

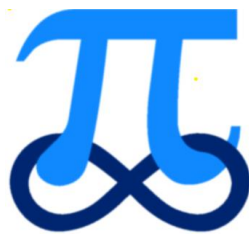
# Edexcel

## Pure Mathematics

### Year 1

## Integration 2

*Past paper questions from Core Maths 2 and IAL C12*



Edited by: K V Kumaran

**Past paper questions from  
Edexcel Core Maths 2 and IAL C12.  
From Jan 2005 to May 2019.**

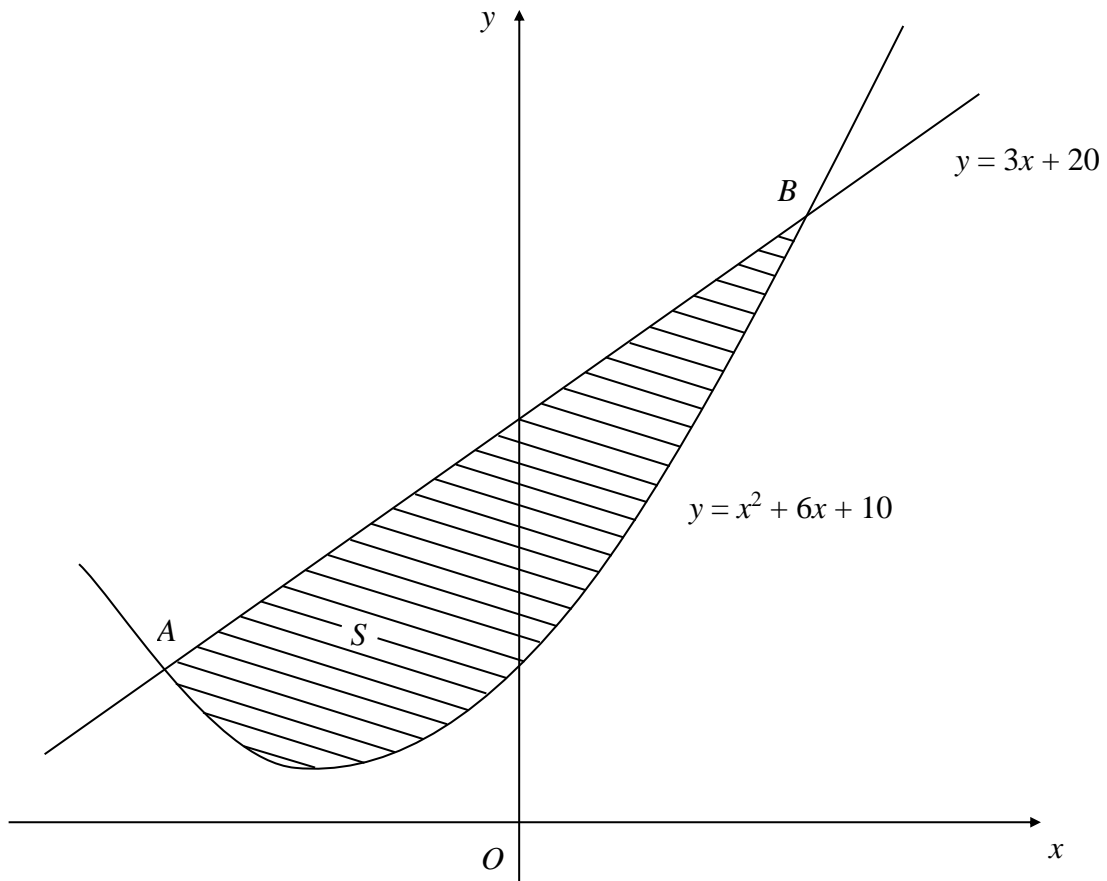
Integration 02

This Section 1 has 54 Questions on application on integration, area under the curves.

*Please check the Edexcel website for the solutions.*

1.

Figure 2



The line with equation  $y = 3x + 20$  cuts the curve with equation  $y = x^2 + 6x + 10$  at the points  $A$  and  $B$ , as shown in Figure 2.

(a) Use algebra to find the coordinates of  $A$  and the coordinates of  $B$ . (5)

The shaded region  $S$  is bounded by the line and the curve, as shown in Figure 2.

(b) Use calculus to find the exact area of  $S$ . (7)

(C2 Jan 2005, Q8)

2. Evaluate  $\int_1^8 \frac{1}{\sqrt{x}} dx$ , giving your answer in the form  $a + b\sqrt{2}$ , where  $a$  and  $b$  are integers. (4)

(C2 May 2007, Q1)

3.

Figure 1

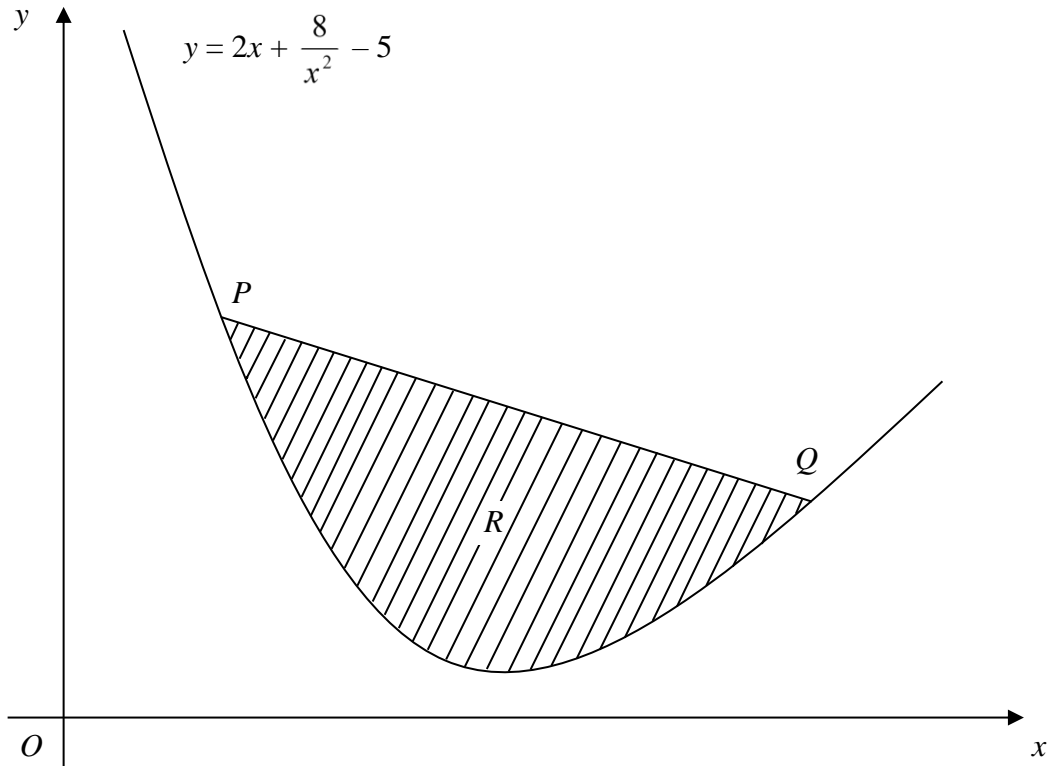


Figure 1 shows part of a curve  $C$  with equation  $y = 2x + \frac{8}{x^2} - 5$ ,  $x > 0$ .

The points  $P$  and  $Q$  lie on  $C$  and have  $x$ -coordinates 1 and 4 respectively. The region  $R$ , shaded in Figure 1, is bounded by  $C$  and the straight line joining  $P$  and  $Q$ .

(a) Find the exact area of  $R$ .

(8)

(b) Use calculus to show that  $y$  is increasing for  $x > 2$ .

(4)

(C2 June 2005, Q10)

4. Use calculus to find the exact value of  $\int_1^2 \left( 3x^2 + 5 + \frac{4}{x^2} \right) dx$ .

(5)

(C2 May 2006, Q2)

5.

Figure 3

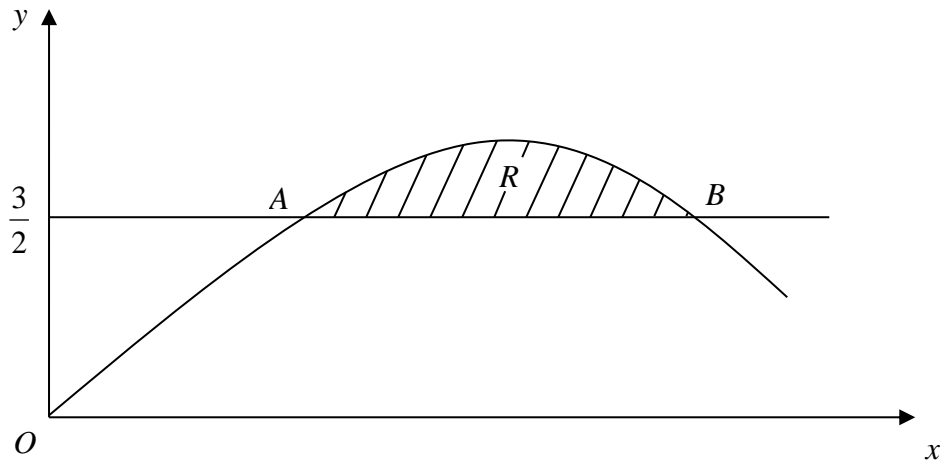


Figure 3 shows the shaded region  $R$  which is bounded by the curve  $y = -2x^2 + 4x$  and the line  $y = \frac{3}{2}$ . The points  $A$  and  $B$  are the points of intersection of the line and the curve.

