Edexcel

Pure Mathematics

Year 2

Parametric Differentiation

Past paper questions from Core Maths 4 and IAL C34

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1. A curve has parametric equations

$$x = 2\cot t, \quad y = 2\sin^2 t, \quad 0 < t \leq \frac{\pi}{2}.$$

(*a*) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of the parameter t. **(4)**

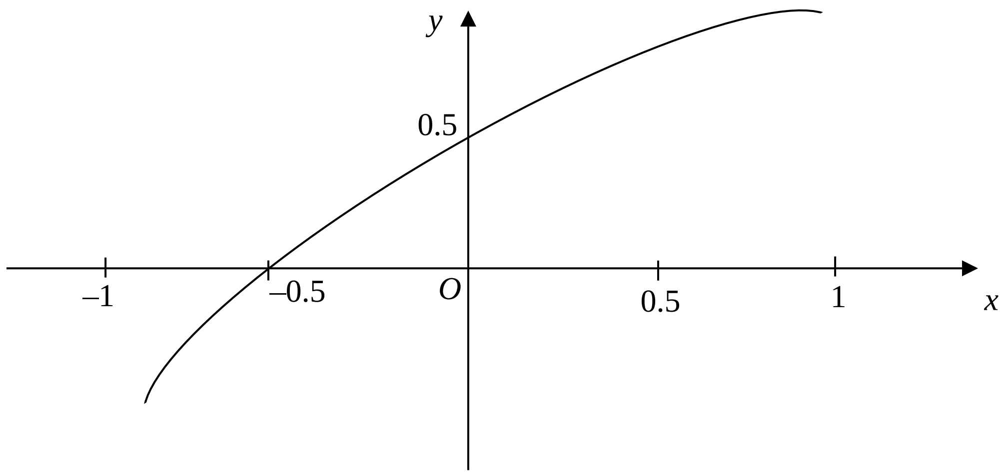
(*b*) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$. **(4)**

(*c*) Find a cartesian equation of the curve in the form $y = \mathrm{f}(x)$. State the domain on which the curve is defined. **(4)**

(C4 June 2005, Q6.)

2.

Figure 2



The curve shown in Figure 2 has parametric equations

$$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right), \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

(*a*) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{6}$.

(6)

(*b*) Show that a cartesian equation of the curve is

$$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1 - x^2)}, \qquad -1 < x < 1.$$

(3)

(C4 June 2006, Q4.)

3. A curve has parametric equations

$$x = 7\cos t - \cos 7t, \quad y = 7\sin t - \sin 7t, \quad \frac{\pi}{8} < t < \frac{\pi}{3}.$$

(*a*) Find an expression for $\frac{dy}{dx}$ in terms of t. You need not simplify your answer.

(3)

(*b*) Find an equation of the normal to the curve at the point where $t = \frac{\pi}{6}$.

Give your answer in its simplest exact form. **(6)**

(C4 Jan 2007, Q3.)

4. A curve has parametric equations

$$x = \tan^2 t, \quad y = \sin t, \quad 0 < t < \frac{\pi}{2}.$$

(*a*) Find an expression for $\frac{dy}{dx}$ in terms of t. You need not simplify your answer.

(3)

(*b*) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

Give your answer in the form $y = ax + b$, where a and b are constants to be determined. **(5)**

(*c*) Find a cartesian equation of the curve in the form $y^2 = f(x)$. **(4)**

(C4 June 2007, Q6.)

5.

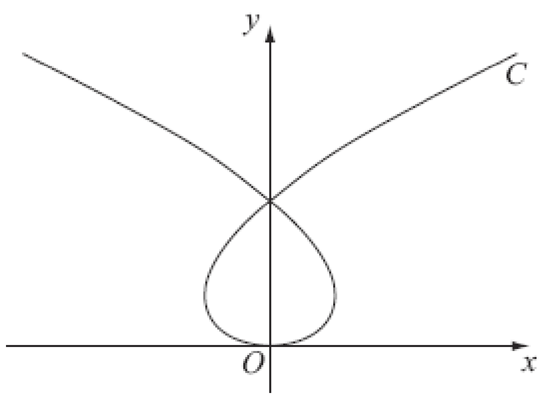


Figure 3

The curve C shown in Figure 3 has parametric equations

$$x = t^3 - 8t, \quad y = t^2$$

where t is a parameter. Given that the point A has parameter $t = -1$,

(*a*) find the coordinates of A. **(1)**

The line l is the tangent to C at A.

(*b*) Show that an equation for l is $2x - 5y - 9 = 0$. **(5)**

The line l also intersects the curve at the point B.

(*c*) Find the coordinates of B. **(6)**

(C4 Jan 2009, Q7.)

6.

