Edexcel

Pure Mathematics

Year 2

Numerical Methods

Past paper questions from Core Maths 3 and IAL C34

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1. $$\mathrm{f}(x) = 3\mathrm{e}^x - \tfrac{1}{2}\ln x - 2, \quad x > 0.$$

(*a*) Differentiate to find $\mathrm{f}'(x)$.

(3)

The curve with equation $y = \mathrm{f}(x)$ has a turning point at P. The x-coordinate of P is α.

(*b*) Show that $\alpha = \tfrac{1}{6}\mathrm{e}^{-\alpha}$.

(2)

The iterative formula

$$x_{n+1} = \tfrac{1}{6}\mathrm{e}^{-x_n}, \quad x_0 = 1,$$

is used to find an approximate value for α.

(*c*) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

(2)

(*d*) By considering the change of sign of $\mathrm{f}'(x)$ in a suitable interval, prove that $\alpha = 0.1443$ correct to 4 decimal places.

(2)

[C3 2005 June Q4]

2. $$\mathrm{f}(x) = 2x^3 - x - 4.$$

(*a*) Show that the equation $\mathrm{f}(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}.$$

(3)

The equation $2x^3 - x - 4 = 0$ has a root between 1.35 and 1.4.

(*b*) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{1}{2}\right)},$$

with $x_0 = 1.35$, to find, to 2 decimal places, the value of x_1, x_2 and x_3.

(3)

The only real root of $\mathrm{f}(x) = 0$ is α.

(*c*) By choosing a suitable interval, prove that $\alpha = 1.392$, to 3 decimal places.

(3)

[C3 2006 Jan Q5]

3.

Figure 2

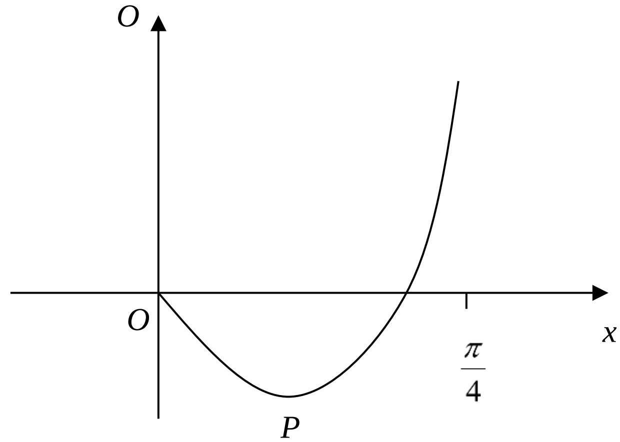

Figure 2 shows part of the curve with equation

$$y = (2x - 1)\tan 2x, \quad 0 \le x < \frac{\pi}{4}.$$

The curve has a minimum at the point P. The x-coordinate of P is k.

(a) Show that k satisfies the equation

$$4k + \sin 4k - 2 = 0.$$

(6)

The iterative formula

$$x_{n+1} = \tfrac{1}{4}(2 - \sin 4x_n), \quad x_0 = 0.3,$$

is used to find an approximate value for k.

(b) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimals places.

(3)

(c) Show that $k = 0.277$, correct to 3 significant figures.

(2)

[C3 2006 June Q5]

4. The function f is defined by

$$f : x \mapsto \ln(4 - 2x), \quad x < 2 \text{ and } x \in \mathbb{R}.$$

(*a*) Show that the inverse function of f is defined by

$$f^{-1} : x \mapsto 2 - \tfrac{1}{2}e^{x}$$

and write down the domain of f^{-1}.

(4)

(*b*) Write down the range of f^{-1}.

(1)

(*c*) Sketch the graph of $y = f^{-1}(x)$. State the coordinates of the points of intersection with the x and y axes.

(4)

The graph of $y = x + 2$ crosses the graph of $y = f^{-1}(x)$ at $x = k$.

The iterative formula

$$x_{n+1} = -\tfrac{1}{2}e^{x_n}, \quad x_0 = -0.3,$$

is used to find an approximate value for k.

(*d*) Calculate the values of x_1 and x_2, giving your answer to 4 decimal places.

(2)

(*e*) Find the values of k to 3 decimal places.

(2)

[C3 2007 Jan Q6]

5. $$f(x) = -x^3 + 3x^2 - 1.$$

(*a*) Show that the equation $f(x) = 0$ can be rewritten as

$$x = \sqrt{\left(\frac{1}{3-x}\right)}.$$

(2)

(*b*) Starting with $x_1 = 0.6$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{1}{3-x_n}\right)}$$

to calculate the values of x_2, x_3 and x_4, giving all your answers to 4 decimal places.

(2)

(*c*) Show that $x = 0.653$ is a root of $f(x) = 0$ correct to 3 decimal places.

(3)

[C3 2007 June Q4]

6. $$f(x) = \ln(x + 2) - x + 1, \qquad x > -2,\ x \in \mathbb{R}.$$

(*a*) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$.

(2)

(*b*) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \qquad x_0 = 2.5,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 5 decimal places.

