Edexcel

Pure Mathematics

Year 1

Exponentials and Natural Logarithms

Past paper questions from C3and IAL C34

Edited by: K V Kumaran

1. A particular species of orchid is being studied. The population p at time t years after the study started is assumed to be

$$p = \frac{2800ae^{0.2t}}{1+ae^{0.2t}}, \text{ where } a \text{ is a constant.}$$

Given that there were 300 orchids when the study started,

(*a*) show that $a = 0.12$,

(3)

(*b*) use the equation with $a = 0.12$ to predict the number of years before the population of orchids reaches 1850.

(4)

(*c*) Show that $p = \dfrac{336}{0.12 + e^{-0.2t}}$.

(1)

(*d*) Hence show that the population cannot exceed 2800.

(2)

(Q7, June 2005)

2. A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T °C, t minutes after it enters the liquid, is given by

$$T = 400e^{-0.05t} + 25, \quad t \geq 0.$$

(*a*) Find the temperature of the ball as it enters the liquid.

(1)

(*b*) Find the value of t for which $T = 300$, giving your answer to 3 significant figures.

(4)

(*c*) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$. Give your answer in °C per minute to 3 significant figures.

(3)

(*d*) From the equation for temperature T in terms of t, given above, explain why the temperature of the ball can never fall to 20 °C.

(1)

(Q4, June 2006)

3. Find the exact solutions to the equations

(*a*) $\ln x + \ln 3 = \ln 6$,

(2)

(*b*) $e^x + 3e^{-x} = 4$.

(4)

(Q1, June 2007)

4. The amount of a certain type of drug in the bloodstream t hours after it has been taken is given by the formula

$$x = De^{-\frac{1}{8}t},$$

where x is the amount of the drug in the bloodstream in milligrams and D is the dose given in milligrams.

A dose of 10 mg of the drug is given.

(*a*) Find the amount of the drug in the bloodstream 5 hours after the dose is given. Give your answer in mg to 3 decimal places.

(2)

A second dose of 10 mg is given after 5 hours.

(*b*) Show that the amount of the drug in the bloodstream 1 hour after the second dose is 13.549 mg to 3 decimal places.

(2)

No more doses of the drug are given. At time T hours after the second dose is given, the amount of the drug in the bloodstream is 3 mg.

(*c*) Find the value of T.

(3)

(Q8, June 2007)

5. The radioactive decay of a substance is given by

$$R = 1000e^{-ct}, \quad t \geq 0.$$

where R is the number of atoms at time t years and c is a positive constant.

(*a*) Find the number of atoms when the substance started to decay.

(1)

It takes 5730 years for half of the substance to decay.

(*b*) Find the value of c to 3 significant figures.

(4)

(*c*) Calculate the number of atoms that will be left when $t = 22\,920$.

(2)

(*d*) Sketch the graph of R against t .

(2)

(Q5, Jan 2008)

6. Rabbits were introduced onto an island. The number of rabbits, P, t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \quad t \in \mathbb{R}, \quad t \geq 0.$$

(*a*) Write down the number of rabbits that were introduced to the island.

(1)

(*b*) Find the number of years it would take for the number of rabbits to first exceed 1000. **(2)**

(*c*) Find $\frac{dP}{dt}$.

(2)

(*d*) Find P when $\frac{dP}{dt} = 50$.

(3)

(Q3, June 2009)

7. (*a*) Simplify fully

$$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}.$$

(3)

Given that

$$\ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15), \quad x \neq -5,$$

(*b*) find x in terms of e.

(4)

(Q8, June 2010)

8. Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, θ °C, of the tea is modelled by the equation

$$\theta = 20 + A\mathrm{e}^{-kt},$$

where A and k are positive constants

Given that the initial temperature of the tea was 90 °C,

(*a*) find the value of A.

(2)

The tea takes 5 minutes to decrease in temperature from 90 °C to 55 °C.

(*b*) Show that $k = \frac{1}{5}\ln 2$. **(3)**

(*c*) Find the rate at which the temperature of the tea is decreasing at the instant when $t = 10$. Give your answer, in °C per minute, to 3 decimal places.

(3)

(Q4, Jan 2011)

9. The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = p\mathrm{e}^{-kt},$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(*a*) Write down the value of p.