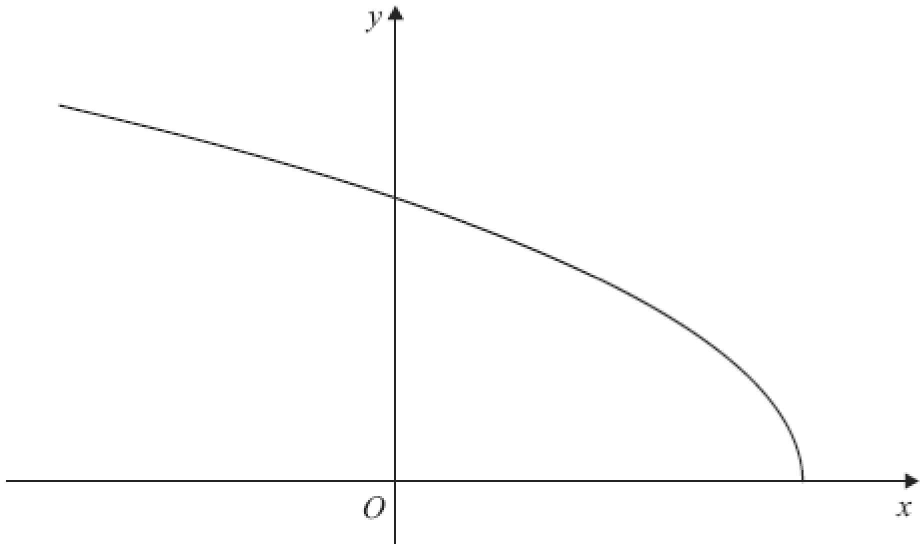


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2\cos 2t, \quad y = 6\sin t, \quad 0 \leq t \leq \frac{\pi}{2}.$$

(*a*) Find the gradient of the curve at the point where $t = \frac{\pi}{3}$.

(4)

(*b*) Find a Cartesian equation of the curve in the form

$$y = \text{f}(x), \quad -k \leq x \leq k,$$

Stating the value of the constant k.

(4)

(*c*) Write down the range of f(x).

(2)

(C4 June 2009, Q5.)

7. A curve C has parametric equations

$$x = \sin^2 t, y = 2\tan t, 0 \leq t < \frac{\pi}{2}.$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

(4)

The tangent to C at the point where $t = \frac{\pi}{3}$ cuts the x-axis at the point P.

(*b*) Find the x-coordinate of P.

(6)

(C4 June 2010, Q4.)

8. The curve C has parametric equations

$$x = \ln t, \qquad y = t^2 - 2, \qquad t > 0.$$

Find

(a) An equation of the normal to C at the point where $t = 3$,

(6)

(b) A Cartesian equation of C.

(3)

(C4 Jan 2011, Q6.)

9.

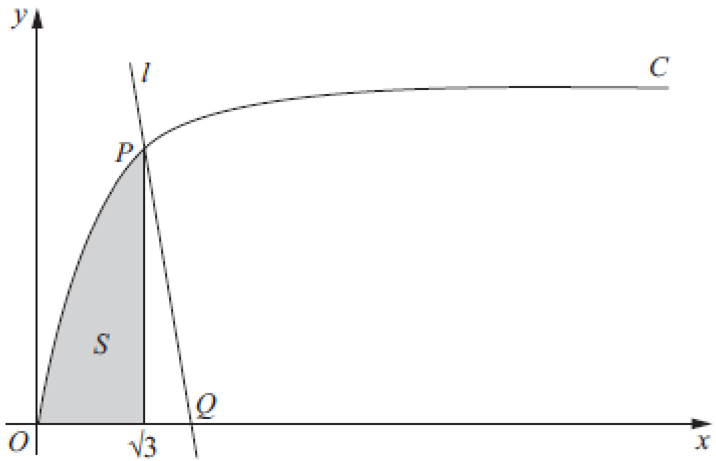


Figure 3

Figure 3 shows part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = \sin\theta, \qquad 0 \leq \theta < \frac{\pi}{2}.$$

The point P lies on C and has coordinates $\left(\sqrt{3},\ \frac{1}{2}\sqrt{3}\right)$.

(a) Find the value of θ at the point P.

(2)

The line l is a normal to C at P. The normal cuts the x-axis at the point Q.

(b) Show that Q has coordinates $(k\sqrt{3}, 0)$, giving the value of the constant k.

(6)

(C4 June 2011, Q7.)

10.

