OCR Core Maths 2

Past paper questions Logarithms

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Exponentials & Logarithms

- (In these notes if I write $\log x$ I *mean* $\log_{10} x$. If I mean a different base, I will write it *explicitly* as $\log_a x$. When we see $\log_a x$ we say "log to the base a of x".)
- We *define* a logarithm to be the solution (in x) to the equation $a^x = b$. It is written $x = \log_a b$. The fundamental relationship is therefore

$$a^x = b \qquad \Leftrightarrow \qquad x = \log_a b. \qquad\qquad \dagger$$

- From † we see that $\log_a b$ *means*: "The number a has to be raised to, to make b". Therefore some simple logarithms can be calculated without a calculator:

$$\begin{aligned}
\log_2 8 &= \text{“the number 2 has to be raised to, to make 8”} &&= 3,\\
\log_{10} 10000 &= \text{“the number 10 has to be raised to, to make 10000”} &&= 4,\\
\log_9 3 &= \text{“the number 9 has to be raised to, to make 3”} &&= \tfrac{1}{2}, \quad (\because 3 = \sqrt{9} = 9^{\frac{1}{2}})\\
\log_a a &= \text{“the number } a \text{ has to be raised to, to make } a\text{”} &&= 1.
\end{aligned}$$

- We see from † that logarithms 'pluck out powers' from equations. Therefore if you ever see an equation with the unknown in the power, then that is the clue that you will need to use logarithms. For example to solve $7^{2x-1} = 22$ we discover

$$\begin{aligned}
7^{2x-1} &= 22,\\
2x - 1 &= \log_7 22,\\
x &= \tfrac{1}{2}\log_7 22 + \tfrac{1}{2}.
\end{aligned}$$

  However we need to build on † because not all equations are this simple (e.g. $3 \times 2^{2x-1} = 5 \times 7^{x+1}$) and not all calculators can calculate $\log_7 22$.
- You can also use † to eliminate logarithms from an equation. Given an equation of the form $\log_a(\cdots) = b$, you can eliminate the logarithm instantly to get $(\cdots) = a^b$. A good way to remember this[2] is 'Girvan's Bullying Base'. So if we have $\log_3 x = 8$, then the bullying base '3' knocks the log out of the way and moves to the other side and squeezes up the 8 to put it in its place; therefore $x = 3^8$.
- Logarithms and exponentials (powers) are the inverse functions of each other (as can be seen from † if one puts one into the other). Therefore

$$\log_{10} 10^x = x \quad \text{and} \quad 10^{\log_{10} x} = x.$$

  So if $\log a = 5.4$ then $a = 10^{5.4}$.

- There are some rules that can be derived from † that *must* be learnt. They are (for all bases):

$$\log(ab) = \log a + \log b \qquad \log 1 = 0$$
$$\log(\frac{a}{b}) = \log a - \log b \qquad \log_a a = 1$$
$$\log(a^n) = n \log a \qquad \log_a b = \frac{\log_c b}{\log_c a}.$$
$$\log(\frac{1}{a}) = -\log a$$

- When we need to solve an equation where the unknown is in the exponent such as $5^{2x-1} = 8$ take $\log_{10}$ of both sides and simplify:

$$5^{2x-1} = 8$$
$$\log_{10}(5^{2x-1}) = \log_{10} 8$$
$$(2x-1)\log_{10} 5 = \log_{10} 8$$
$$2x - 1 = \frac{\log_{10} 8}{\log_{10} 5}$$
$$x = \frac{1}{2} \times \left(\frac{\log_{10} 8}{\log_{10} 5} + 1\right)$$
$$x = 1.15 \text{ (3sf)}.$$

1.

(i) Evaluate $\log_5 15 + \log_5 20 - \log_5 12$. [3]

(ii) Given that $y = 3 \times 10^{2x}$, show that $x = a\log_{10}(by)$, where the values of the constants a and b are to be found. [4]

Q7 June 2005

2.

(i) Express each of the following in terms of $\log_{10} x$ and $\log_{10} y$.

(a) $\log_{10}\left(\frac{x}{y}\right)$ [1]

(b) $\log_{10}(10x^2y)$ [3]

(ii) Given that

$$2\log_{10}\left(\frac{x}{y}\right) = 1 + \log_{10}(10x^2y),$$

find the value of y correct to 3 decimal places. [4]

Q7 Jan 2006

3.

