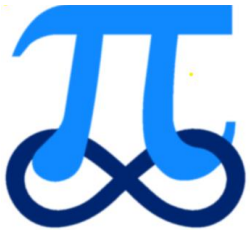
Edexcel

Pure Mathematics

Year 2

Newton-Raphson Methods

Past paper questions from FP1and IAL F1



Edited by: K V Kumaran

1. $f(x) = x^3 + x - 3.$

The equation $f(x) = 0$ has a root, α, between 1 and 2.

(*a*) By considering $f'(x)$, show that α is the only real root of the equation $f(x) = 0$.

(3)

(*b*) Taking 1.2 as your first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 significant figures.

(2)

(*c*) Prove that your answer to part (*b*) gives the value of α correct to 3 significant figures.

(2)

[P4 January 2002 Qn 4]

2. $f(x) = x^3 + 8x - 19.$

(*a*) Show that the equation $f(x) = 0$ has only one real root.

(3)

(*b*) Show that the real root of $f(x) = 0$ lies between 1 and 2.

(2)

(*c*) Obtain an approximation to the real root of $f(x) = 0$ by performing two applications of the Newton-Raphson procedure to $f(x)$, using $x = 2$ as the first approximation. Give your answer to 3 decimal places.

(4)

(*d*) By considering the change of sign of $f(x)$ over an appropriate interval, show that your answer to part (*c*) is accurate to 3 decimal places.

(2)

[FP1 June 2007 Qn 4]

3. $f(x) = 4\cos x + e^{-x}$.

(*a*) Show that the equation $f(x) = 0$ has a root α between 1.6 and 1.7

(2)

(*b*) Taking 1.6 as your first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 significant figures. **(4)**

[FP1 June 2008Qn 2]

4. $$f(x) = 3\sqrt{x} + \frac{18}{\sqrt{x}} - 20.$$

(*a*) Show that the equation f(x) = 0 has a root α in the interval [1.1, 1.2].

(2)

(*b*) Find f ′(x).

(3)

(*c*) Using x_0 = 1.1 as a first approximation to α, apply the Newton-Raphson procedure once to f(x) to find a second approximation to α, giving your answer to 3 significant figures.

(4)

[FP1 January 2009 Qn 5]

5. Given that α is the only real root of the equation

$$x^3 - x^2 - 6 = 0,$$

(*a*) show that 2.2 < α < 2.3

(2)

(*b*) Taking 2.2 as a first approximation to α, apply the Newton-Raphson procedure once to f(x) = $x^3 - x^2 - 6$ to obtain a second approximation to α, giving your answer to 3 decimal places.

(5)

(*c*) Use linear interpolation once on the interval [2.2, 2.3] to find another approximation to α, giving your answer to 3 decimal places.

(3)

[FP1 June 2009 Qn 4]

6. $$f(x) = x^3 - \frac{7}{x} + 2, \; x > 0.$$

(*a*) Show that f (x) = 0 has a root α between 1.4 and 1.5. **(2)**

(*b*) Taking 1.45 as a first approximation to α, apply the Newton-Raphson procedure once to f(x) = $x^3 - \frac{7}{2} + 2$, $x > 0$ to obtain a second approximation to α, giving your answer to 3 decimal places. **(5)**

[FP1 June 2010 Qn 3]

7. $f(x) = 5x^2 - 4x^{\frac{3}{2}} - 6, \quad x \geq 0.$

The root α of the equation f $(x) = 0$ lies in the interval [1.6,1.8].

(a) Use linear interpolation once on the interval [1.6, 1.8] to find an approximation to α.

Give your answer to 3 decimal places.

(4)

(b) Differentiate f(x) to find f $'(x)$.

(2)

(c) Taking 1.7 as a first approximation to α, apply the Newton-Raphson process once to f(x) to obtain a second approximation to α. Give your answer to 3 decimal places.

(4)

[FP1 January 2011 Qn 3]

8. $f(x) = x^2 + \frac{5}{2x} - 3x - 1, \quad x \neq 0.$

(a) Use differentiation to find f $'(x)$.

(2)

The root α of the equation f(x) = 0 lies in the interval [0.7, 0.9].

(b) Taking 0.8 as a first approximation to α, apply the Newton-Raphson process once to f(x) to obtain a second approximation to α. Give your answer to 3 decimal places.

(4)

[FP1 June 2011 Qn 4]

9. (a) Show that $f(x) = x^4 + x - 1$ has a real root α in the interval [0.5, 1.0].

(2)

(b) Taking 0.75 as a first approximation, apply the Newton Raphson process twice to f(x) to obtain an approximate value of α. Give your answer to 3 decimal places.