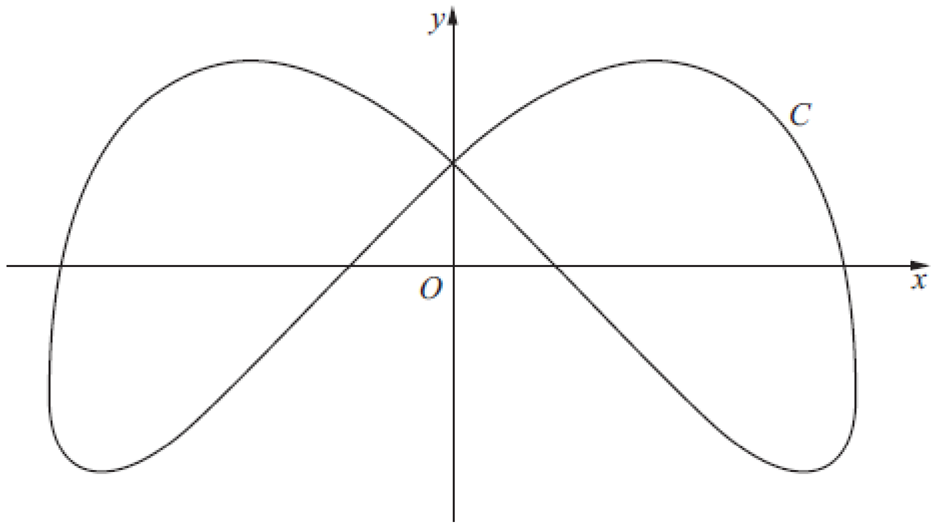


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\sin\left(t + \frac{\pi}{6}\right), \qquad y = 3\cos 2t, \qquad 0 \le t < 2\pi.$$

(*a*) Find an expression for $\frac{dy}{dx}$ in terms of t. **(3)**

(*b*) Find the coordinates of all the points on C where $\frac{dy}{dx} = 0$. **(5)**

(C4 Jan 2012, Q5.)

11.

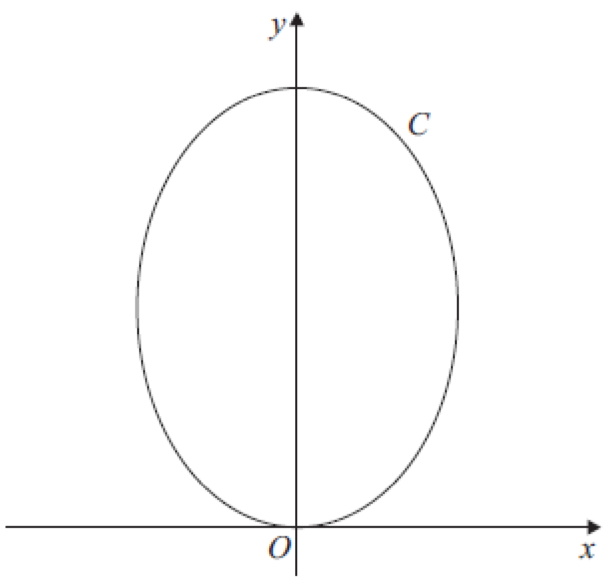


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = \sqrt{3}\sin 2t, \qquad y = 4\cos^2 t, \qquad 0 \leq t \leq \pi.$$

(a) Show that $\frac{dy}{dx} = k\sqrt{3}\tan 2t$, where k is a constant to be determined.

(5)

(b) Find an equation of the tangent to C at the point where $t = \frac{\pi}{3}$.

Give your answer in the form $y = ax + b$, where a and b are constants.

(4)

(c) Find a Cartesian equation of C.

(3)

(C4 June 2012, Q6.)

12.

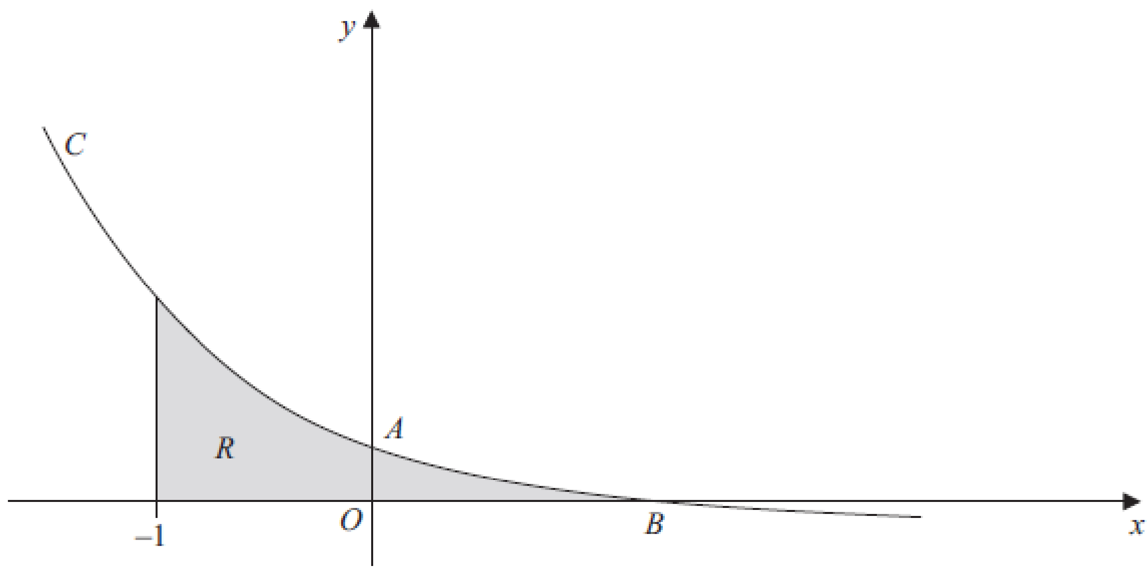


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \qquad y = 2^t - 1.$$

The curve crosses the y-axis at the point A and crosses the x-axis at the point B.