(3)

(*c*) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places.

(2)

[C3 2008 Jan Q3]

7. $$f(x) = 3x^3 - 2x - 6.$$

(*a*) Show that $f(x) = 0$ has a root, α, between $x = 1.4$ and $x = 1.45$.

(2)

(*b*) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0.$$

(3)

(*c*) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(*d*) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.

(3)

[C3 2008 June Q7]

8. $$f(x) = 3xe^x - 1.$$

The curve with equation $y = f(x)$ has a turning point P.

(*a*) Find the exact coordinates of P.

(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$.

(*b*) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}.$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3.

`

(3)

(*c*) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)

[C3 2009 Jan Q7]

9.

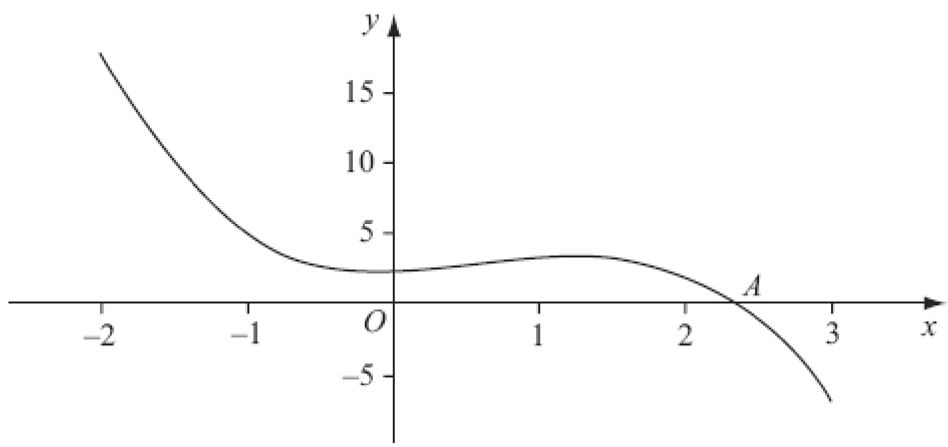

Figure 1

shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the x-axis at the point A where $x = \alpha$.

To find an approximation to α, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2 \qquad \text{is used.}$$

(*a*) Taking $x_0 = 2.5$, find the values of x_1, x_2, x_3 and x_4.
Give your answers to 3 decimal places where appropriate.

(3)

(*b*) Show that $\alpha = 2.359$ correct to 3 decimal places.

(3)

[C3 2009 June Q1]

10. $$f(x) = x^3 + 2x^2 - 3x - 11$$

(*a*) Show that f(x) = 0 can be rearranged as

(2)

$$x = \sqrt{\left(\frac{3x+11}{x+2}\right)}, \quad x \neq -2.$$

The equation f(x) = 0 has one positive root α.

The iterative formula $x_{n+1} = \sqrt{\left(\frac{3x_n+11}{x_n+2}\right)}$ is used to find an approximation to α .

(*b*) Taking $x_1 = 0$, find, to 3 decimal places, the values of x_2 , x_3 and x_4 .

(3)

(*c*) Show that α = 2.057 correct to 3 decimal places.

(3)

[C3 2010 Jan Q2]

11. $$f(x) = 4 \operatorname{cosec} x - 4x + 1, \quad \text{where } x \text{ is in radians.}$$

(*a*) Show that there is a root α of f(x) = 0 in the interval [1.2, 1.3].

(2)

(*b*) Show that the equation f(x) = 0 can be written in the form

$$x = \frac{1}{\sin x} + \frac{1}{4}$$

(2)

(*c*) Use the iterative formula

$$x_{n+1} = \frac{1}{\sin x_n} + \frac{1}{4}, \quad x_0 = 1.25,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(*d*) By considering the change of sign of f(x) in a suitable interval, verify that α = 1.291 correct to 3 decimal places.

(2)

[C3 2010 June Q3]

12.

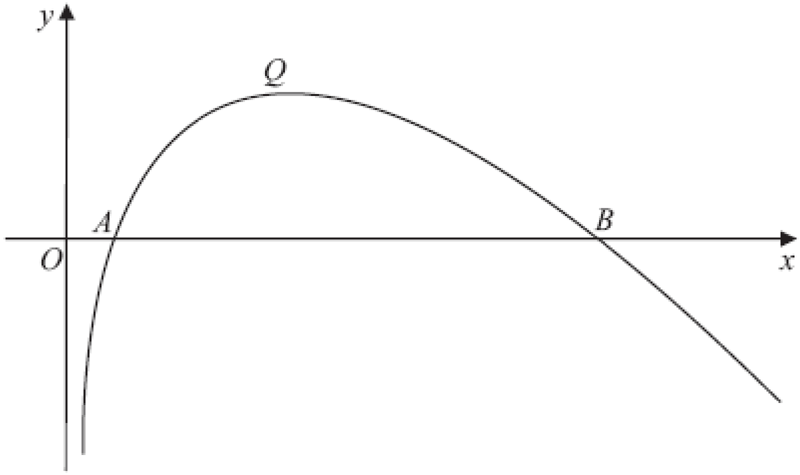

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (8 - x) \ln x, \qquad x > 0.$$

The curve cuts the x-axis at the points A and B and has a maximum turning point at Q, as shown in Figure

(*a*) Write down the coordinates of A and the coordinates of B.