(i) Sketch the curve $y = \left(\frac{1}{2}\right)^x$, and state the coordinates of any point where the curve crosses an axis. [3]

(ii) Use the trapezium rule, with 4 strips of width 0.5, to estimate the area of the region bounded by the curve $y = \left(\frac{1}{2}\right)^x$, the axes, and the line $x = 2$. [4]

(iii) The point P on the curve $y = \left(\frac{1}{2}\right)^x$ has y-coordinate equal to $\frac{1}{6}$. Prove that the x-coordinate of P may be written as

$$1 + \frac{\log_{10} 3}{\log_{10} 2}.$$ [4]

Q9 June 2006

4.

(a) **(i)** Express $\log_3(4x + 7) - \log_3 x$ as a single logarithm. [1]

(ii) Hence solve the equation $\log_3(4x + 7) - \log_3 x = 2$. [3]

(b) Use the trapezium rule, with two strips of width 3, to find an approximate value for

$$\int_3^9 \log_{10} x \, dx,$$

giving your answer correct to 3 significant figures. [4]

Q5 Jan 2007

5.

Use logarithms to solve the equation $3^{2x+1} = 5^{200}$, giving the value of x correct to 3 significant figures. [5]

Q3 June 2007

6.

The polynomial $f(x)$ is given by

$$f(x) = x^3 + 6x^2 + x - 4.$$

(i) **(a)** Show that $(x + 1)$ is a factor of $f(x)$. [1]

(b) Hence find the exact roots of the equation $f(x) = 0$. [6]

(ii) **(a)** Show that the equation

$$2\log_2(x + 3) + \log_2 x - \log_2(4x + 2) = 1$$

can be written in the form $f(x) = 0$. [5]

(b) Explain why the equation

$$2\log_2(x + 3) + \log_2 x - \log_2(4x + 2) = 1$$

has only one real root and state the exact value of this root. [2]

Q9 June 2007

7.

Express each of the following as a single logarithm:

(i) $\log_a 2 + \log_a 3$, [1]

(ii) $2\log_{10} x - 3\log_{10} y$. [3]

Q3 Jan 2008

8.

(i) Sketch the curve $y = 2 \times 3^x$, stating the coordinates of any intersections with the axes. [3]

(ii) The curve $y = 2 \times 3^x$ intersects the curve $y = 8^x$ at the point P. Show that the x-coordinate of P may be written as

$$\frac{1}{3 - \log_2 3}.$$ [5]

Q8 June 2008

9.

(a) Given that $\log_a x = p$ and $\log_a y = q$, express the following in terms of p and q.

(i) $\log_a(xy)$ **[1]**

(ii) $\log_a\left(\frac{a^2x^3}{y}\right)$ **[3]**

(b) **(i)** Express $\log_{10}(x^2 - 10) - \log_{10} x$ as a single logarithm. **[1]**

(ii) Hence solve the equation $\log_{10}(x^2 - 10) - \log_{10} x = 2\log_{10} 3$. **[5]**

Q8 Jan 2009

10.

Use logarithms to solve the equation $7^x = 2^{x+1}$, giving the value of x correct to 3 significant figures. **[5]**

Q3 June 2009

11.

(i) Sketch the curve $y = 6 \times 5^x$, stating the coordinates of any points of intersection with the axes. **[3]**

(ii) The point P on the curve $y = 9^x$ has y-coordinate equal to 150. Use logarithms to find the x-coordinate of P, correct to 3 significant figures. **[3]**

(iii) The curves $y = 6 \times 5^x$ and $y = 9^x$ intersect at the point Q. Show that the x-coordinate of Q can be written as $x = \frac{1 + \log_3 2}{2 - \log_3 5}$. **[5]**

Q9 Jan 2010

12.

(a) Use logarithms to solve the equation $5^{3w-1} = 4^{250}$, giving the value of w correct to 3 significant figures. **[5]**

(b) Given that $\log_x(5y + 1) - \log_x 3 = 4$, express y in terms of x. **[4]**

Q10 June 2010

13.

(a) Use logarithms to solve the equation $5^{x-1} = 120$, giving your answer correct to 3 significant figures. **[4]**

(b) Solve the equation $\log_2 x + 2\log_2 3 = \log_2(x + 5)$. **[4]**

Q4 Jan 2011

14.

(a) Use logarithms to solve the equation $7^{w-3} - 4 = 180$, giving your answer correct to 3 significant figures. **[4]**

(b) Solve the simultaneous equations

$$\log_{10}x + \log_{10}y = \log_{10}3, \qquad \log_{10}(3x + y) = 1.$$ **[6]**

Q8 Jan 2012

15.

(a) An arithmetic progression has first term $\log_2 27$ and common difference $\log_2 x$.