(a) Show that A has coordinates (0, 3).

(2)

(b) Find the x-coordinate of the point B.

(2)

(c) Find an equation of the normal to C at the point A.

(5)

(C4 Jan 2013, part of Q5.)

13. A curve C has parametric equations

$$x = 2\sin t, \qquad y = 1 - \cos 2t, \qquad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

(*a*) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at the point where $t = \dfrac{\pi}{6}$.

(4)

(*b*) Find a cartesian equation for C in the form

$$y = \mathrm{f}(x), -k \leq x \leq k,$$

stating the value of the constant k.

(3)

(*c*) Write down the range of f(x).

(2)

(C4 June 2013, Q4)

14.

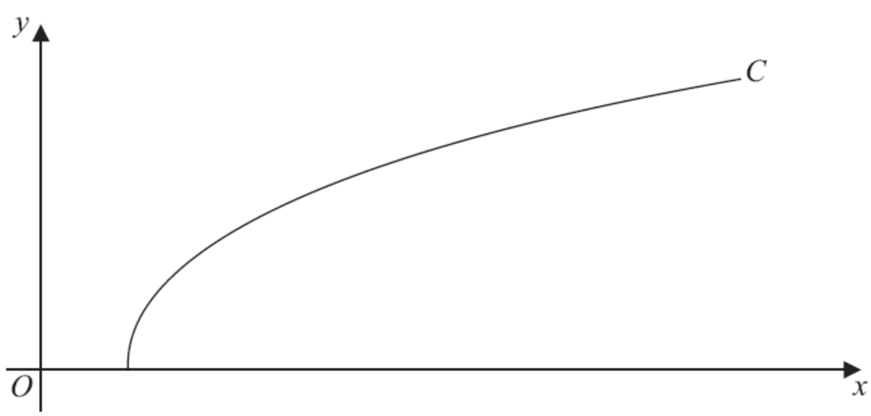


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 27\sec^3 t, \quad y = 3\tan t, \qquad 0 \leq t \leq \frac{\pi}{3}$$

(*a*) Find the gradient of the curve C at the point where $t = \dfrac{\pi}{6}$.

(4)

(*b*) Show that the cartesian equation of C may be written in the form

$$y = (x^{\frac{2}{3}} - 9)^{\frac{1}{2}}, \qquad a \leq x \leq b$$

stating values of a and b.

(3)

(C4 June 2013_R, part of Q7)

15.

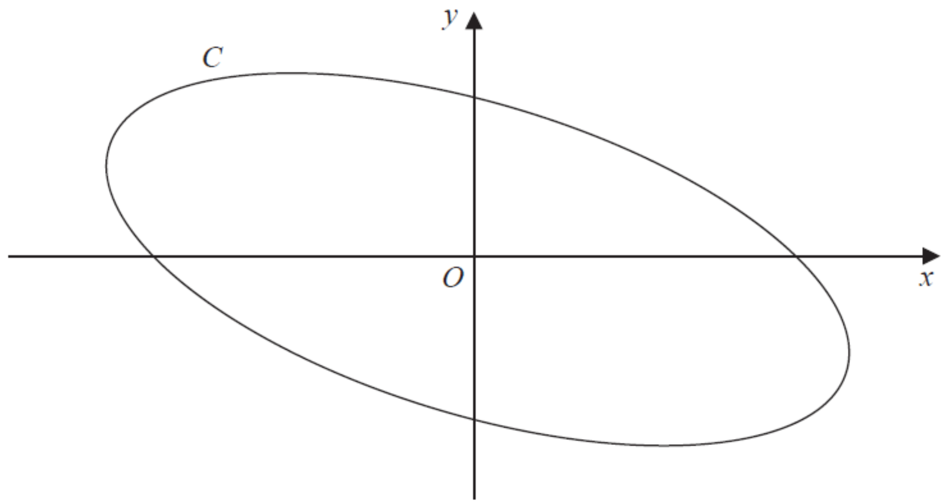


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \qquad y = 2\sin t, \qquad 0 \leq t \leq 2\pi$$

(a) Show that

$$x + y = 2\sqrt{3}\cos t$$ **(3)**

(b) Show that a cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be determined. **(2)**

(C4 June 2014, Q5)

16.