Find

(a) the  $x$ -coordinates of the points  $A$  and  $B$ ,

(4)

(b) the exact area of  $R$ .

(6)

(C2 Jan 2006, Q9)

6.

$$f(x) = x^3 + 3x^2 + 5.$$

Find

(a)  $f''(x)$ ,

(3)

(b)  $\int_1^2 f(x) \, dx$ .

(4)

(C2 Jan 2007, Q1)

7.

Figure 3

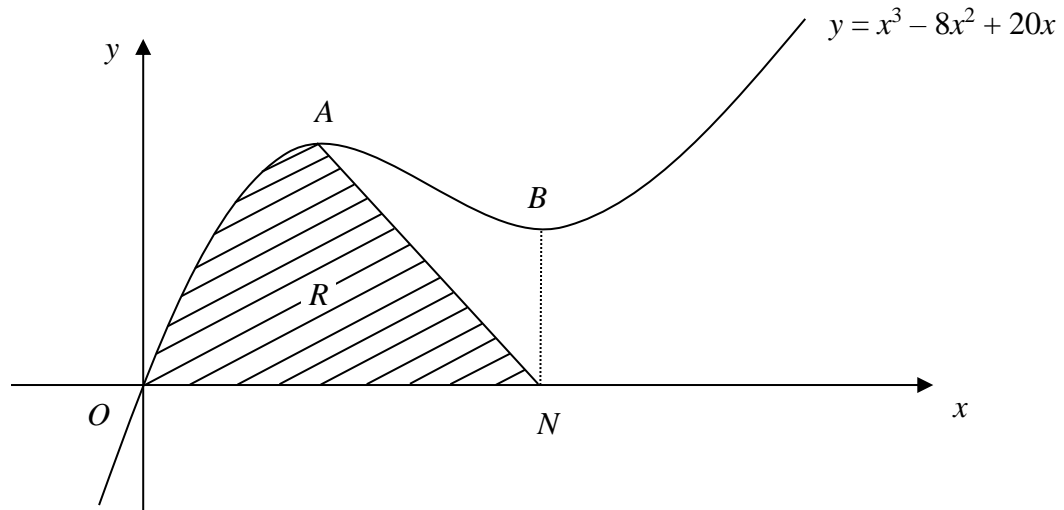


Figure 3 shows a sketch of part of the curve with equation  $y = x^3 - 8x^2 + 20x$ . The curve has stationary points A and B.

(a) Use calculus to find the  $x$ -coordinates of A and B. (4)

(b) Find the value of  $\frac{d^2y}{dx^2}$  at A, and hence verify that A is a maximum. (2)

The line through B parallel to the  $y$ -axis meets the  $x$ -axis at the point N. The region R, shown shaded in Figure 3, is bounded by the curve, the  $x$ -axis and the line from A to N.

(c) Find  $\int (x^3 - 8x^2 + 20x) dx$ . (3)

(d) Hence calculate the exact area of R. (5)

(C2 May 2006, Q10)

8. Use calculus to find the value of

$$\int_1^4 (2x + 3\sqrt{x}) dx.$$

(5)

(C2 June 2009, Q1)

9.

Figure 1

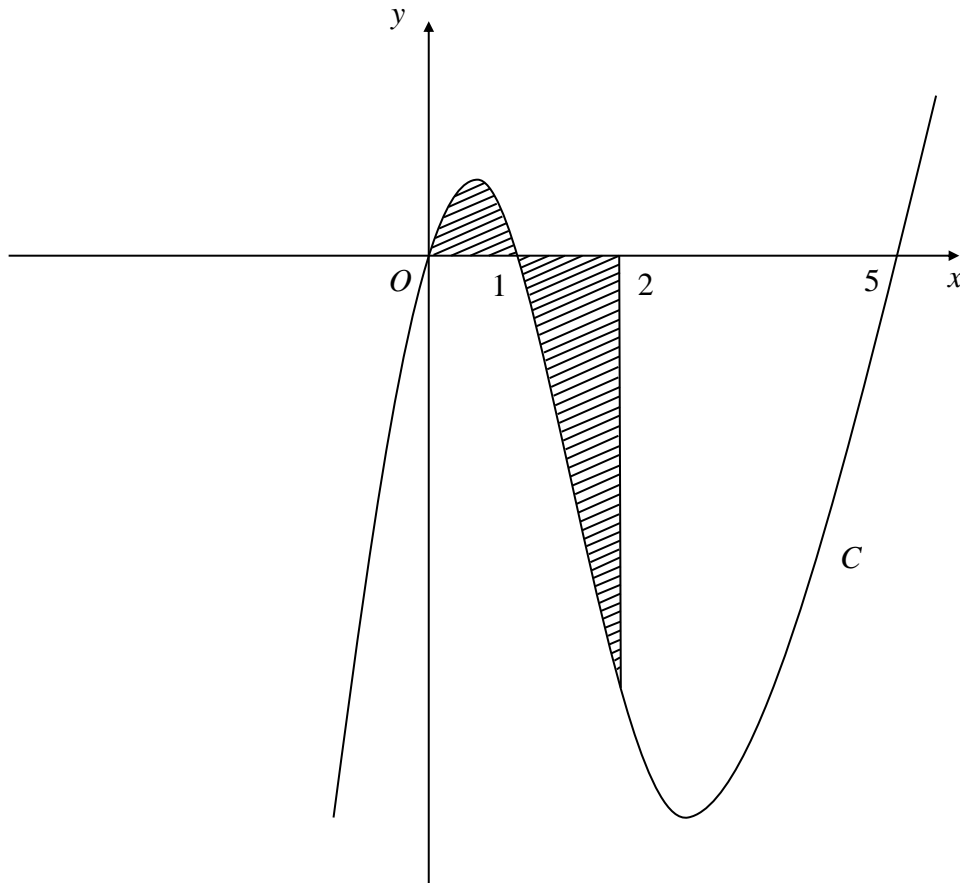


Figure 1 shows a sketch of part of the curve  $C$  with equation

$$y = x(x - 1)(x - 5).$$

Use calculus to find the total area of the finite region, shown shaded in Figure 1, that is between  $x = 0$  and  $x = 2$  and is bounded by  $C$ , the  $x$ -axis and the line  $x = 2$ .

(9)

(C2 Jan 2007, Q7)

10. Use integration to find

$$\int_1^{\sqrt{3}} \left( \frac{x^3}{6} + \frac{1}{3x^2} \right) dx,$$

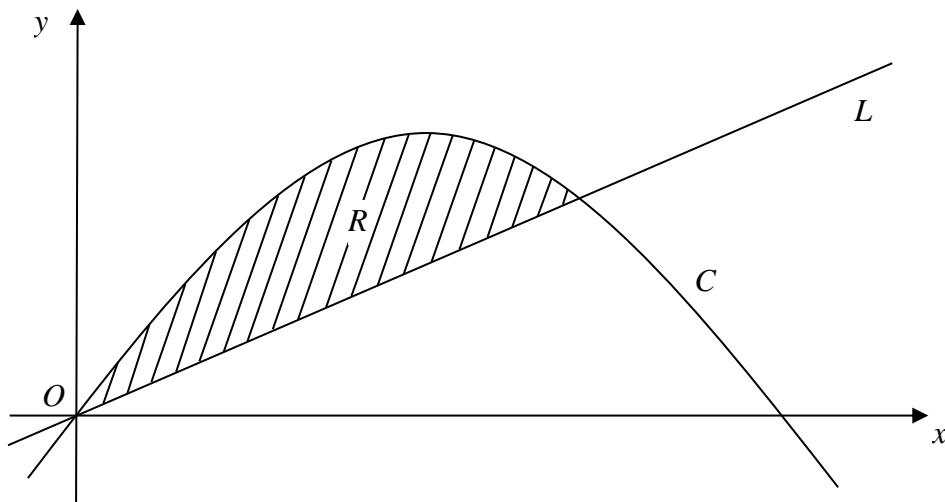
giving your answer in the form  $a + b\sqrt{3}$ , where  $a$  and  $b$  are constants to be determined.

(5)

(C2 May 2014, Q4)

11.

Figure 2



In Figure 2 the curve  $C$  has equation  $y = 6x - x^2$  and the line  $L$  has equation  $y = 2x$ .

(a) Show that the curve  $C$  intersects with the  $x$ -axis at  $x = 0$  and  $x = 6$ . (1)

(b) Show that the line  $L$  intersects the curve  $C$  at the points  $(0, 0)$  and  $(4, 8)$ . (3)

The region  $R$ , bounded by the curve  $C$  and the line  $L$ , is shown shaded in Figure 2.