(1)

(*b*) Show that $k = \frac{1}{4}\ln 3$.

(4)

(*c*) Find the value of t when $\frac{\mathrm{d}m}{\mathrm{d}t} = -0\ 6\ln 3$.

(6)

(Q5, June 2011)

10. The area, A mm^2, of a bacterial culture growing in milk, t hours after midday, is given by

$$A = 20e^{1.5t}, \qquad t \geq 0.$$

(a) Write down the area of the culture at midday.

(1)

(b) Find the time at which the area of the culture is twice its area at midday. Give your answer to the nearest minute.

(5)

(Q5, Jan 2012)

11. The value of Bob's car can be calculated from the formula

$$V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500.$$

where V is the value of the car in pounds (£) and t is the age in years.

(a) Find the value of the car when $t = 0$.

(1)

(b) Calculate the exact value of t when $V = 9500$.

(4)

(c) Find the rate at which the value of the car is decreasing at the instant when $t = 8$. Give your answer in pounds per year to the nearest pound.

(4)

(Q8, Jan 2013)

12. Find algebraically the exact solutions to the equations

(a) $\ln(4 - 2x) + \ln(9 - 3x) = 2\ln(x + 1), \qquad -1 < x < 2$

(5)

(b) $2^x e^{3x+1} = 10$

Give your answer to (b) in the form $\dfrac{a + \ln b}{c + \ln d}$ where a, b, c and d are integers.

(5)

(Q6, June 2013)

13.

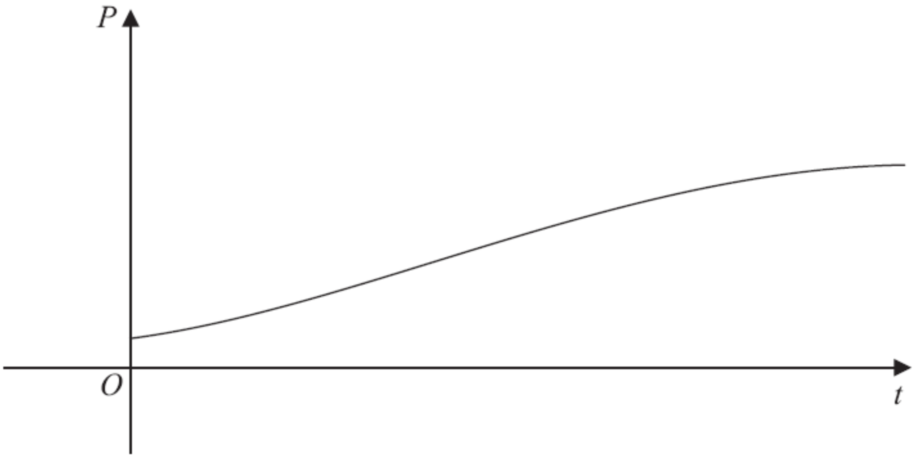

Figure 3

The population of a town is being studied. The population P, at time t years from the start of the study, is assumed to be

$$P = \frac{8000}{1 + 7e^{-kt}}, \qquad t \geq 0$$

where k is a positive constant.

The graph of P against t is shown in Figure 3.

Use the given equation to

(*a*) find the population at the start of the study,

(2)

(*b*) find a value for the expected upper limit of the population.

(1)

Given also that the population reaches 2500 at 3 years from the start of the study,

(*c*) calculate the value of k to 3 decimal places.

(5)

Using this value for k,

(*d*) find the population at 10 years from the start of the study, giving your answer to 3 significant figures.

(2)

(*e*) Find, using $\frac{\mathrm{d}P}{\mathrm{d}t}$, the rate at which the population is growing at 10 years from the start of the study.

(3)

(Q8, June 2013_R)

14. Find the exact solutions, in their simplest form, to the equations

(a) $2 \ln (2x + 1) - 10 = 0$

(2)

(b) $3^x e^{4x} = e^7$

(4)

(Q2, June 2014)

15. A rare species of primrose is being studied. The population, P, of primroses at time t years after the study started is modelled by the equation

$$P = \frac{800e^{0.1t}}{1+3e^{0.1t}}, \qquad t \geq 0, \quad t \in \square$$

(a) Calculate the number of primroses at the start of the study.