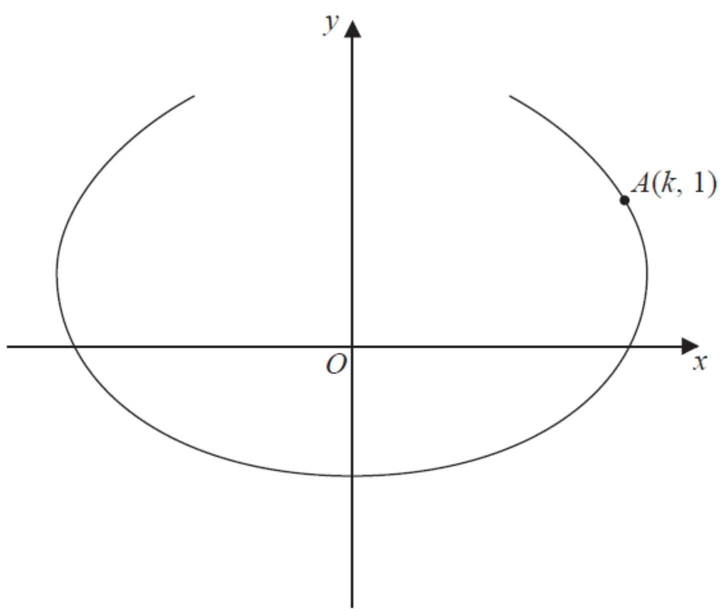


Figure 3

The curve shown in Figure 3 has parametric equations

$$x = t - 4 \sin t, \quad y = 1 - 2 \cos t, \qquad -\frac{2\pi}{3} \leq t \leq \frac{2\pi}{3}$$

The point A, with coordinates $(k, 1)$, lies on the curve.

Given that $k > 0$

(*a*) find the exact value of k,

(2)

(*b*) find the gradient of the curve at the point A.

(4)

There is one point on the curve where the gradient is equal to $-\frac{1}{2}$.

(*c*) Find the value of t at this point, showing each step in your working and giving your answer to 4 decimal places.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

(C4 June 2014_R, Q8)

17. A curve C has parametric equations

$$x = 4t + 3, \qquad y = 4t + 8 + \frac{5}{2t}, \qquad t \neq 0.$$

(*a*) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point on C where $t = 2$, giving your answer as a fraction in its simplest form.

(3)

(*b*) Show that the Cartesian equation of the curve C can be written in the form

$$y = \frac{x^2 + ax + b}{x - 3}, \qquad x \neq 3,$$

where a and b are integers to be determined.

(3)

(C4 June 2015, Q5)

18.

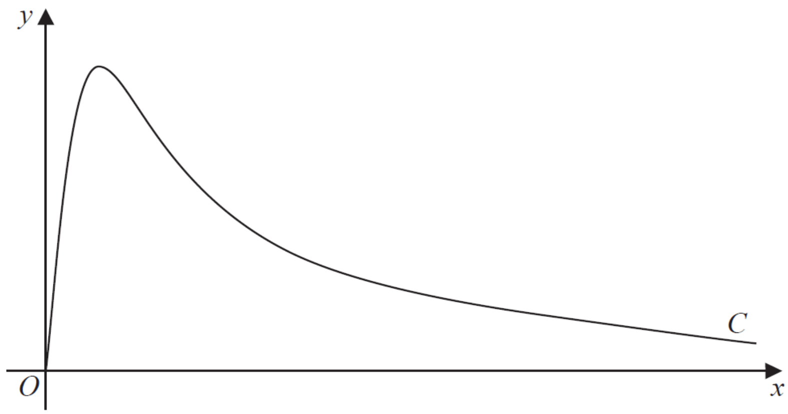


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\tan t, \qquad y = 5\sqrt{3}\sin 2t, \quad 0 \leq t < \frac{\pi}{2}.$$

The point P lies on C and has coordinates $\left(4\sqrt{3}, \frac{15}{2}\right)$.

(*a*) Find the exact value of $\frac{dy}{dx}$ at the point P.

Give your answer as a simplified surd. **(4)**

The point Q lies on the curve C, where $\frac{dy}{dx} = 0$.

(*b*) Find the exact coordinates of the point Q. **(2)**

(C4 June 2016, Q5)

19. The curve C has parametric equations

$$x = 3t - 4, \ y = 5 - \frac{6}{t}, \ t > 0$$

(*a*) Find $\frac{dy}{dx}$ in terms of t

(2)

The point P lies on C where $t = \frac{1}{2}$

(*b*) Find the equation of the tangent to C at the point P. Give your answer in the form $y = px + q$, where p and q are integers to be determined.