(c) Use calculus to find the area of  $R$ . (6)

(C2 Jan 2008, Q7)

12.

$$f(x) = \frac{8}{x^2} - 4\sqrt{x} + 3x - 1, \quad x > 0$$

Giving your answers in their simplest form, find

(a)  $f'(x)$  (3)

(b)  $\int f(x) dx$  (4)

(IAL C12 Jan 2014, Q2)



13.

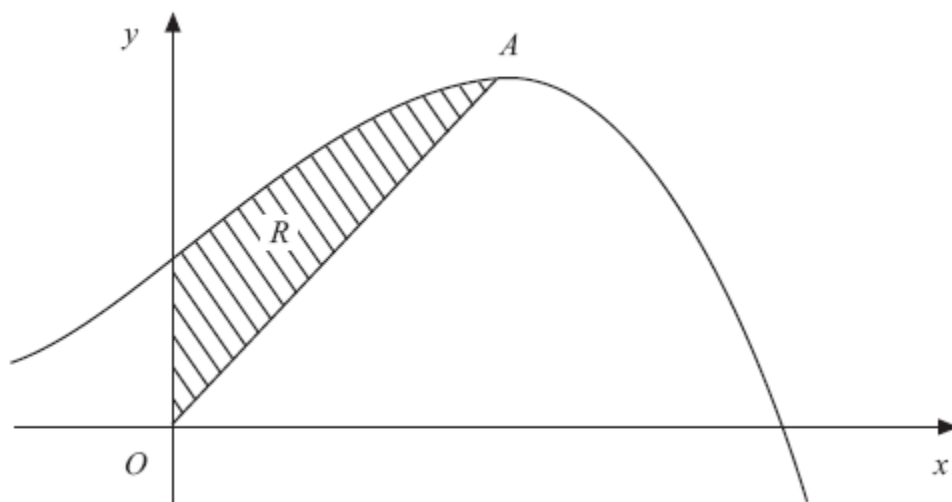


Figure 2

Figure 2 shows a sketch of part of the curve with equation  $y = 10 + 8x + x^2 - x^3$ .

The curve has a maximum turning point  $A$ .

- (a) Using calculus, show that the  $x$ -coordinate of  $A$  is 2. (3)

The region  $R$ , shown shaded in Figure 2, is bounded by the curve, the  $y$ -axis and the line from  $O$  to  $A$ , where  $O$  is the origin.

- (b) Using calculus, find the exact area of  $R$ . (8)

(C2 June 2008, Q8)

14. (i) A curve with equation  $y = f(x)$  passes through the point  $(2, 3)$ .

Given that

$$f'(x) = \frac{4}{x^3} + 2x - 1$$

find the value of  $f(1)$ .

(5)

- (ii) Given that

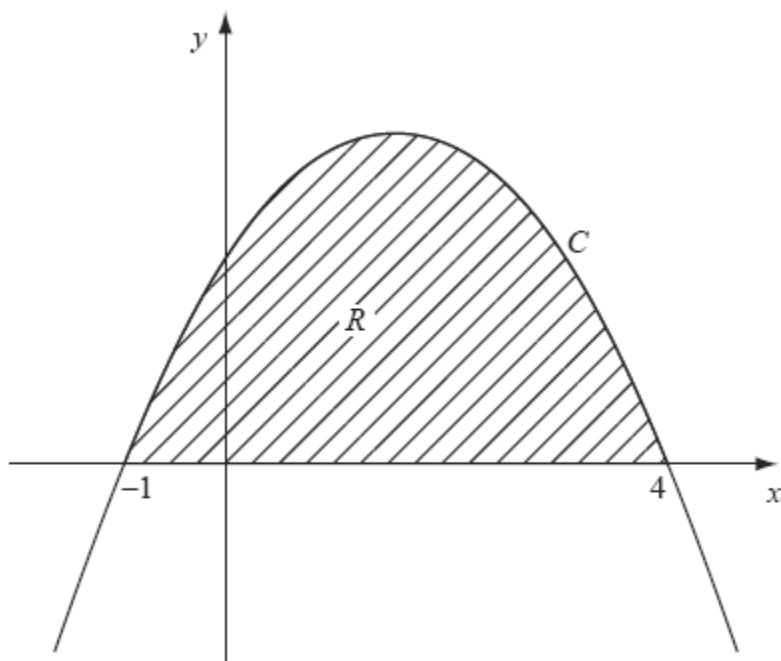
$$\int_1^4 (3\sqrt{x+A}) \, dx = 21$$

find the exact value of the constant  $A$ .

(5)

(IAL C12 May 2014, Q7)

15.



**Figure 1**

Figure 1 shows part of the curve  $C$  with equation  $y = (1 + x)(4 - x)$ .

The curve intersects the  $x$ -axis at  $x = -1$  and  $x = 4$ . The region  $R$ , shown shaded in Figure 1, is bounded by  $C$  and the  $x$ -axis.

Use calculus to find the exact area of  $R$ .

(5)

(C2 Jan 2009, Q2)

16. The curve  $C$  has equation  $y = f(x)$ ,  $x > 0$ , where

$$f'(x) = 3\sqrt{x} - \frac{9}{\sqrt{x}} + 2$$

Given that the point  $P(9, 14)$  lies on  $C$ ,

(a) find  $f(x)$ , simplifying your answer,

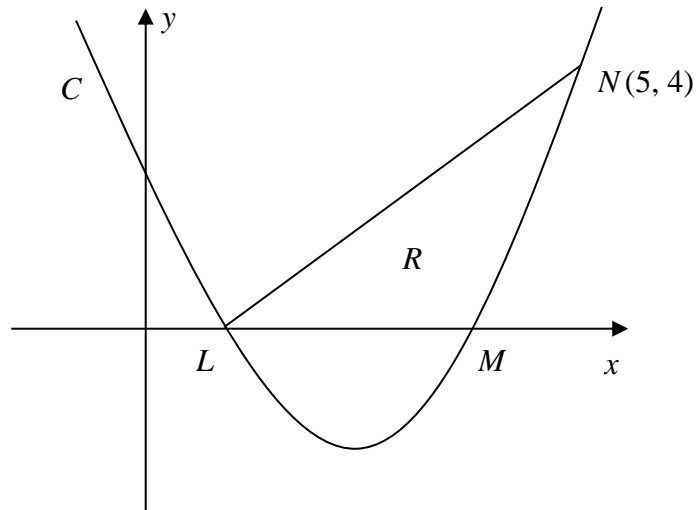
(6)

(b) find an equation of the normal to  $C$  at the point  $P$ , giving your answer in the form  $ax + by + c = 0$  where  $a$ ,  $b$  and  $c$  are integers.

(5)

(IAL C12 May 2015, Q11)

17.



**Figure 2**

The curve  $C$  has equation  $y = x^2 - 5x + 4$ . It cuts the  $x$ -axis at the points  $L$  and  $M$  as shown in Figure 2.

(a) Find the coordinates of the point  $L$  and the point  $M$ . (2)

(b) Show that the point  $N(5, 4)$  lies on  $C$ . (1)

(c) Find  $\int (x^2 - 5x + 4) \, dx$ . (2)

The finite region  $R$  is bounded by  $LN$ ,  $LM$  and the curve  $C$  as shown in Figure 2.

(d) Use your answer to part (c) to find the exact value of the area of  $R$ . (5)

(C2 Jan 2010, Q7)

18. Find, using calculus and showing each step of your working,

$$\int_1^4 \left( 6x - 3 - \frac{2}{\sqrt{x}} \right) dx$$

(5)

(IAL C12 Jan 2016, Q3)

19.

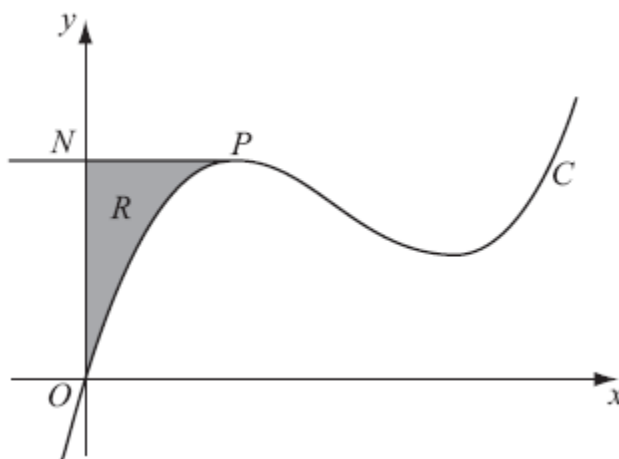


Figure 2

Figure 2 shows a sketch of part of the curve  $C$  with equation

$$y = x^3 - 10x^2 + kx,$$

where  $k$  is a constant.

The point  $P$  on  $C$  is the maximum turning point.

Given that the  $x$ -coordinate of  $P$  is 2,

- (a) show that  $k = 28$ . (3)

The line through  $P$  parallel to the  $x$ -axis cuts the  $y$ -axis at the point  $N$ .  
The region  $R$  is bounded by  $C$ , the  $y$ -axis and  $PN$ , as shown shaded in Figure 2.