(2)

(b) Find the exact value of t when $P = 250$, giving your answer in the form $a \ln(b)$ where
a and b are integers.

(4)

(c) Explain why the population of primroses can never be 270.

(1)

(Q8, June 2014)

16. Water is being heated in an electric kettle. The temperature, θ °C, of the water t seconds after the kettle is switched on, is modelled by the equation

$$\theta = 120 - 100e^{-\lambda t}, \qquad 0 \leq t \leq T.$$

(a) State the value of θ when $t = 0$.

(1)

Given that the temperature of the water in the kettle is 70 °C when $t = 40$,

(b) find the exact value of λ, giving your answer in the form $\frac{\ln a}{b}$, where a and b are integers.

(4)

When $t = T$, the temperature of the water reaches 100 °C and the kettle switches off.

(c) Calculate the value of T to the nearest whole number.

(2)

(Q4, June 2015)

17. The amount of an antibiotic in the bloodstream, from a given dose, is modelled by the formula

$$x = De^{-0.2t},$$

where x is the amount of the antibiotic in the bloodstream in milligrams, D is the dose given in milligrams and t is the time in hours after the antibiotic has been given.

A first dose of 15 mg of the antibiotic is given.

(*a*) Use the model to find the amount of the antibiotic in the bloodstream 4 hours after the dose is given. Give your answer in mg to 3 decimal places.

(2)

A second dose of 15 mg is given 5 hours after the first dose has been given. Using the same model for the second dose,

(*b*) show that the **total** amount of the antibiotic in the bloodstream 2 hours after the second dose is given is 13.754 mg to 3 decimal places.

(2)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the total amount of the antibiotic in the bloodstream is 7.5 mg.

(*c*) Show that $T = a \ln\left(b + \frac{b}{e}\right)$, where a and b are integers to be determined.

(4)

(Q9, June 2016)

18. Find the exact solutions, in their simplest form, to the equations

(*a*) $e^{3x-9} = 8$

(3)

(*b*) $\ln(2y + 5) = 2 + \ln(4 - y)$

(4)

(Q1, June 2017)

19.

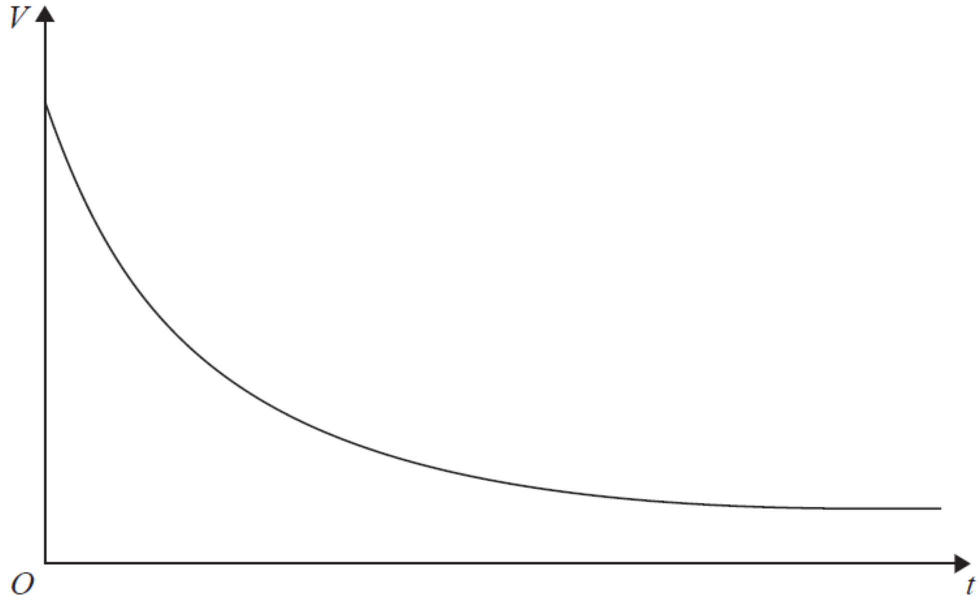

Figure 1

The value of Lin's car is modelled by the formula

$$V = 18\,000\mathrm{e}^{-0.2t} + 4000\mathrm{e}^{-0.1t} + 1000, \qquad t \geq 0$$

where the value of the car is V pounds when the age of the car is t years.