(3)

(*c*) Show that the cartesian equation for C can be written in the form

$$y = \frac{ax + b}{x + 4}, \qquad x > -4$$

where a and b are integers to be determined.

(3)

(C4 June 2017, Q1)

20.

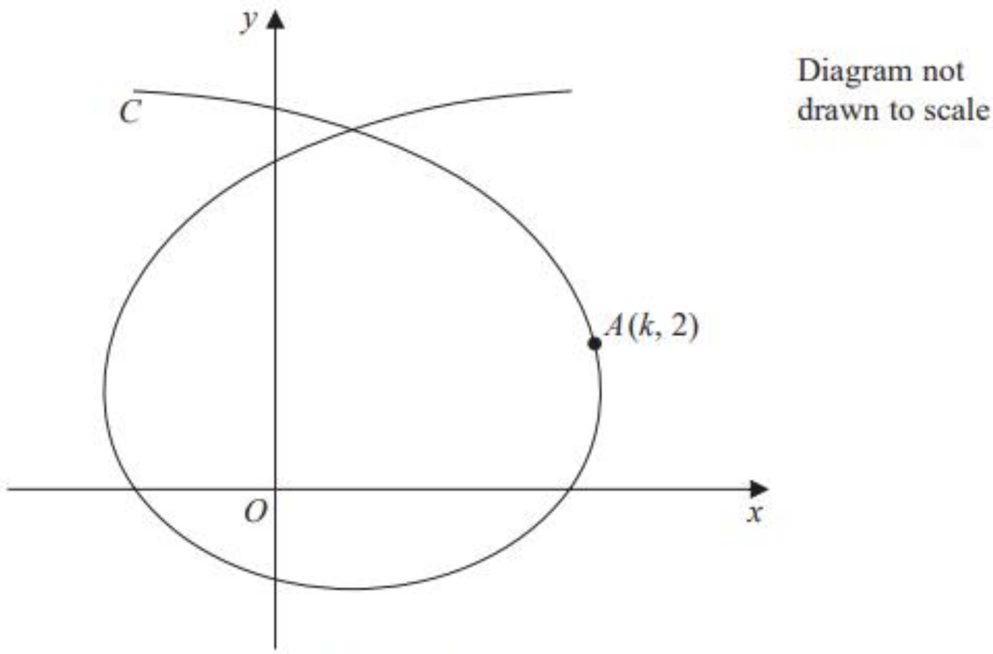


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 1 + t - 5\sin t, \quad y = 2 - 4\cos t, \quad -\pi \leqslant t \leqslant \pi$$

The point A lies on the curve C.

Given that the coordinates of A are $(k, 2)$, where $k > 0$

(a) find the exact value of k, giving your answer in a fully simplified form.

(2)

(b) Find the equation of the tangent to C at the point A.
Give your answer in the form $y = px + q$, where p and q are exact real values.

(5)

(C4 June 2018, Q5)

21. The curve C has parametric equations

$$x = -3 + 6\sin\theta \qquad y = 4\sqrt{3}\cos 2\theta \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

where θ is a parameter.

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ.

(2)

The curve C cuts the y-axis at the point A.

The line l is the normal to C at the point A.

(b) Show that an equation for l is

$$\sqrt{3}x - 4y + 8\sqrt{3} = 0$$

(6)

The line l intersects the curve C again at the point B.

(c) Find the coordinates of B. Give your answer in the form $(p, q\sqrt{3})$, where p and q are rational constants.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(C4 June 2019, Q7)

22. The curve C has parametric equations

$$x = 10\cos 2t, \quad y = 6\sin t, \qquad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

The point A with coordinates (5, 3) lies on C.

(*a*) Find the value of t at the point A.

(1)

(*b*) Show that an equation of the normal to C at A is

$$3y = 10x - 41$$

(6)

The normal to C at A cuts C again at the point B.

(*c*) Find the exact coordinates of B.

(8)

(IAL C34 June 2014, Q11)

23.

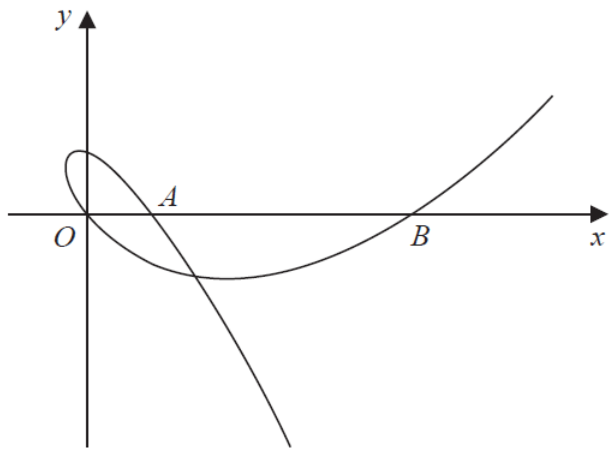


Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = t^2 + 2t, \qquad y = t^3 - 9t, \qquad t \in \mathbb{R}$$

The curve cuts the x-axis at the origin and at the points A and B as shown in Figure 3.

