each digit may be included at most twice? **[5]**

Q4 Jan 2013

27.

The diagram shows five cards, each with a letter on it.

A	B	C	D	E

The letters A and E are vowels; the letters B, C and D are consonants.

(i) Two of the five cards are chosen at random, without replacement. Find the probability that they both have vowels on them. **[2]**

(ii) The two cards are replaced. Now three of the five cards are chosen at random, without replacement. Find the probability that they include exactly one card with a vowel on it. **[3]**

(iii) The three cards are replaced. Now four of the five cards are chosen at random without replacement. Find the probability that they include the card with the letter B on it. **[2]**

Q6 June 2013

28.

(i) A bag contains 12 black discs, 10 white discs and 5 green discs. Three discs are drawn at random from the bag, without replacement. Find the probability that all three discs are of different colours. **[3]**

(ii) A bag contains 30 red discs and 20 blue discs. A second bag contains 50 discs, each of which is either red or blue. A disc is drawn at random from each bag. The probability that these two discs are of different colours is 0.54. Find the number of red discs that were in the second bag at the start. **[4]**

Q8 June 2013

29.

A group of 8 people, including Kathy, David and Harpreet, are planning a theatre trip.

(i) Four of the group are chosen at random, without regard to order, to carry the refreshments. Find the probability that these 4 people include Kathy and David but not Harpreet. **[3]**

(ii) The 8 people sit in a row. Kathy and David sit next to each other and Harpreet sits at the left-hand end of the row. How many different arrangements of the 8 people are possible? **[3]**

(iii) The 8 people stand in a line to queue for the exit. Kathy and David stand next to each other and Harpreet stands next to them. How many different arrangements of the 8 people are possible? **[3]**

Q7 June 2014

30.

The table shows the numbers of members of a swimming club in certain categories.

	Male	Female
Adults	78	45
Children	52	n

It is given that $\frac{5}{8}$ of the female members are children.

(i) Find the value of n. **[2]**

(ii) Find the probability that a member chosen at random is either female or a child (or both). **[2]**

The table below shows the corresponding numbers for an athletics club.

	Male	Female
Adults	6	4
Children	5	10

(iii) Two members of the athletics club are chosen at random for a photograph.

(a) Find the probability that one of these members is a female child and the other is an adult male. **[2]**

(b) Find the probability that exactly one of these members is female and exactly one is a child. **[2]**

Q7 June 2014

31.

(i) The seven digits 1, 1, 2, 3, 4, 5, 6 are arranged in a random order in a line. Find the probability that they form the number 1 452 163. **[3]**

(ii) Three of the seven digits 1, 1, 2, 3, 4, 5, 6 are chosen at random, without regard to order.

(a) How many possible groups of three digits contain two 1s? **[1]**

(b) How many possible groups of three digits contain exactly one 1? **[2]**

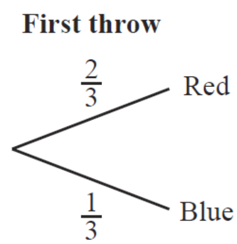
(c) How many possible groups of three digits can be formed altogether? **[2]**

Q6 June 2015

32.

A game is played with a fair, six-sided die which has 4 red faces and 2 blue faces. One turn consists of throwing the die repeatedly until a blue face is on top or until the die has been thrown 4 times.

(i) In the answer book, complete the probability tree diagram for one turn.



[2]

(ii) Find the probability that in one particular turn the die is thrown 4 times. **[2]**

(iii) Adnan and Beryl each have one turn. Find the probability that Adnan throws the die more times than Beryl. **[4]**

(iv) State one change that needs to be made to the rules so that the number of throws in one turn will have a geometric distribution. **[1]**

Q8 June 2015