A sketch of t against V is shown in Figure 1.

(a) State the range of V.

(2)

According to this model,

(b) find the rate at which the value of the car is decreasing when $t = 10$.
Give your answer in pounds per year.

(3)

(c) Calculate the exact value of t when $V = 15\,000$.

(4)

(Q8, IAL Jan 2015)

20. The mass, m grams, of a radioactive substance t years after first being observed, is modelled by the equation

$$m = 25e^{1-kt}$$

where k is a positive constant.

(a) State the value of m when the radioactive substance was first observed.

(1)

Given that the mass is 50 grams, 10 years after first being observed,

(b) show that $k = \frac{1}{10}\ln\left(\frac{1}{2}e\right)$

(4)

(c) Find the value of t when $m = 20$, giving your answer to the nearest year.

(3)

(Q6, IAL June 2015)

21. A hot piece of metal is dropped into a cool liquid. As the metal cools, its temperature T degrees Celsius, t minutes after it enters the liquid, is modelled by

$$T = 300e^{-0.04t} + 20, \quad t \geq 0$$

(a) Find the temperature of the piece of metal as it enters the liquid.

(1)

(b) Find the value of t for which $T = 180$, giving your answer to 3 significant figures.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

(c) Show, by differentiation, that the rate, in degrees Celsius per minute, at which the temperature of the metal is changing, is given by the expression

$$\frac{20 - T}{25}$$

(3)

(Q6, IAL Jan 2016)

22.

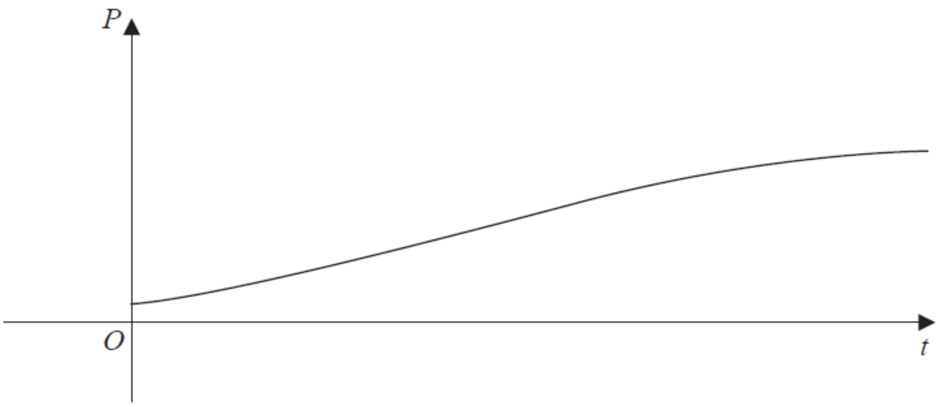

Figure 2

The population of a species of animal is being studied. The population P, at time t years
from the start of the study, is assumed to be

$$P = \frac{9000e^{kt}}{3e^{kt}+7}, \qquad t \geq 0$$

where k is a positive constant.
A sketch of the graph of P against t is shown in Figure 2.
Use the given equation to

(*a*) find the population at the start of the study,

(2)

(*b*) find the value for the upper limit of the population.

(1)

Given that $P = 2500$ when $t = 4$

(*c*) calculate the value of k, giving your answer to 3 decimal places.

(5)

(Q9, IAL June 2016)

23. A population of insects is being studied. The number of insects, N, in the population, is

modelled by the equation

$$N = \frac{300}{3+17e^{-0.2t}} \qquad t \in \mathbb{R}, \ \ t \geq 0$$

where t is the time, in weeks, from the start of the study.
Using the model,

(*a*) find the number of insects at the start of the study,

(1)

(*b*) find the number of insects when $t = 10$,

(2)

(*c*) find the time from the start of the study when there are 82 insects.