(5)

[FP1 January 2012 Qn 3]

10. $$f(x) = x^2 + \frac{3}{4\sqrt{x}} - 3x - 7, \quad x > 0.$$

A root α of the equation f(x) = 0 lies in the interval [3, 5].

Taking 4 as a first approximation to α, apply the Newton-Raphson process once to f(x) to obtain a second approximation to α. Give your answer to 2 decimal places.

(6)

[FP1 June 2012 Qn 3]

11. $$f(x) = 2x^{\frac{1}{2}} + x^{-\frac{1}{2}} - 5, \qquad x > 0.$$

(a) Find f′(x).

(2)

The equation f(x) = 0 has a root α in the interval [4.5, 5.5].

(b) Using $x_0 = 5$ as a first approximation to α, apply the Newton-Raphson procedure once to f(x) to find a second approximation to α, giving your answer to 3 significant figures.

(4)

[FP1 Jan 2013 Qn 3]

12. $$f(x) = \frac{1}{2}x^4 - x^3 + x - 3$$

(a) Show that the equation f(x) = 0 has a root α between x = 2 and x = 2.5.

(2)

The equation f(x) = 0 has a root β in the interval [–2, –1].

(b) Taking –1.5 as a first approximation to β, apply the Newton-Raphson process once to f(x) to obtain a second approximation to β.

Give your answer to 2 decimal places.

(5)

[FP1_R June 2013 Qn 3]

13.

$$f(x) = x^3 - \frac{5}{2x^{\frac{3}{2}}} + 2x - 3, \qquad x > 0$$

(*a*) Show that the equation f(x) = 0 has a root α in the interval [1.1, 1.5].

(2)

(*b*) Find f′(x).

(2)

(*c*) Using x_0 = 1.1 as a first approximation to α, apply the Newton-Raphson procedure once to f(x) to find a second approximation to α, giving your answer to 3 decimal places.

(3)

[FP1 June 2014 Qn 2]

14. f(x) = $3x^{\frac{3}{2}} - 25x^{-\frac{1}{2}} - 125$ $x > 0$.

(*a*) Find f′(x)

(2)

The equation f(x) = 0 has a root α in the interval [12, 13].

(*b*) Using x_0 = 12.5 as a first approximation to α, apply the Newton-Raphson procedure once to f(x) to find a second approximation to α, giving your answer to 3 decimal places.

(4)

[FP1 June 2016 Qn 2]

15.

$$f(x) = \frac{1}{3}x^2 + \frac{4}{x^2} - 2x - 1, \quad x > 0$$

(*a*) Show that the equation f(x) = 0 has a root α in the interval [6,7]

(2)

(*b*) Taking 6 as a first approximation to α, apply the Newton-Raphson process once to f (x) to obtain a second approximation to α. Give your answer to 2 decimal places.

(5)

[FP1 June 2017 Qn 1]

16. $$f(x) = \frac{3}{2}x^2 + \frac{4}{3x} + 2x - 5, \qquad x < 0$$

The equation f $(x) = 0$ has a single root α.

(a) Show that α lies in the interval $[-3, -2.5]$

(2)

(b) Taking -3 as a first approximation to α, apply the Newton-Raphson procedure once to f (x) to obtain a second approximation to α. Give your answer to 3 decimal places.

(5)

[FP1 June 2018 Qn 2]

17. (i) $$f(x) = 3x^2 - \frac{1}{2\sqrt{x}} - 5x, \qquad x > 0$$

(a) Show that the equation f $(x) = 0$ has a root α in the interval $[1, 2]$.

(2)

(b) Find f′ (x).

(2)

(c) Using $x_0 = 2$ as a first approximation to α, apply the Newton-Raphson procedure once to f (x) to find a second approximation to α, giving your answer to 2 decimal places.

(2)

[FP1 June 2019 Qn 4]

18. $$f(x) = 6\sqrt{x} - x^2 - \frac{1}{2x}, \qquad x > 0$$

(a) Show that the equation f$(x) = 0$ has a root α in the interval $[3, 4]$.

(2)

(b) Taking 3 as a first approximation to α, apply the Newton-Raphson process once to f(x) to obtain a second approximation to α. Give your answer to 3 decimal places.

(5)

[IAL F1 Jan 2014 Qn 1]

19. $$f(x) = x^{\frac{3}{2}} - 3x^{\frac{1}{2}} - 3, \qquad x > 0$$

Given that α is the only real root of the equation f $(x) = 0$,

(a) show that $4 < \alpha < 5$.

(2)

(b) Taking 4.5 as a first approximation to α, apply the Newton-Raphson procedure once to f(x) to find a second approximation to α, giving your answer to 3 decimal places.