- (b) Use calculus to find the exact area of  $R$ . (6)

(C2 June 2010, Q8)

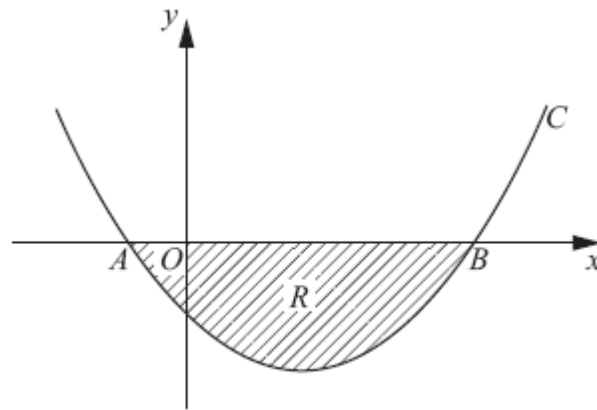
20. (a) Show that  $\frac{x^2 - 4}{2\sqrt{x}}$  can be written in the form  $Ax^p + Bx^q$ , where  $A$ ,  $B$ ,  $p$  and  $q$  are constants to be determined. (3)
- (b) Hence find

$$\int \frac{x^2 - 4}{2\sqrt{x}} dx, \quad x > 0$$

giving your answer in its simplest form. (4)

(IAL C12 May 2016, Q6)

21.



**Figure 1**

Figure 1 shows a sketch of part of the curve  $C$  with equation

$$y = (x + 1)(x - 5).$$

The curve crosses the  $x$ -axis at the points  $A$  and  $B$ .

(a) Write down the  $x$ -coordinates of  $A$  and  $B$ .

(1)

The finite region  $R$ , shown shaded in Figure 1, is bounded by  $C$  and the  $x$ -axis.

(b) Use integration to find the area of  $R$ .

(6)

(C2 Jan 2011, Q4)

22.

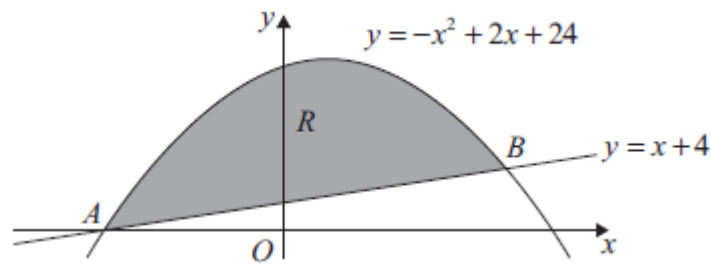
$$f(x) = 3x^2 + x - \frac{4}{\sqrt{x}} + 6x^{-3}, \quad x > 0$$

Find  $\int f(x)dx$ , simplifying each term.

(5)

(IAL C12 Oct 2016, Q1)

23.



**Figure 3**

The straight line with equation  $y = x + 4$  cuts the curve with equation  $y = -x^2 + 2x + 24$  at the points  $A$  and  $B$ , as shown in Figure 3.

(a) Use algebra to find the coordinates of the points  $A$  and  $B$ . (4)

The finite region  $R$  is bounded by the straight line and the curve and is shown shaded in Figure 3.

(b) Use calculus to find the exact area of  $R$ . (7)  
(C2 May 2011, Q9)

24. (i) Find

$$\int \frac{2 + 4x^3}{x^2} dx$$

giving each term in its simplest form. (4)

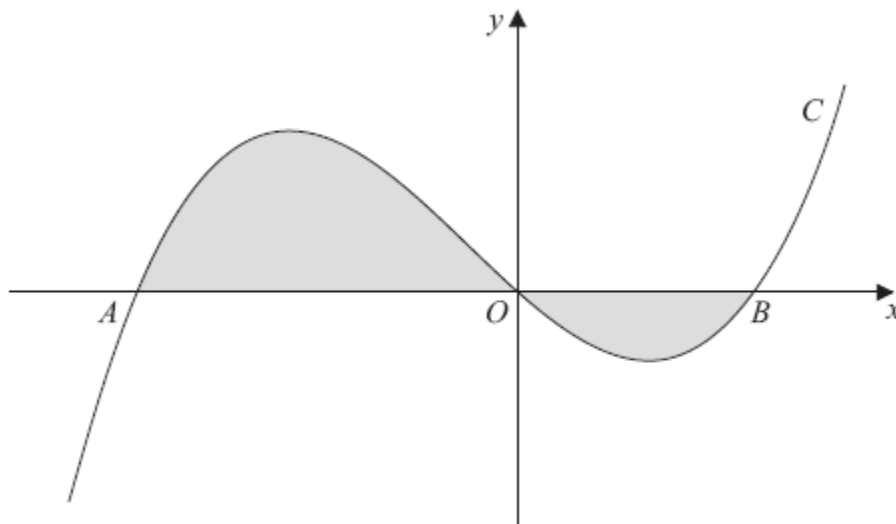
(ii) Given that  $k$  is a constant and

$$\int_2^4 \left( \frac{4}{\sqrt{x}} + k \right) dx = 30$$

find the exact value of  $k$ . (5)

(IAL C12 Jan2016, Q7)

25.



**Figure 3**

Figure 3 shows a sketch of part of the curve  $C$  with equation

$$y = x(x + 4)(x - 2).$$

The curve  $C$  crosses the  $x$ -axis at the origin  $O$  and at the points  $A$  and  $B$ .

- (a) Write down the  $x$ -coordinates of the points  $A$  and  $B$ . (1)

The finite region, shown shaded in Figure 3, is bounded by the curve  $C$  and the  $x$ -axis.

- (b) Use integration to find the total area of the finite region shown shaded in Figure 3. (7)

(C2 May 2013, Q6)

26. (a) Find  $\int (3x^2 + 4x - 15)dx$ , simplifying each term. (3)

Given that  $b$  is a constant and

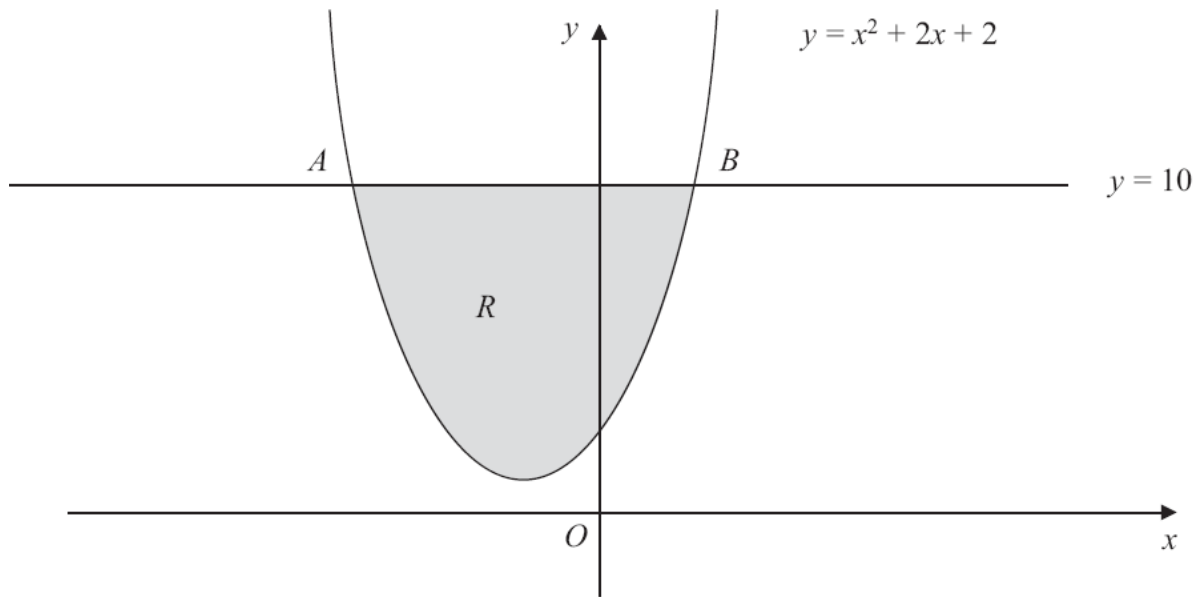
$$\int_b^4 (3x^2 + 4x - 15)dx = 36$$

- (b) show that  $b^3 + 2b^2 - 15b = 0$  (2)

- (c) Hence find the possible values of  $b$ . (3)

(IAL C12 May 2017, Q8)

27.



**Figure 1**

The line with equation  $y = 10$  cuts the curve with equation  $y = x^2 + 2x + 2$  at the points  $A$  and  $B$  as shown in Figure 1. The figure is not drawn to scale.

- (a) Find by calculation the  $x$ -coordinate of  $A$  and the  $x$ -coordinate of  $B$ . (2)

The shaded region  $R$  is bounded by the line with equation  $y = 10$  and the curve as shown in Figure 1.