(2)

(*b*) Find f '(x).

(3)

(*c*) Show that the x-coordinate of Q lies between 3.5 and 3.6

(2)

(*d*) Show that the x-coordinate of Q is the solution of

$$x = \frac{8}{1 + \ln x}.$$

(3)

To find an approximation for the x-coordinate of Q, the iteration formula

$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(*e*) Taking $x_0 = 3.55$, find the values of x_1, x_2 and x_3.

Give your answers to 3 decimal places.

(3)

[C3 2011 Jan Q5]

13. $$f(x) = 2\sin(x^2) + x - 2, \qquad 0 \leq x < 2\pi.$$

(a) Show that f $(x) = 0$ has a root α between $x = 0.75$ and $x = 0.85$.

(2)

The equation f $(x) = 0$ can be written as $x = \left[\arcsin(1 - 0.5x)\right]^{\frac{1}{2}}$.

(b) Use the iterative formula

$$x_{n+1} = \left[\arcsin(1 - 0.5x_n)\right]^{\frac{1}{2}}, \qquad x_0 = 0.8,$$

to find the values of x_1, x_2 and x_3, giving your answers to 5 decimal places.

(3)

(c) Show that $\alpha = 0.80157$ is correct to 5 decimal places.

(3)

[C3 2011 June Q2]

14. $$f(x) = x^2 - 3x + 2\cos\left(\tfrac{1}{2}x\right), \qquad 0 \leq x \leq \pi.$$

(a) Show that the equation f$(x) = 0$ has a solution in the interval $0.8 < x < 0.9$.

(2)

The curve with equation $y =$ f(x) has a minimum point P.

(b) Show that the x-coordinate of P is the solution of the equation

$$x = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}.$$

(4)

(c) Using the iteration formula

$$x_{n+1} = \frac{3 + \sin\left(\frac{1}{2}x_n\right)}{2}, \qquad x_0 = 2,$$

find the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(d) By choosing a suitable interval, show that the x-coordinate of P is 1.9078 correct to 4 decimal places. **(3)**

[C3 2012Jan Q6]

15. $$f(x) = x^3 + 3x^2 + 4x - 12$$

(*a*) Show that the equation f(*x*) = 0 can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)}, \quad x \neq -3.$$

(3)

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2.

(*b*) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, \quad n \geq 0,$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3.

(3)

The root of f(*x*) = 0 is α.

(*c*) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

(3)

[C3 2012June Q2]

16. $$g(x) = e^{x-1} + x - 6$$

(*a*) Show that the equation g(*x*) = 0 can be written as

$$x = \ln(6 - x) + 1, \qquad x < 6.$$

(2)

The root of g(*x*) = 0 is α.

The iterative formula

$$x_{n+1} = \ln(6 - x_n) + 1, \qquad x_0 = 2.$$

is used to find an approximate value for α.

(*b*) Calculate the values of x_1, x_2 and x_3 to 4 decimal places.

(3)

(*c*) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places.

(3)

[C3 2013Jan Q2]

17. $$f(x) = 25x^2e^{2x} - 16, \qquad x \in \mathbb{R}$$

(*a*) Using calculus, find the exact coordinates of the turning points on the curve with equation $y = f(x)$.

(5)

(*b*) Show that the equation $f(x) = 0$ can be written as $x = \pm\frac{4}{5}e^{-x}$.

(1)

The equation $f(x) = 0$ has a root α, where $\alpha = 0.5$ to 1 decimal place.

(*c*) Starting with $x_0 = 0.5$, use the iteration formula

$$x_{n+1} = \frac{4}{5}e^{-x_n}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(*d*) Give an accurate estimate for α to 2 decimal places, and justify your answer.

(2)

[C3 2013June Q2]

18.

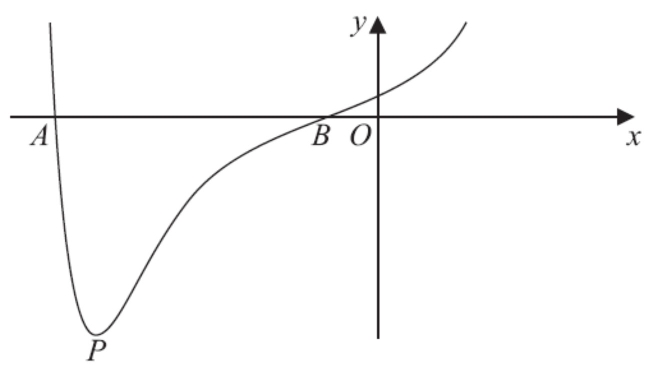

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = (x^2 + 3x + 1)\,e^{x^2}$$

The curve cuts the x-axis at points A and B as shown in Figure 2.