(i) Show that the fourth term can be written as $\log_2 (27x^3)$. **[3]**

(ii) Given that the fourth term is 6, find the exact value of x. **[2]**

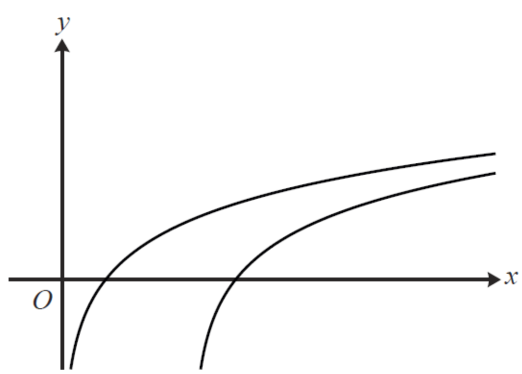
(b) A geometric progression has first term $\log_2 27$ and common ratio $\log_2 y$.

(i) Find the set of values of y for which the geometric progression has a sum to infinity. **[2]**

(ii) Find the exact value of y for which the sum to infinity of the geometric progression is 3. **[5]**

Q9 June 2012

16.



The diagram shows the curves $y = \log_2 x$ and $y = \log_2 (x - 3)$.

(i) Describe the geometrical transformation that transforms the curve $y = \log_2 x$ to the curve $y = \log_2 (x - 3)$. **[2]**

(ii) The curve $y = \log_2 x$ passes through the point $(a, 3)$. State the value of a. **[1]**

(iii) The curve $y = \log_2 (x - 3)$ passes through the point $(b, 1.8)$. Find the value of b, giving your answer correct to 3 significant figures. **[2]**

(iv) The point P lies on $y = \log_2 x$ and has an x-coordinate of c. The point Q lies on $y = \log_2 (x - 3)$ and also has an x-coordinate of c. Given that the distance PQ is 4 units find the exact value of c. **[4]**

Q8 Jan 2013

17.

Sarah is carrying out a series of experiments which involve using increasing amounts of a chemical. In the first experiment she uses 6 g of the chemical and in the second experiment she uses 7.8 g of the chemical.

(i) Given that the amounts of the chemical used form an arithmetic progression, find the total amount of chemical used in the first 30 experiments. **[3]**

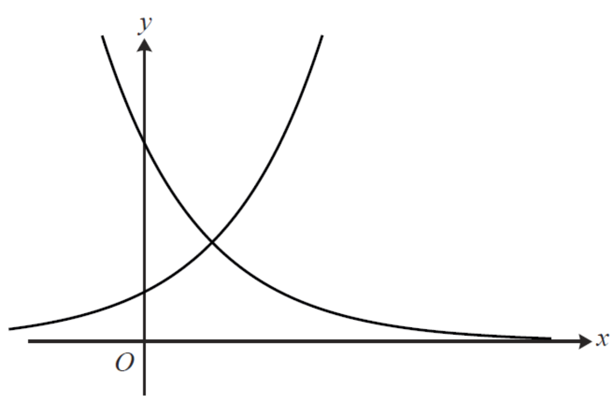
(ii) Instead it is given that the amounts of the chemical used form a geometric progression. Sarah has a total of 1800 g of the chemical available. Show that N, the greatest number of experiments possible, satisfies the inequality

$$1.3^N \leqslant 91,$$

and use logarithms to calculate the value of N. **[6]**

Q6 June 2013

18.



The diagram shows the curves $y = a^x$ and $y = 4b^x$.

(i) **(a)** State the coordinates of the point of intersection of $y = a^x$ with the y-axis. **[1]**

(b) State the coordinates of the point of intersection of $y = 4b^x$ with the y-axis. **[1]**

(c) State a possible value for a and a possible value for b. **[2]**

(ii) It is now given that $ab = 2$. Show that the x-coordinate of the point of intersection of $y = a^x$ and $y = 4b^x$ can be written as

$$x = \frac{2}{2\log_2 a - 1}.$$

[5]

Q8 June 2013

19.

Solve the equation $2^{4x-1} = 3^{5-2x}$, giving your answer in the form $x = \dfrac{\log_{10} a}{\log_{10} b}$. **[6]**

Q5 June 2014

20.

(a) Use logarithms to solve the equation

$$2^{n-3} = 18000,$$

giving your answer correct to 3 significant figures. **[4]**

(b) Solve the simultaneous equations

$$\log_2 x + \log_2 y = 8, \qquad \log_2\left(\frac{x^2}{y}\right) = 7.$$ **[5]**

Q8 June 2015