(4)

(Q10, IAL Jan 2017)

24. The value of a car is modelled by the formula

$$V = 16\,000\text{e}^{-kt} + A, \qquad t \geqslant 0, t \in \mathbb{R}$$

where V is the value of the car in pounds, t is the age of the car in years, and k and A are positive constants.
Given that the value of the car is £17 500 when new and £13 500 two years later,
(a) find the value of A,

(1)

(b) show that $k = \ln\left(\dfrac{2}{\sqrt{3}}\right)$

(4)

(c) Find the age of the car, in years, when the value of the car is £6000.
Give your answer to 2 decimal places.

(4)

(Q3, C3 June 2018)

25. A study is being carried out on two colonies of ants.

The number of ants N_A in colony A, t years after the start of the study, is modelled by the
equation

$$N_A = 3000 + 600\text{e}^{0.12t} \qquad t \in \mathbb{R}, t \geq 0$$

Using the model,

(a) find the time taken, from the start of the study, for the number of ants in colony A to double. Give your answer, in years, to 2 decimal places.

(5)

(b) Show that $\dfrac{\text{d}N_A}{\text{d}t} = pN_A + q$, where p and q are constants to be determined.

(3)

The number of ants N_B in colony B, t years after the start of the study, is modelled by the
equation

$$N_B = 2900 + C\text{e}^{kt} \qquad t \in \mathbb{R}, t \geq 0$$

where C and k are positive constants.

According to this model, there will be 3100 ants in colony B one year after the start of the
study and 3400 ants in colony B two years after the start of the study.

(c) (i) Show that $k = \ln\left(\dfrac{5}{2}\right)$

(ii) Find the value of C.

(4)

(Q8, C3 June 2019)

26. The number of bacteria in a liquid culture is modelled by the formula

$$N = 3500(1.035)^t, \qquad t \geqslant 0$$

where N is the number of bacteria t hours after the start of a scientific study.

(*a*) State the number of bacteria at the start of the scientific study.

(1)

(*b*) Find the time taken from the start of the study for the number of bacteria to reach 10 000

Give your answer in hours and minutes, to the nearest minute.

(4)

(Q13, IAL C34 Oct 2017)

27. A rare species of mammal is being studied. The population P, t years after the study started, is modelled by the formula

$$P = \frac{900e^{\frac{1}{4}t}}{3e^{\frac{1}{4}t} - 1}, \qquad t \in \mathbb{R}, \qquad t \geqslant 0$$

Using the model,

(*a*) calculate the number of mammals at the start of the study,

(1)

(*b*) calculate the exact value of t when $P = 315$

Give your answer in the form $a \ln k$, where a and k are integers to be determined.

(4)

(Q9, IAL C34 Oct 2018)

28. A scientist is studying a population of insects. The number of insects, N, in the population,

t days after the start of the study is modelled by the equation

$$N = \frac{240}{1 + ke^{-\frac{t}{16}}}$$

where k is a constant.

Given that there were 50 insects at the start of the study,

(*a*) find the value of k

(2)

(*b*) use the model to find the value of t when $N = 100$

(3)

(Q13, IAL C34 Jan2019)

29. A scientist is studying a population of fish in a lake. The number of fish, N, in the population, t years after the start of the study, is modelled by the equation

$$N = \frac{250e^{0.2t}}{1 + 0.25e^{0.2t}} \qquad t \geq 0$$

(*a*) Find, according to the model, the number of fish in the lake at the start of the study.

(1)

(*b*) Find, according to the model, the value of t when there are 800 fish in the lake, giving

your answer to the nearest integer.

(3)

(Q12, IAL C34 Nov 2019)

30. A bath is filled with hot water. The temperature, θ°C, of the water in the bath, t minutes
after the bath has been filled, is given by

$$\theta = 20 + Ae^{-kt}$$

where A and k are positive constants.

Given that the temperature of the water in the bath is initially 38° C,

(*a*) find the value of A.

(2)

The temperature of the water in the bath 16 minutes after the bath has been filled is 24.5° C.

(*b*) Show that $k = \frac{1}{8}\ln 2$

(4)

Using the values for k and A,

(*c*) find the temperature of the water 40 minutes after the bath has been filled, giving your
answer to 3 significant figures.

(2)

(*d*) Explain why the temperature of the water in the bath cannot fall to 19° C.

(1)

(Q5, IAL C34 June 2019)