- (b) Use calculus to find the exact area of  $R$ . (7)

(C2 May 2013\_R, Q7)

28. (a) Express  $\frac{x^3 + 4}{2x^2}$  in the form  $Ax^p + Bx^q$ , where  $A$ ,  $B$ ,  $p$  and  $q$  are constants. (3)

- (b) Hence find

$$\int \frac{x^3 + 4}{2x^2} dx$$

simplifying your answer.

(3)

(IAL C12 Oct 2017, Q3)



29.

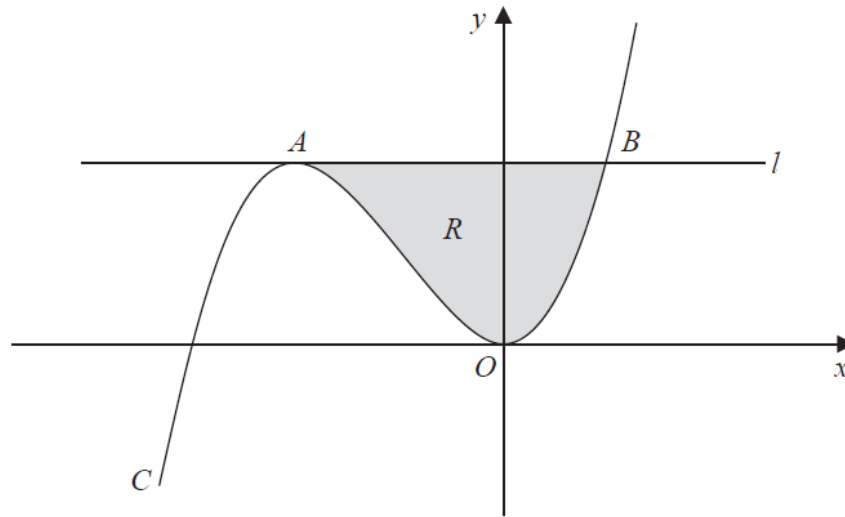


Figure 3

Figure 3 shows a sketch of part of the curve  $C$  with equation

$$y = \frac{1}{8}x^3 + \frac{3}{4}x^2, \quad x \in \mathbb{R}$$

The curve  $C$  has a maximum turning point at the point  $A$  and a minimum turning point at the origin  $O$ .

The line  $l$  touches the curve  $C$  at the point  $A$  and cuts the curve  $C$  at the point  $B$ .

The  $x$  coordinate of  $A$  is  $-4$  and the  $x$  coordinate of  $B$  is  $2$ .

The finite region  $R$ , shown shaded in Figure 3, is bounded by the curve  $C$  and the line  $l$ .

Use integration to find the area of the finite region  $R$ .

(7)  
(C2 May 2014\_R, Q6)

30. Given that

$$y = \frac{2x^{\frac{2}{3}} + 3}{6}, \quad x > 0$$

find, in the simplest form,

(a)  $\frac{dy}{dx}$  (2)

(b)  $\int y dx$  (3)

(IAL C12 Jan 2018, Q1)

31. (a) Find

$$\int 10x(x^{\frac{1}{2}} - 2) \, dx,$$

giving each term in its simplest form.

(4)

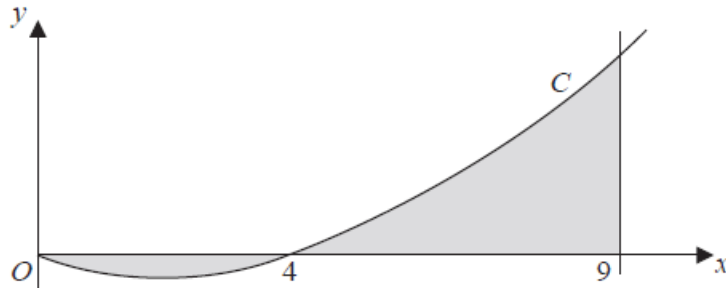


Figure 2

Figure 2 shows a sketch of part of the curve  $C$  with equation

$$y = 10x(x^{\frac{1}{2}} - 2), \quad x \geq 0.$$

The curve  $C$  starts at the origin and crosses the  $x$ -axis at the point  $(4, 0)$ .

The area, shown shaded in Figure 2, consists of two finite regions and is bounded by the curve  $C$ , the  $x$ -axis and the line  $x = 9$ .

(b) Use your answer from part (a) to find the total area of the shaded regions.

(5)

(C2 May 2015, Q6)

32. The curve  $C$  has equation  $y = f(x)$ ,  $x > 0$ , where

$$f'(x) = \frac{5x^2 + 4}{2\sqrt{x}} - 5$$

It is given that the point  $P(4, 14)$  lies on  $C$ .

(a) Find  $f(x)$ , writing each term in a simplified form.

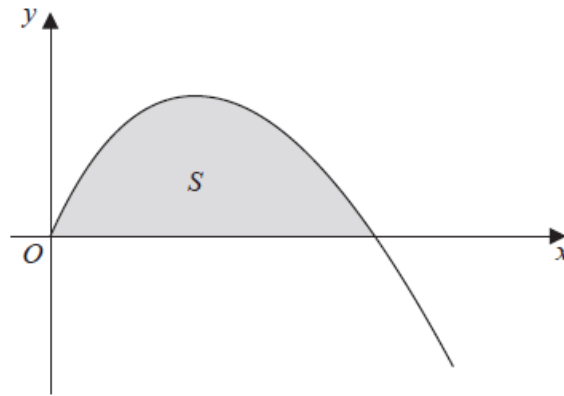
(6)

(b) Find the equation of the tangent to  $C$  at the point  $P$ , giving your answer in the form  $y = mx + c$ , where  $m$  and  $c$  are constants.

(4)

(IAL C12 May 2018, Q11)

33.



**Figure 3**

Figure 3 shows a sketch of part of the curve with equation

$$y = 3x - x^{\frac{3}{2}} \quad x \geq 0.$$

The finite region  $S$ , bounded by the  $x$ -axis and the curve, is shown shaded in Figure 3.

(a) Find

$$\int \left( 3x - x^{\frac{3}{2}} \right) dx.$$

(3)

(b) Hence find the area of  $S$ .

(3)

(C2 May 2016, Q7)

34. Given that  $y = 2x^3 - \frac{5}{3x^2} + 7$ ,  $x \neq 0$ , find in its simplest form

(a)  $\frac{dy}{dx}$ ,

(3)

(b)  $\int y \, dx$ .

(4)

(IAL C12 Oct 2018, Q3)

35.

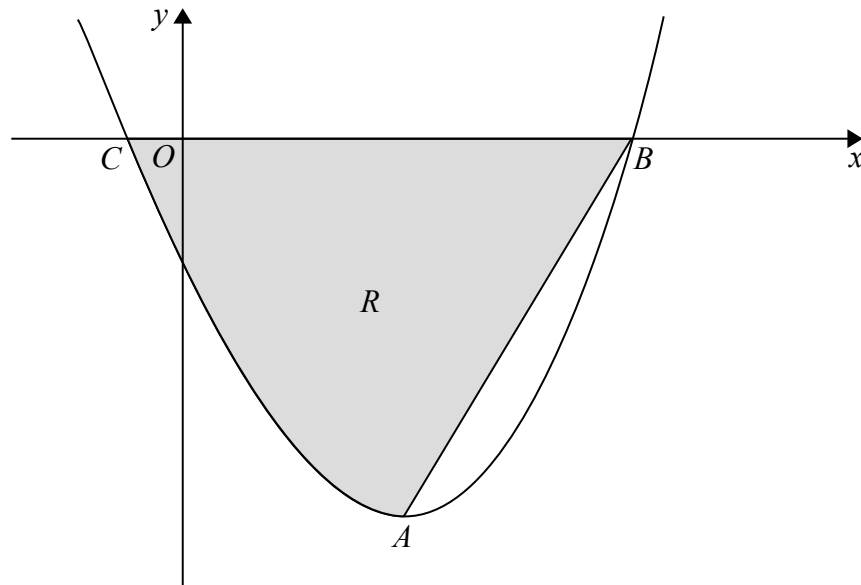


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 4x^3 + 9x^2 - 30x - 8, \quad -0.5 \leq x \leq 2.2$$

The curve has a turning point at the point A.

(a) Using calculus, show that the  $x$  coordinate of A is 1

(3)

The curve crosses the  $x$ -axis at the points  $B(2, 0)$  and  $C\left(-\frac{1}{4}, 0\right)$

The finite region  $R$ , shown shaded in Figure 2, is bounded by the curve, the line  $AB$ , and the  $x$ -axis.

(b) Use integration to find the area of the finite region  $R$ , giving your answer to 2 decimal places.

(7)

(C2 May 2017, Q10)

36. Given  $k > 3$  and

$$\int_3^k \left( 2x + \frac{6}{x^2} \right) dx = 10k$$

show that  $k^3 - 10k^2 - 7k - 6 = 0$

(5)

(IAL C12 Jan 2019, Q8)

37.