(*a*) Calculate the x-coordinate of A and the x-coordinate of B, giving your answers to 3 decimal places.

(2)

(*b*) Find $f'(x)$.

(3)

The curve has a minimum turning point P as shown in Figure 2.

(c) Show that the x-coordinate of P is the solution of

$$x = -\frac{3(2x^2+1)}{2(x^2+2)}$$

(3)

(d) Use the iteration formula

$$x_{n+1} = -\frac{3(2x_n^2+1)}{2(x_n^2+2)}, \quad \text{with } x_0 = -2.4,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

The x-coordinate of P is α.

(e) By choosing a suitable interval, prove that $\alpha = -2.43$ to 2 decimal places.

(2)

[C3 2013June Q2_R]

19.

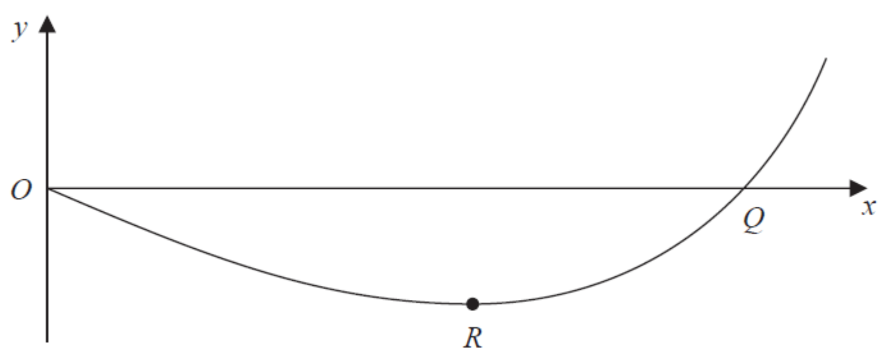

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 2\cos\left(\frac{1}{2}x^2\right) + x^3 - 3x - 2$$

The curve crosses the x-axis at the point Q and has a minimum turning point at R.

(a) Show that the x coordinate of Q lies between 2.1 and 2.2.

(2)

(b) Show that the x coordinate of R is a solution of the equation

$$x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$$

(4)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{2}{3}x_n \sin\left(\frac{1}{2}x_n{}^2\right)}, \qquad x_0 = 1.3$$

(*c*) find the values of x_1 and x_2 to 3 decimal places.

(2)

[C3 2014June Q6]

20. A curve C has equation $y = e^{4x} + x^4 + 8x + 5$.

(*a*) Show that the x coordinate of any turning point of C satisfies the equation

$$x^3 = -2 - e^{4x}$$

(3)

(*b*) On a pair of axes, sketch, on a single diagram, the curves with equations

(i) $y = x^3$,

(ii) $y = -2 - e^{4x}$

On your diagram give the coordinates of the points where each curve crosses the y-axis and state the equation of any asymptotes.

(4)

(*c*) Explain how your diagram illustrates that the equation $x^3 = -2 - e^{4x}$ has only one root.

(1)

The iteration formula

$$x_{n+1} = \left(-2 - e^{4x_n}\right)^{\frac{1}{3}}, \qquad x_0 = -1$$

can be used to find an approximate value for this root.

(*d*) Calculate the values of x_1 and x_2, giving your answers to 5 decimal places.

(2)

(*e*) Hence deduce the coordinates, to 2 decimal places, of the turning point of the curve C.

(2)

[C3 2014June Q2_R]

21.

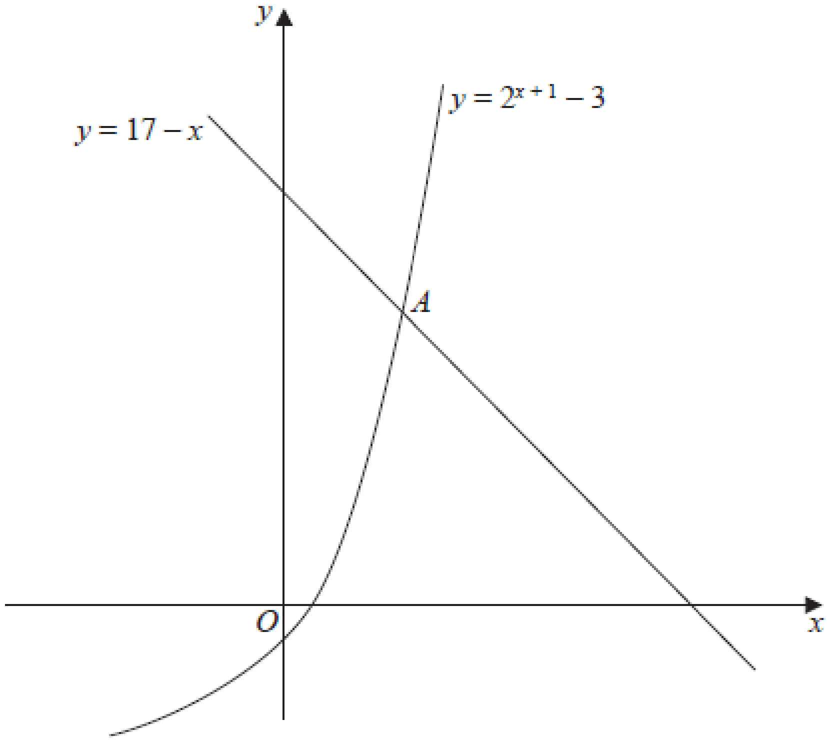

Figure 1

Figure 1 is a sketch showing part of the curve with equation $y = 2^{x+1} - 3$ and part of the line with equation $y = 17 - x$.