(a) Find the coordinates of point A and show that point B has coordinates (15, 0).

(3)

(b) Show that the equation of the tangent to the curve at B is $9x - 4y - 135 = 0$.

(5)

The tangent to the curve at B cuts the curve again at the point X.

(c) Find the coordinates of X.

(5)

(IAL C34 June 2015, Q9)

24. A curve C has parametric equations

$$x = 6\cos 2t, \qquad y = 2\sin t, \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

(a) Show that $\frac{dy}{dx} = \lambda \operatorname{cosec} t$, giving the exact value of the constant λ. **(4)**

(b) Find an equation of the normal to C at the point where $t = \frac{\pi}{3}$

Give your answer in the form $y = mx + c$, where m and c are simplified surds. **(6)**

The cartesian equation for the curve C can be written in the form

$$x = f(y), -k < y < k$$

where f(y) is a polynomial in y and k is a constant.

(c) Find f(y). **(3)**

(d) State the value of k. **(1)**

(IAL C34 Jan 2016, Q9)

25.

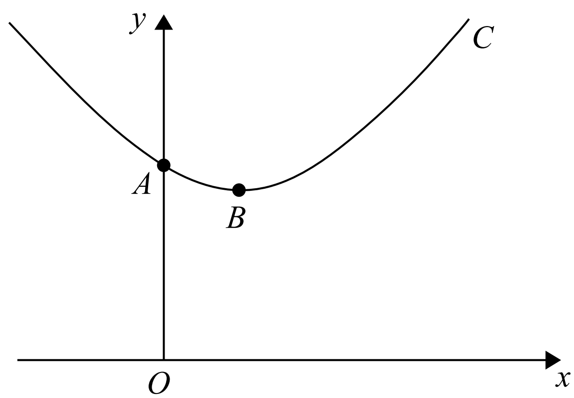


Figure 4

The curve C shown in Figure 4 has parametric equations

$$x = 1 + \sqrt{3}\tan\theta, \qquad y = 5\sec\theta, \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

The curve C crosses the y-axis at A and has a minimum turning point at B, as shown in Figure 4.

(a) Find the exact coordinates of A. **(3)**

(b) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \lambda \sin\theta$, giving the exact value of the constant λ. **(4)**

(c) Find the coordinates of B. **(2)**

(d) Show that the cartesian equation for the curve C can be written in the form

$$y = k\sqrt{\left(x^2 - 2x + 4\right)}$$

where k is a simplified surd to be found. **(3)**

(IAL C34 Jan 2017, Q13)

26.

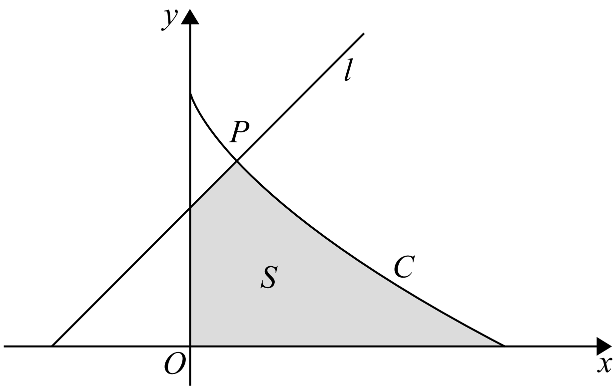


Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 8\cos^3\theta, \quad y = 6\sin^2\theta, \quad 0 \leqslant \theta \leqslant \frac{p}{2}$$

Given that the point P lies on C and has parameter $\theta = \frac{p}{3}$

(*a*) find the coordinates of P.

(2)

The line l is the normal to C at P.

(*b*) Show that an equation of l is $y = x + 3.5$

(5)

(IAL C34 June 2017, Q14)

27. A curve C has parametric equations

$$x = \frac{3}{2}t - 5, \quad y = 4 - \frac{6}{t} \qquad t \neq 0$$

(*a*) Find the value of $\frac{dy}{dx}$ at $t = 3$, giving your answer as a fraction in its simplest form.

(3)

(*b*) Show that a cartesian equation of C can be expressed in the form

$$y = \frac{ax + b}{x + 5} \qquad x \neq k$$

where a, b and k are integers to be found.

(4)

(IAL C34 June 2018, Q2)

28.