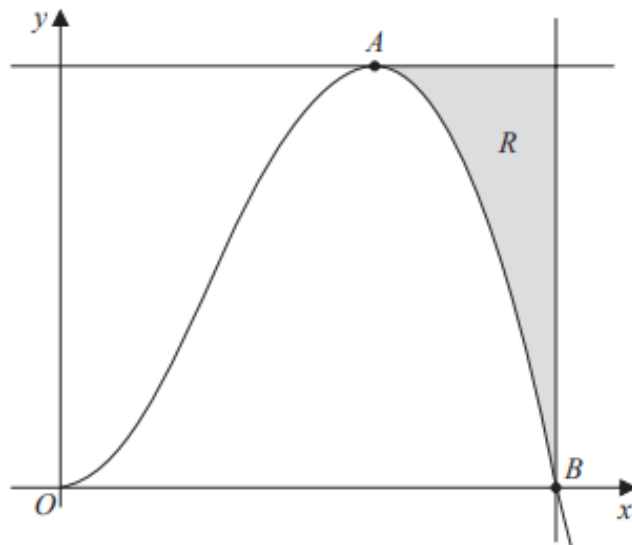


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 7x^2(5 - 2\sqrt{x}), \quad x \geq 0$$

The curve has a turning point at the point  $A$ , where  $x > 0$ , as shown in Figure 3.

- (a) Using calculus, find the coordinates of the point  $A$ . (5)

The curve crosses the  $x$ -axis at the point  $B$ , as shown in Figure 3.

- (b) Use algebra to find the  $x$  coordinate of the point  $B$ . (2)

The finite region  $R$ , shown shaded in Figure 3, is bounded by the curve, the line through  $A$  parallel to the  $x$ -axis and the line through  $B$  parallel to the  $y$ -axis.

- (c) Use integration to find the area of the region  $R$ , giving your answer to 2 decimal places. (5)

(C2 May 2018, Q9)

38. Find

$$\int \left( \frac{1}{2x^3} + 3x^{\frac{1}{2}} - 6 \right) dx \quad x > 0$$

writing each term as simply as possible.

(4)

(IAL C12 Oct 2019, Q1)

39.

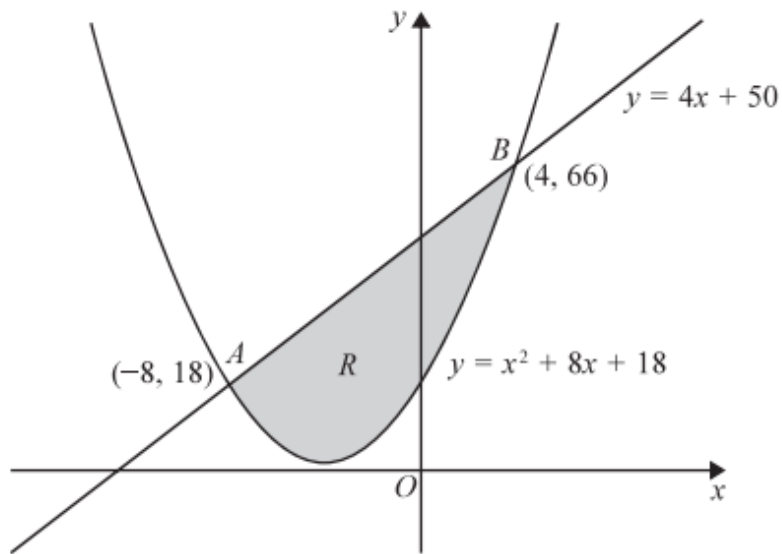


Figure 2

Figure 2 shows the line with equation  $y = 4x + 50$  and the curve with equation  $y = x^2 + 8x + 18$ . The line cuts the curve at the points  $A(-8, 18)$  and  $B(4, 66)$ .

The shaded region  $R$  is bounded by the line and the curve, as shown in Figure 2.

Using calculus, find the area of  $R$ .

(6)

(C2 May 2019, Q6)

40.

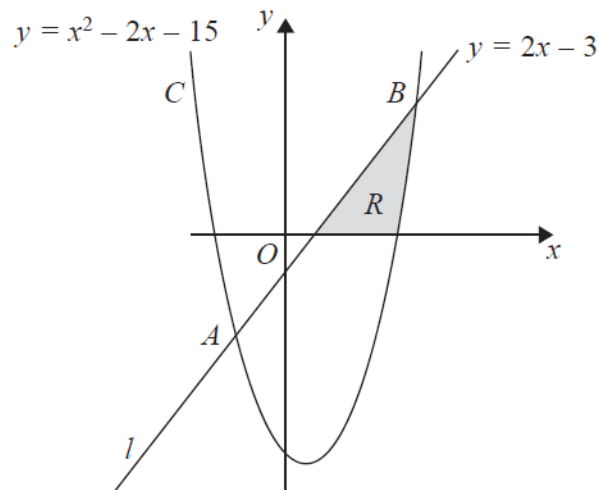


Diagram **NOT**  
drawn to scale

**Figure 2**

Figure 2 shows part of the line  $l$  with equation  $y = 2x - 3$  and part of the curve  $C$  with equation  $y = x^2 - 2x - 15$ .

The line  $l$  and the curve  $C$  intersect at the points  $A$  and  $B$  as shown.

(a) Use algebra to find the coordinates of  $A$  and the coordinates of  $B$ . (5)

In Figure 2, the shaded region  $R$  is bounded by the line  $l$ , the curve  $C$  and the positive  $x$ -axis.

(b) Use integration to calculate an exact value for the area of  $R$ . (7)

**(IAL C12 Jan 2014, Q14)**

41.

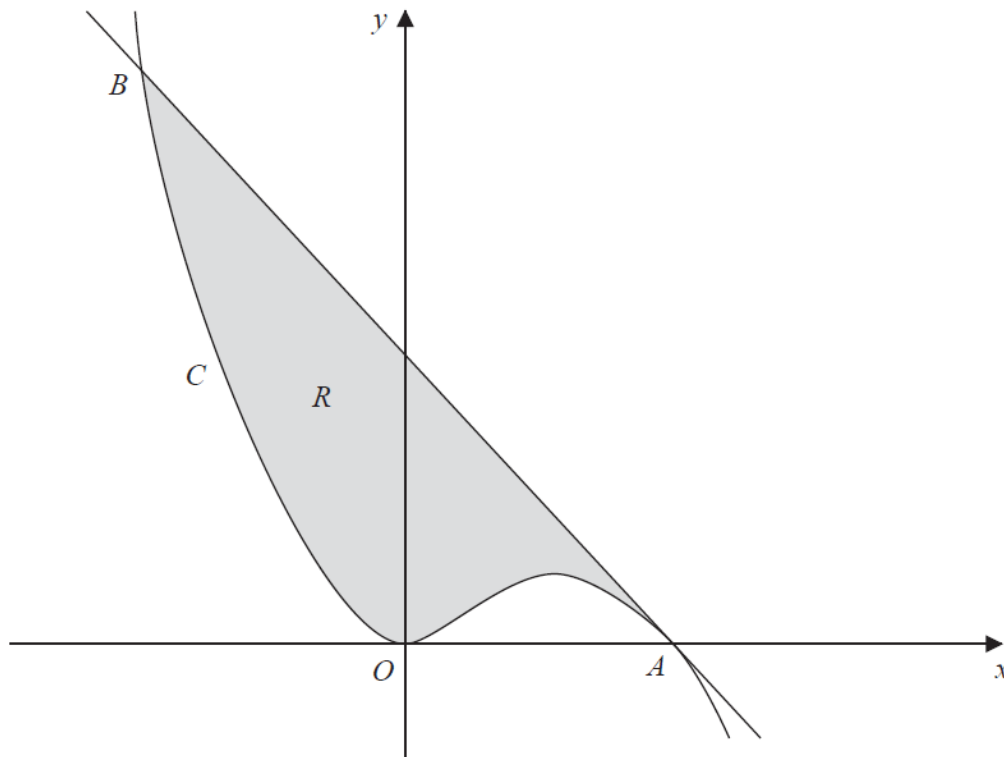


Figure 5

Figure 5 shows a sketch of part of the curve  $C$  with equation  $y = x^2 - \frac{1}{3}x^3$ .

$C$  touches the  $x$ -axis at the origin and cuts the  $x$ -axis at the point  $A$ .

(a) Show that the coordinates of  $A$  are  $(3, 0)$ . (1)

(b) Show that the equation of the tangent to  $C$  at the point  $A$  is  $y = -3x + 9$ . (5)

The tangent to  $C$  at  $A$  meets  $C$  again at the point  $B$ , as shown in Figure 5.

(c) Use algebra to find the  $x$  coordinate of  $B$ . (4)

The region  $R$ , shown shaded in Figure 5, is bounded by the curve  $C$  and the tangent to  $C$  at  $A$ .

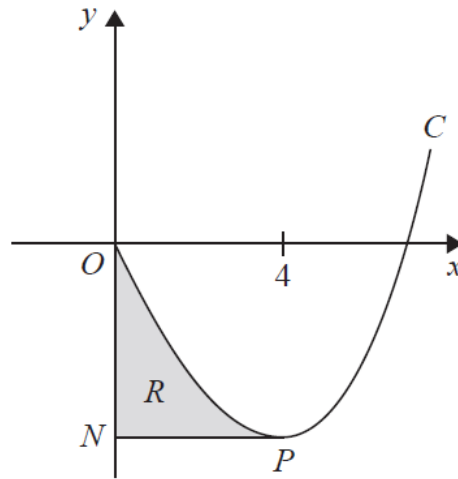
(d) Find, by using calculus, the area of region  $R$ .