The curve and the line intersect at the point A.

(*a*) Show that the x-coordinate of A satisfies the equation

$$x = \frac{\ln(20 - x)}{\ln 2} - 1.$$

(3)

(*b*) Use the iterative formula

$$x_{n+1} = \frac{\ln(20 - x_n)}{\ln 2} - 1, \qquad x_0 = 3,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(*c*) Use your answer to part (*b*) to deduce the coordinates of the point A, giving your answers to one decimal place.

(2)

[C3 2015 June Q6]

22.

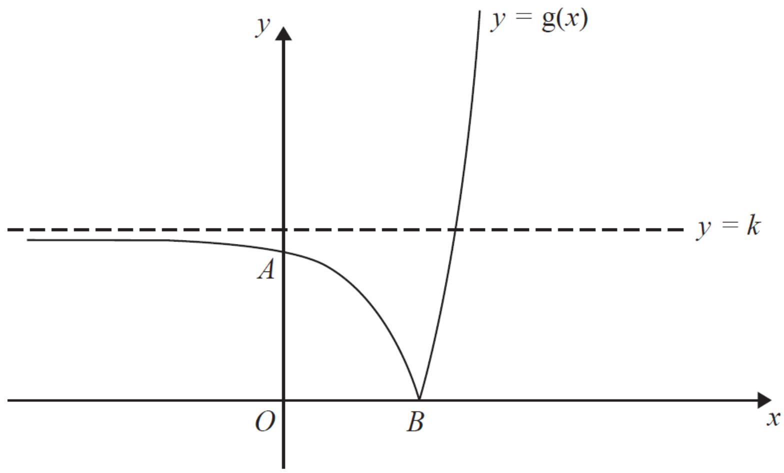

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = |4e^{2x} - 25|, \qquad x \in \mathbb{R}.$$

The curve cuts the y-axis at the point A and meets the x-axis at the point B. The curve has an asymptote $y = k$, where k is a constant, as shown in Figure 1.

(*a*) Find, giving each answer in its simplest form,

(i) the y coordinate of the point A,

(ii) the exact x coordinate of the point B,

(iii) the value of the constant k.

(5)

The equation $\mathrm{g}(x) = 2x + 43$ has a positive root at $x = \alpha$.

(*b*) Show that α is a solution of $x = \frac{1}{2}\ln\left(\frac{1}{2}x + 17\right)$.

(2)

The iteration formula

$$x_{n+1} = \frac{1}{2}\ln\left(\frac{1}{2}x_n + 17\right)$$

can be used to find an approximation for α.

(*c*) Taking $x_0 = 1.4$, find the values of x_1 and x_2. Give each answer to 4 decimal places.

(2)

(d) By choosing a suitable interval, show that $\alpha = 1.437$ to 3 decimal places.

(2)

[C3 2016 June Q4]

23.

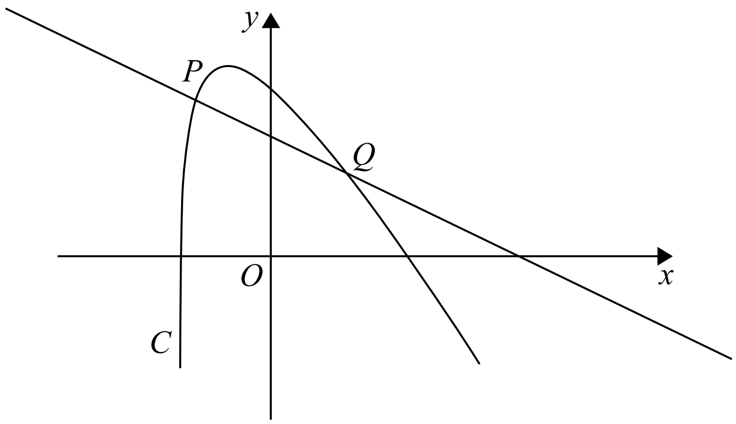

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = 2\ln(2x + 5) - \frac{3x}{2}, \qquad x > -2.5$$

The point P with x coordinate –2 lies on C.

(*a*) Find an equation of the normal to C at P. Write your answer in the form $ax + by = c$, where a, b and c are integers.

(5)

The normal to C at P cuts the curve again at the point Q, as shown in Figure 2.

(*b*) Show that the x coordinate of Q is a solution of the equation

$$x = \frac{20}{11}\ln(2x + 5) - 2$$

(3)

The iteration formula

$$x_{n+1} = \frac{20}{11}\ln(2x_n + 5) - 2$$

can be used to find an approximation for the x coordinate of Q.