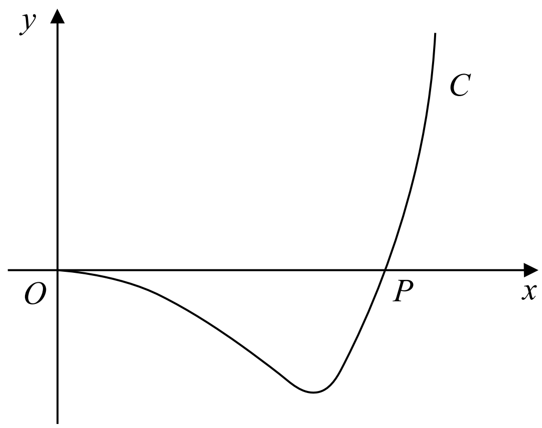


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = \frac{20t}{2t+1} \qquad y = t(t-4), \qquad t > 0$$

The curve cuts the x-axis at the point P.

(a) Find the x coordinate of P.

(2)

(b) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{(t-A)(2t+1)^2}{B}$ where A and B are constants to be found.

(5)

(c) (i) Make t the subject of the formula

$$x = \frac{20t}{2t+1}$$

(ii) Hence find a cartesian equation of the curve C. Write your answer in the form

$$y = \mathrm{f}(x), \qquad 0 < x < k$$

where $\mathrm{f}(x)$ is a single fraction and k is a constant to be found.

(6)

(IAL C34 Nov 2017, Q10)

29.

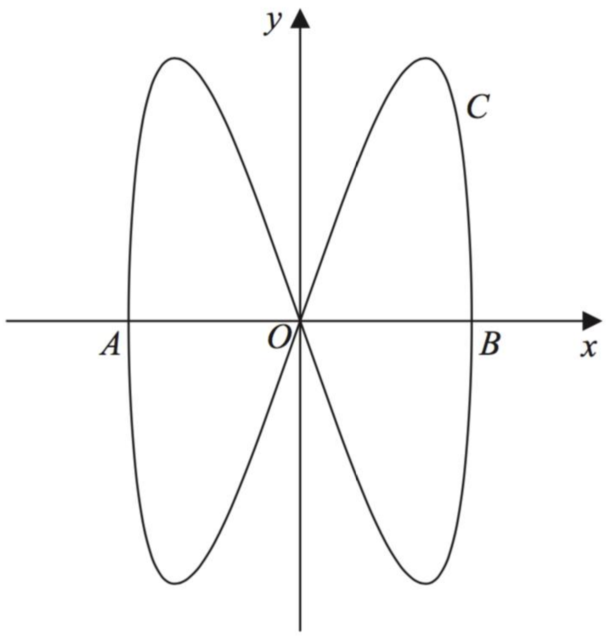


Figure 3

The curve C shown in Figure 3 has parametric equations

$$x = 3\cos t, \qquad y = 9\sin 2t, \qquad 0 \leqslant t \leqslant 2\pi$$

The curve C meets the x-axis at the origin and at the points A and B, as shown in Figure 3.

(a) Write down the coordinates of A and B.

(2)

(b) Find the values of t at which the curve passes through the origin.

(2)

(c) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t, and hence find the gradient of the curve when $t = \frac{p}{6}$

(4)

(d) Show that the cartesian equation for the curve C can be written in the form

$$y^2 = ax^2(b - x^2)$$

where a and b are integers to be determined.

(4)

(IAL C34 Jan 2018, Q11)

30. A curve has parametric equations

$$x = t^2 - t, \qquad y = \frac{4t}{1-t} \qquad t \neq 1$$

(*a*) Find $\frac{dy}{dx}$ in terms of t, giving your answer as a simplified fraction.

(4)

(*b*) Find an equation for the tangent to the curve at the point P where $t = -1$, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(4)

The tangent to the curve at P cuts the curve at the point Q.

(*c*) Use algebra to find the coordinates of Q.

(5)

(IAL C34 Jan 2019, Q8)

31.

. A curve C has parametric equations

$$x = \sqrt{3}\tan\theta, \qquad y = \sec^2\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{3}$$

The cartesian equation of C is

$$y = \mathrm{f}(x), \qquad 0 \leqslant x \leqslant k, \qquad \text{where } k \text{ is a constant}$$

(a) State the value of k.

(1)

(b) Find $\mathrm{f}(x)$ in its simplest form.

(2)

(c) Hence, or otherwise, find the gradient of the curve at the point where $\theta = \frac{\pi}{6}$

(3)

(IAL C34 June 2019, Q3)

32. The curve C_1 has parametric equations

$$x = t^2 - 1, \quad y = t^3 - t \qquad t \in \mathbb{R}$$

The line l is the normal to C_1 at the point where $t = 2$

(*a*) Show that an equation of l is

$$4x + 11y - 78 = 0$$

(5)

The curve C_2 has parametric equations

$$x = 12.5 + a\cos t, \quad y = 15 + a\sin t \qquad 0 \leq t < 2\pi$$

where a is a constant.

(*b*) Find the range of values of a for which the curve C_2 does not cross or touch the line l.

(5)

(IAL C34 Nov 2019, Q14)