*(Solutions based entirely on graphical or numerical methods are not acceptable.)* (5)

(IAL C12 May 2014, Q12)



42.



**Figure 5**

Figure 5 shows a sketch of part of the curve  $C$  with equation

$$y = x^3 + 10x^{\frac{3}{2}} + kx, \quad x \geq 0$$

where  $k$  is a constant.

- (a) Find  $\frac{dy}{dx}$  (2)

The point  $P$  on the curve  $C$  is a minimum turning point.  
Given that the  $x$  coordinate of  $P$  is 4,

- (b) show that  $k = -78$ . (2)

The line through  $P$  parallel to the  $x$ -axis cuts the  $y$ -axis at the point  $N$ .

The finite region  $R$ , shown shaded in Figure 5, is bounded by  $C$ , the  $y$ -axis and  $PN$ .

- (c) Use integration to find the area of  $R$ . (7)

(IAL C12 Jan 2015, Q15)

43.

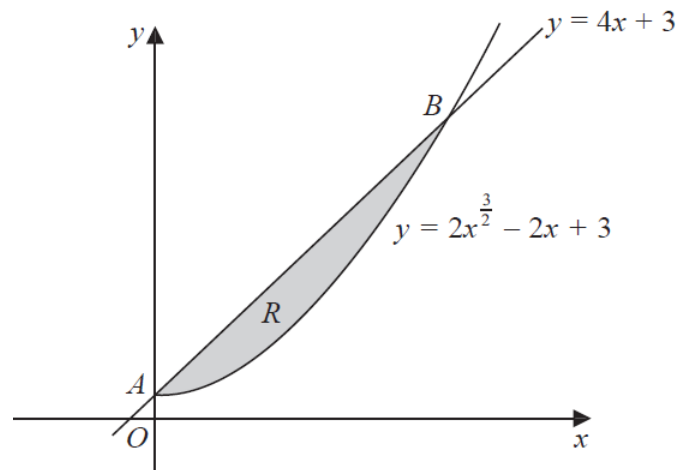


Figure 3

The finite region  $R$ , which is shown shaded in Figure 3, is bounded by the straight line  $l$  with equation  $y = 4x + 3$  and the curve  $C$  with equation

$$y = 2x^{\frac{3}{2}} - 2x + 3, x \geq 0$$

The line  $l$  meets the curve  $C$  at the point  $A$  on the  $y$ -axis and  $l$  meets  $C$  again at the point  $B$ , as shown in Figure 3.

(a) Use algebra to find the coordinates of  $A$  and  $B$ .

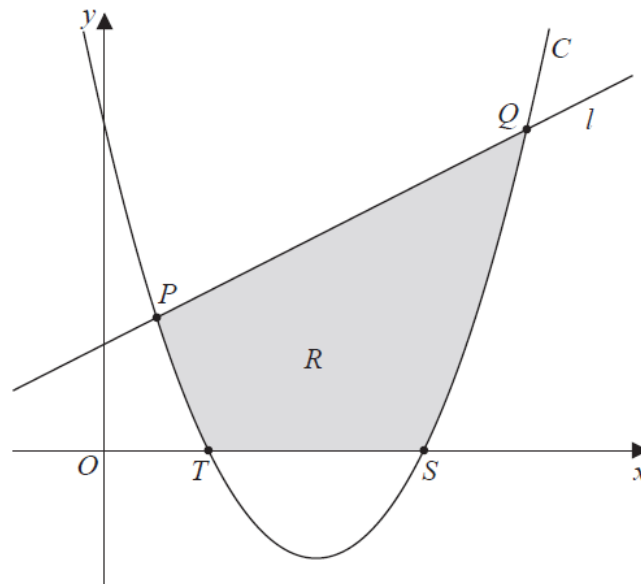
(4)

(b) Use integration to find the area of the shaded region  $R$ .

(6)

(IAL C12 May 2015, Q14)

44.



**Figure 2**

The straight line  $l$  with equation  $y = \frac{1}{2}x + 1$  cuts the curve  $C$ , with equation  $y = x^2 - 4x + 3$ , at the points  $P$  and  $Q$ , as shown in Figure 2.

(a) Use algebra to find the coordinates of the points  $P$  and  $Q$ .

(5)

The curve  $C$  crosses the  $x$ -axis at the points  $T$  and  $S$ .

(b) Write down the coordinates of the points  $T$  and  $S$ .

(2)

The finite region  $R$  is shown shaded in Figure 2. This region  $R$  is bounded by the line segment  $PQ$ , the line segment  $TS$ , and the arcs  $PT$  and  $SQ$  of the curve.

(c) Use integration to find the exact area of the shaded region  $R$ .

(8)

(IAL C12 Jan 2016, Q6)

45.

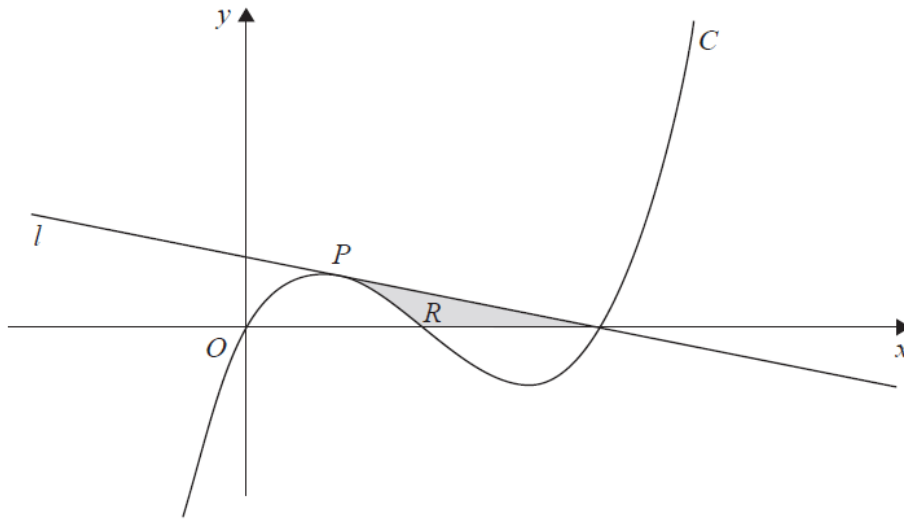


Figure 6

Figure 6 shows a sketch of part of the curve  $C$  with equation

$$y = x(x - 1)(x - 2)$$

The point  $P$  lies on  $C$  and has  $x$  coordinate  $\frac{1}{2}$

The line  $l$ , as shown on Figure 6, is the tangent to  $C$  at  $P$ .

(a) Find  $\frac{dy}{dx}$  (2)

(b) Use part (a) to find an equation for  $l$  in the form  $ax + by = c$ , where  $a$ ,  $b$  and  $c$  are integers. (4)

The finite region  $R$ , shown shaded in Figure 6, is bounded by the line  $l$ , the curve  $C$  and the  $x$ -axis.

The line  $l$  meets the curve again at the point  $(2, 0)$

(c) Use integration to find the exact area of the shaded region  $R$ . (6)

(IAL C12 May 2016, Q16)

46.

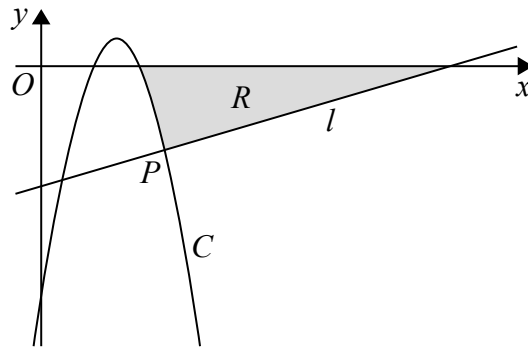


Figure 3

Figure 3 shows a sketch of the curve  $C$  with equation  $y = -x^2 + 6x - 8$ . The normal to  $C$  at the point  $P(5, -3)$  is the line  $l$ , which is also shown in Figure 3.

- (a) Find an equation for  $l$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

(5)

The finite region  $R$ , shown shaded in Figure 3, is bounded below by the line  $l$  and the curve  $C$ , and is bounded above by the  $x$ -axis.

- (b) Find the exact value of the area of  $R$ .

(6)

(IAL C12 Oct 2016, Q14)

47.