(*c*) Taking $x_1 = 2$, find the values of x_2 and x_3, giving each answer to 4 decimal places.

(2)

[C3 2017 June Q5]

24. $$f(x) = 2x^3 + x - 10$$

(*a*) Show that the equation f(*x*) = 0 has a root *α* in the interval [1.5, 2].

(2)

The only real root of f(*x*) = 0 is *α*.

The iterative formula

$$x_{n+1} = \left(5 - \frac{1}{2}x_n\right)^{\frac{1}{3}}, \qquad x_0 = 1.5$$

can be used to find an approximate value for *α*.

(*b*) Calculate x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(*c*) By choosing a suitable interval, show that *α* = 1.6126 correct to 4 decimal places.

(2)

[2014 June, IAL Q1]

25. $$f(x) = -x^3 + 4x^2 - 6$$

(*a*) Show that the equation f(*x*) = 0 has a root between *x* = 1 and *x* = 2

(2)

(*b*) Show that the equation f(*x*) = 0 can be rewritten as

$$x = \sqrt{\left(\frac{6}{4-x}\right)}$$

(2)

(*c*) Starting with $x_1 = 1.5$ use the iteration $x_{n+1} = \sqrt{\left(\frac{6}{4-x_n}\right)}$ to calculate the values of x_2, x_3 and x_4 giving all your answers to 4 decimal places.

(3)

(*d*) Using a suitable interval, show that 1.572 is a root of f(*x*) = 0 correct to 3 decimal places.

(2)

[2016 Jan, IAL Q5]

26.

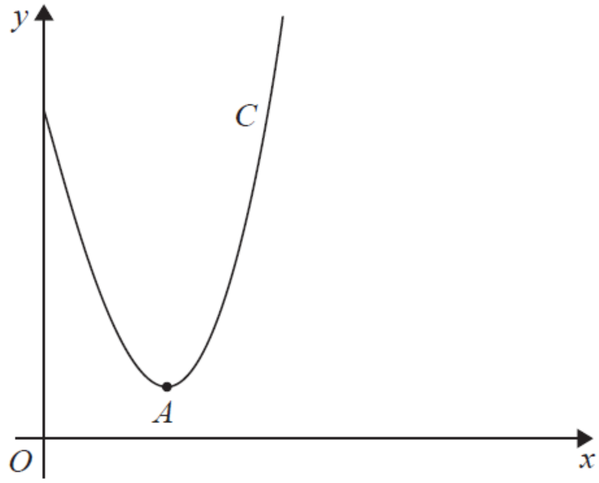

Figure 3

Figure 3 shows a sketch of part of the curve *C* with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 4, \quad x > 0$$

Point *A* is the minimum turning point on the curve.

(*a*) Show, by using calculus, that the *x* coordinate of point *A* is a solution of

$$x = \frac{6}{1 + \ln(x^2)}$$

(5)

(*b*) Starting with $x_0 = 2.27$, use the iteration

$$x_{n+1} = \frac{6}{1 + \ln(x_n^{\ 2})}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(*c*) Use your answer to part (*b*) to deduce the coordinates of point *A* to one decimal place.

(2)

[2015 Jan, IAL Q10]

27. (*a*) Given that $-\frac{\pi}{2} < \text{g}(x) < \frac{\pi}{2}$, sketch the graph of $y = \text{g}(x)$ where

$$\text{g}(x) = \arctan x, \qquad x \in \mathbb{R}$$

(2)

(*b*) Find the exact value of x for which

$$3\text{g}(x + 1) - \pi = 0$$

(3)

The equation $\arctan x - 4 + \frac{1}{2}x = 0$ has a positive root at $x = \alpha$ radians.

(*c*) Show that $5 < \alpha < 6$

(2)

The iteration formula

$$x_{n+1} = 8 - 2 \arctan x_n$$

can be used to find an approximation for α

(*d*) Taking $x_0 = 5$, use this formula to find x_1 and x_2, giving each answer to 3 decimal places.

(2)

[2016 June, IAL Q10]

28.

$$\text{f}(x) = x^3 - 5x + 16$$

(*a*) Show that the equation $\text{f}(x) = 0$ can be rewritten as

$$x = (ax + b)^{\frac{1}{3}}$$

giving the values of the constants a and b.

(2)

The equation f $(x) = 0$ has exactly one real root α, where $\alpha = -3$ to one significant figure.

(*b*) Starting with $x_1 = -3$, use the iteration

$$x_{n+1} = (ax_n + b)^{\frac{1}{3}}$$

with the values of a and b found
in part (*a*), to calculate the values of x_2, x_3 and x_4,
giving all your answers to 3 decimal places.

(3)

(*c*) Using a suitable interval, show that $\alpha = -3.17$ correct to 2 decimal places.