(5)

[IAL F1 June 2014 Qn 4]

20. $$f(x) = x^3 - 3x^2 + \frac{1}{2\sqrt{x^5}} + 2, \quad x > 0$$

(*a*) Show that the equation f(*x*) = 0 has a root α in the interval [2,3].

(2)

(*b*) Taking 3 as a first approximation to α, apply the Newton-Raphson process once to f(*x*) to find a second approximation to α. Give your answer to 3 decimal places.

(5)

[IAL F1 Jan 2015 Qn 2]

21. $$f(x) = x^2 - \frac{3}{\sqrt{x}} - \frac{4}{3x^2}, \quad x > 0$$

(*a*) Show that the equation f(*x*) = 0 has a root α in the interval [1.6, 1.7]

(2)

(*b*) Taking 1.6 as a first approximation to α, apply the Newton-Raphson process once to f(*x*) to find a second approximation to α. Give your answer to 3 decimal places.

(5)

[IAL F1 Jan 2016 Qn 2]

22.

$$f(x) = x^2 + \frac{3}{x} - 1, \quad x < 0$$

The only real root, α, of the equation f(*x*) = 0 lies in the interval [−2, −1].

(*a*) Taking −1.5 as a first approximation to α, apply the Newton-Raphson procedure once to f(*x*) to find a second approximation to α, giving your answer to 2 decimal places.

(5)

(*b*) Show that your answer to part (*a*) gives α correct to 2 decimal places.

(2)

[IAL F1 May 2016 Qn 3]

23.

$$f(x) = x^3 - \frac{1}{2x} + x^{\frac{3}{2}}, \quad x > 0,$$

The root α of the equation f(*x*) = 0 lies in the interval [0.6, 0.7].

(*a*) Taking 0.6 as a first approximation to α, apply the Newton-Raphson process once to f(*x*) to obtain a second approximation to α. Give your answer to 3 decimal places.

(5)

(*b*) Show that your answer to part (*a*) is correct to 3 decimal places.

(2)

[IAL F1 Jan 2017 Qn 6]

24.

$$f(x) = 30 + \frac{7}{\sqrt{x}} - x^5, \qquad x > 0$$

Taking 2 as a first approximation to α, apply the Newton-Raphson process once to f (x) to find a second approximation to α, giving your answer to 2 decimal places.

(5)

[IAL F1 May 2017 Qn 5]

25.

$$f(x) = 3x^2 - \frac{5}{3\sqrt{x}} - 6, \qquad x > 0$$

The single root α of the equation f $(x) = 0$ lies in the interval [1.5, 1.6].

(a) Taking 1.5 as a first approximation to α, apply the Newton-Raphson process once to f (x) to obtain a second approximation to α. Give your answer to 3 decimal places.

(4)

26.

$$f(x) = x^3 - \frac{1}{2x} + x^{\frac{3}{2}}, \quad x > 0,$$

The root α of the equation f$(x) = 0$ lies in the interval [0.6, 0.7].

(a) Taking 0.6 as a first approximation to α, apply the Newton-Raphson process once to f(x) to obtain a second approximation to α. Give your answer to 3 decimal places.

(5)

(b) Show that your answer to part (a) is correct to 3 decimal places.

(2)

[IAL F1 Jan 2018 Qn 6]

27.

$$f(x) = \frac{2(x^3+3)}{\sqrt{x}} - 9, \qquad x > 0$$

The equation f$(x) = 0$ has two real roots α and β, where $0.4 < \alpha < 0.5$ and $1.2 < \beta < 1.3$

(a) Taking 0.45 as a first approximation to α, apply the Newton-Raphson procedure once to f (x) to find a second approximation to α, giving your answer to 3 decimal places.

(5)

[IAL F1 May 2018 Qn 6]

28.

$$f(x) = 2x^3 - \frac{7}{x^2} + 16, \qquad x \neq 0$$

The equation f $(x) = 0$ also has a single root β in the interval [0.6, 0.7].

Taking 0.65 as a first approximation to β, apply the Newton-Raphson procedure once to f (x) to obtain a second approximation to β, giving your answer to 4 decimal places.

(4)

[IAL F1 Jan 2019 Qn 4]

29.

$$f(x) = 5 + 4x^2 - \frac{4}{3}x^3 - \frac{7}{2x} \quad x > 0$$

(*a*) Find $f'(x)$

(2)

A root α of the equation f$(x) = 0$ lies in the interval [0.5, 0.6].

(*b*)Using 0.5 as a first approximation to α, apply the Newton-Raphson process once to f(x) to find a second approximation to α. Give your answer to 3 decimal places.

(3)

(*c*) Show that the equation f$(x) = 0$ has a root β in the interval [3, 3.5].

(2)

[IAL F1 May 2019 Qn 1]