(2)

[2017 Jan, IAL Q2]

29. $$y = (2x^2 - 3)\tan\left(\frac{1}{2}x\right), \qquad 0 < x < \pi$$

(*a*) Find the exact value of x when $y = 0$

(1)

Given that $\frac{dy}{dx} = 0$ when $x = \alpha$,

(*b*) show that

$$2\alpha^2 - 3 + 4\alpha\sin\alpha = 0$$

(6)

The iterative formula

$$x_{n+1} = \frac{3}{(2x_n + 4\sin x_n)}$$

can be used to find an approximation for α.

(*c*) Taking $x_1 = 0.7$, find the values of x_2 and x_3, giving each answer to 4 decimal places.

(2)

(*d*) By choosing a suitable interval, show that $\alpha = 0.7283$ to 4 decimal places.

(2)

[2017 June, IAL Q11]

30. $$f(x) = x^5 + x^3 - 12x^2 - 8, \qquad x \in \mathbb{R}$$

(*a*) Show that the equation f $(x) = 0$ can be written as

$$x = \sqrt[3]{\frac{4(3x^2 + 2)}{x^2 + 1}}$$

(3)

(*b*) Use the iterative formula

$$x_{n+1} = \sqrt[3]{\frac{4(3x_n^2 + 2)}{x_n^2 + 1}}$$

with $x_0 = 2$, to find x_1, x_2 and x_3 giving your answers to 3 decimal places.

(3)

The equation f $(x) = 0$ has a single root, α.

(*c*) By choosing a suitable interval, prove that $\alpha = 2.247$ to 3 decimal places.

(2)

[2017 Oct, IAL Q1]

31. $$f(x) = \frac{x^2}{4} + \ln(2x), \qquad x > 0$$

(*a*) Show that the equation f(*x*) = 0 can be rewritten as

$$x = \frac{1}{2}e^{-\frac{1}{4}x^2}$$

(2)

The equation f(*x*) = 0 has a root near 0.5

(*b*) Starting with $x_1 = 0.5$ use the iterative formula

$$x_{n+1} = \frac{1}{2}e^{-\frac{1}{4}x_n^2}$$

to calculate the values of x_2 , x_3 and x_4, giving your answers to 4 decimal places.

(3)

(*c*) Using a suitable interval, show that 0.473 is a root of f(*x*) = 0 correct to 3 decimal places.

(2)

[2018 Jan, IAL Q3]

32. $$f(x) = 2^{x-1} - 4 + 1.5x \qquad x \in \mathbb{R}$$

(*a*) Show that the equation f(*x*) = 0 can be written as

$$x = \frac{1}{3}(8 - 2^x)$$

(2)

The equation f(*x*) = 0 has a root *α*, where *α* = 1.6 to one decimal place.

(*b*) Starting with $x_0 = 1.6$, use the iteration formula

$$x_{n+1} = \frac{1}{3}(8 - 2^{x_n})$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(*c*) By choosing a suitable interval, prove that *α* = 1.633 to 3 decimal places.

(2)

[2018 June, IAL Q3]

33.

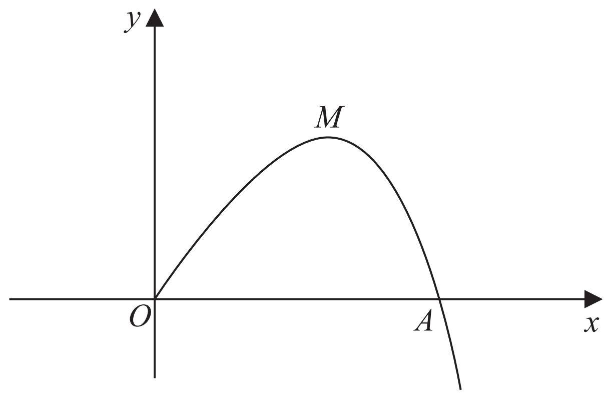

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = 8x - xe^{3x}$, $x \geqslant 0$

The curve meets the x-axis at the origin and cuts the x-axis at the point A.

(*a*) Find the exact x coordinate of A, giving your answer in its simplest form. **(2)**

The curve has a maximum turning point at the point M.

(*b*) Show, by using calculus, that the x coordinate of M is a solution of

$$x = \frac{1}{3}\ln\left(\frac{8}{1+3x}\right)$$ **(5)**

(*c*) Use the iterative formula

$$x_{n+1} = \frac{1}{3}\ln\left(\frac{8}{1+3x_n}\right)$$

with $x_0 = 0.4$ to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places. **(3)**

[2018 Oct, IAL Q4]

34. (*a*) Given that $0 \leqslant f(x) \leqslant \pi$, sketch the graph of $y = f(x)$ where

$$f(x) = \arccos(x - 1), \qquad 0 \leqslant x \leqslant 2$$ **(2)**

The equation $\arccos(x - 1) - \tan x = 0$ has a single root α.

(*b*) Show that $0.9 < \alpha < 1.1$ **(2)**

The iteration formula

$$x_{n+1} = \arctan(\arccos(x_n - 1))$$

can be used to find an approximation for α.

(*c*) Taking $x_0 = 1.1$ find, to 3 decimal places, the values of x_1 and x_2 **(2)**

[2019 Jan, IAL Q11]

35.

$$f(x) = 2x^3 + x - 20$$

(a) Show that the equation $f(x) = 0$ can be rewritten as

$$x = \sqrt[3]{a - bx}$$

where a and b are positive constants to be determined.

(2)

(b) Starting with $x_1 = 2.1$ use the iteration formula $x_{n+1} = \sqrt[3]{a - bx_n}$, with the numerical values of a and b, to calculate the values of x_2 and x_3 giving your answers to 3 decimal places.

(2)

(c) Using a suitable interval, show that 2.077 is a root of the equation $f(x) = 0$ correct to 3 decimal places.

(2)

(d) Hence state a root, to 3 decimal places, of the equation

$$2(x + 2)^3 + x - 18 = 0$$

(1)

[2019 June, IAL Q1]

36. The curve C has equation

$$y = x\cos 2x \qquad 0 \leq x \leq \frac{\pi}{4}$$

The curve has a turning point at the point P.

(*a*) Show, using calculus, that the x coordinate of P is a solution of the equation

$$x = \frac{1}{2}\arctan\left(\frac{1}{2x}\right)$$

(4)

(*b*) Starting with $x_0 = 0.5$ use

$$x_{n+1} = \frac{1}{2}\arctan\left(\frac{1}{2x_n}\right)$$

to calculate the value of x_1 and the value of x_2, giving your answers to 4 decimal places.

(3)

[2019 Oct, IAL Q4]

37.

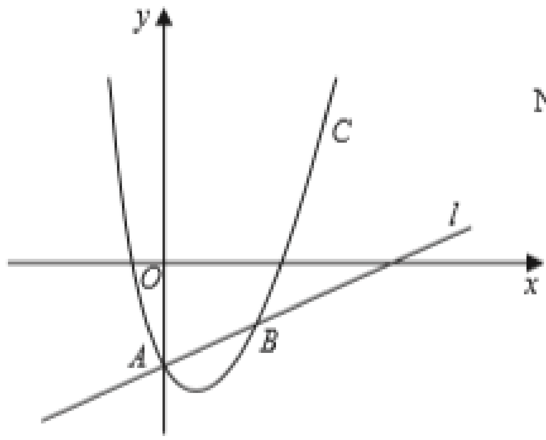

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = e^{-2x} + x^2 - 3$$

The curve C crosses the y-axis at the point A.

The line l is the normal to C at the point A.

(a) Find the equation of l, writing your answer in the form $y = mx + c$, where m and c are constants.

(5)

The line l meets C again at the point B, as shown in Figure 1.

(b) Show that the x coordinate of B is a solution of

$$x = \sqrt{1 + \frac{1}{2}x - e^{-2x}}$$

(2)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{1}{2}x_n - e^{-2x_n}}$$

with $x_1 = 1$

(c) find x_2 and x_3 to 3 decimal places.

(2)

[2018 June, C3 Q4]

38.

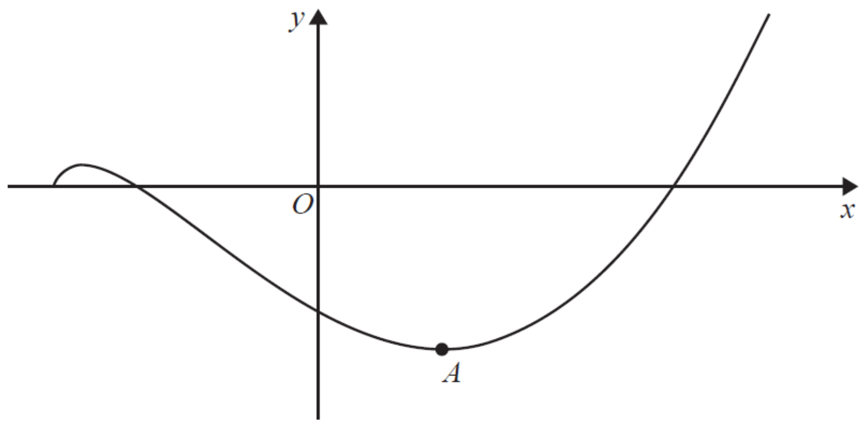

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (x^2 - x - 12)\ln(x+3), \qquad x \in \mathbb{R}, x > -3$$

(*a*) Find $\mathrm{f}'(x)$.

(2)

The curve has a minimum turning point at *A*.

(*b*) Show that the *x* coordinate of *A* is a solution of the equation

$$x = \frac{\ln(x+3)+4}{2\ln(x+3)+1}$$

(3)

(*c*) Use the iteration formula

$$x_{n+1} = \frac{\ln(x_n+3)+4}{2\ln(x_n+3)+1}$$

with $x_0 = 1$ to find the values of x_1, x_2 and x_3 giving your answers to 3 decimal places.

(3)

A different curve with equation $y = 2\mathrm{f}(x) + k$, where *k* is a constant, passes through the origin.

(*d*) Find the exact value of *k*.

(2)

[2019 June, C3 Q6]