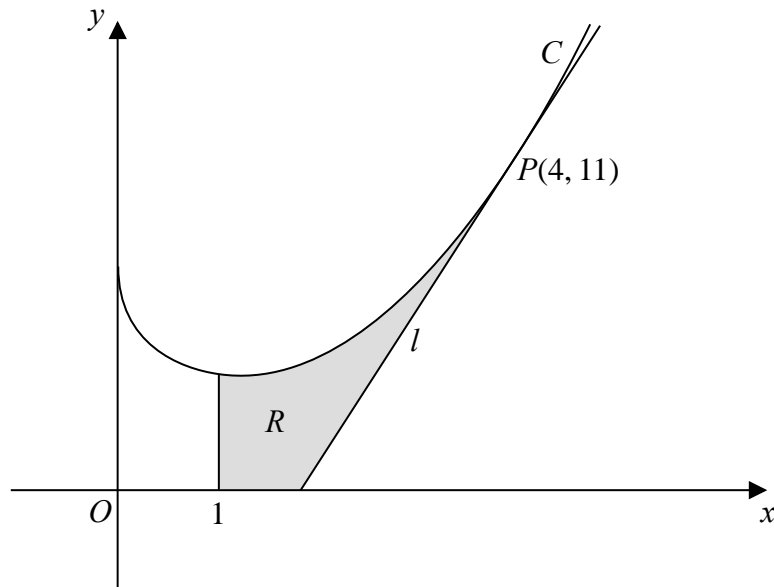


Figure 4

Figure 4 shows a sketch of part of the curve  $C$  with equation

$$y = \frac{3}{4}x^2 - 4\sqrt{x} + 7, \quad x > 0$$

The point  $P$  lies on  $C$  and has coordinates  $(4, 11)$ .

Line  $l$  is the tangent to  $C$  at the point  $P$ .

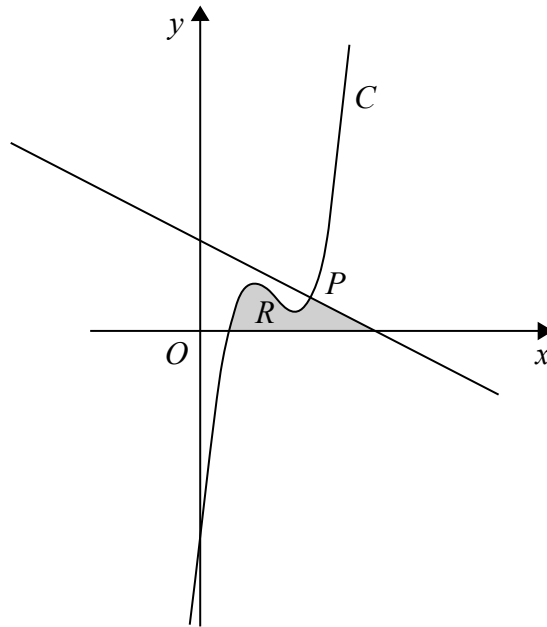
- (a) Use calculus to show that  $l$  has equation  $y = 5x - 9$  (5)

The finite region  $R$ , shown shaded in Figure 4, is bounded by the curve  $C$ , the line  $x = 1$ , the  $x$ -axis and the line  $l$ .

- (b) Find, by using calculus, the area of  $R$ , giving your answer to 2 decimal places. (6)

(IAL C12 Jan 2017, Q12)

48.



**Figure 4**

Figure 4 shows a sketch of part of the curve  $C$  with equation

$$y = x^3 - 9x^2 + 26x - 18$$

The point  $P(4, 6)$  lies on  $C$ .

(a) Use calculus to show that the normal to  $C$  at the point  $P$  has equation

$$2y + x = 16$$

**(5)**

The region  $R$ , shown shaded in Figure 4, is bounded by the curve  $C$ , the  $x$ -axis and the normal to  $C$  at  $P$ .

(b) Show that  $C$  cuts the  $x$ -axis at  $(1, 0)$

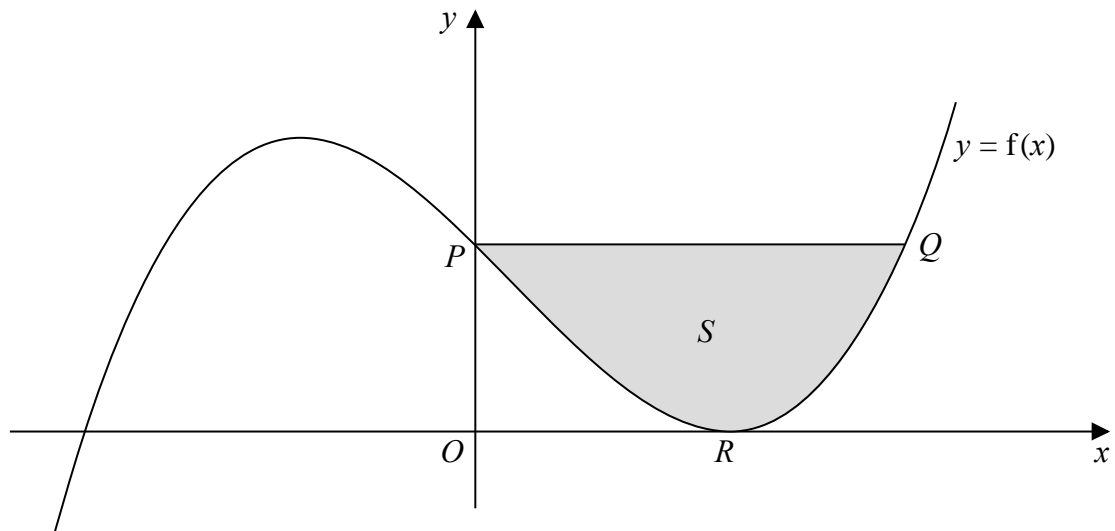
**(1)**

(c) Showing all your working, use calculus to find the exact area of  $R$ .

**(6)**

**(IAL C12 May 2017, Q12)**

49.



**Figure 5**

Figure 5 shows a sketch of part of the graph  $y = f(x)$ , where

$$f(x) = \frac{(x-3)^2(x+4)}{2}, \quad x \in \mathbb{R}$$

The graph cuts the  $y$ -axis at the point  $P$  and meets the positive  $x$ -axis at the point  $R$ , as shown in Figure 5.

(a) (i) State the  $y$  coordinate of  $P$ .

(ii) State the  $x$  coordinate of  $R$ .

(2)

The line segment  $PQ$  is parallel to the  $x$ -axis. Point  $Q$  lies on  $y = f(x)$ ,  $x > 0$

(b) Use algebra to show that the  $x$  coordinate of  $Q$  satisfies the equation

$$x^2 - 2x - 15 = 0$$

(3)

(c) Use part (b) to find the coordinates of  $Q$ .

(3)

The region  $S$ , shown shaded in Figure 5, is bounded by the curve  $y = f(x)$  and the line segment  $PQ$ .

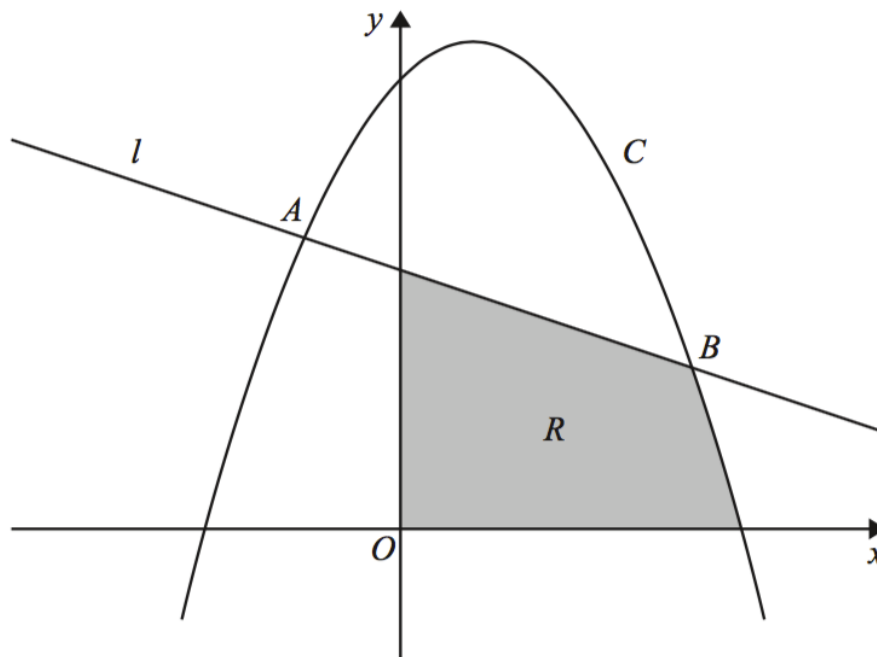
(d) Use calculus to find the exact area of  $S$ .

(6)

(IAL C12 Oct 2017, Q15)



50.



**Figure 5**

Figure 5 shows a sketch of part of the line  $l$  with equation  $y = 8 - x$  and part of the curve  $C$  with equation  $y = 14 + 3x - 2x^2$

The line  $l$  and the curve  $C$  intersect at the point  $A$  and the point  $B$  as shown.

(a) Use algebra to find the coordinates of  $A$  and the coordinates of  $B$ .

(5)

The region  $R$ , shown shaded in Figure 5, is bounded by the coordinate axes, the line  $l$ , and the curve  $C$ .

(b) Use algebraic integration to calculate the exact area of  $R$ .

(8)

(IAL C12 Jan 2018, Q14)

51.

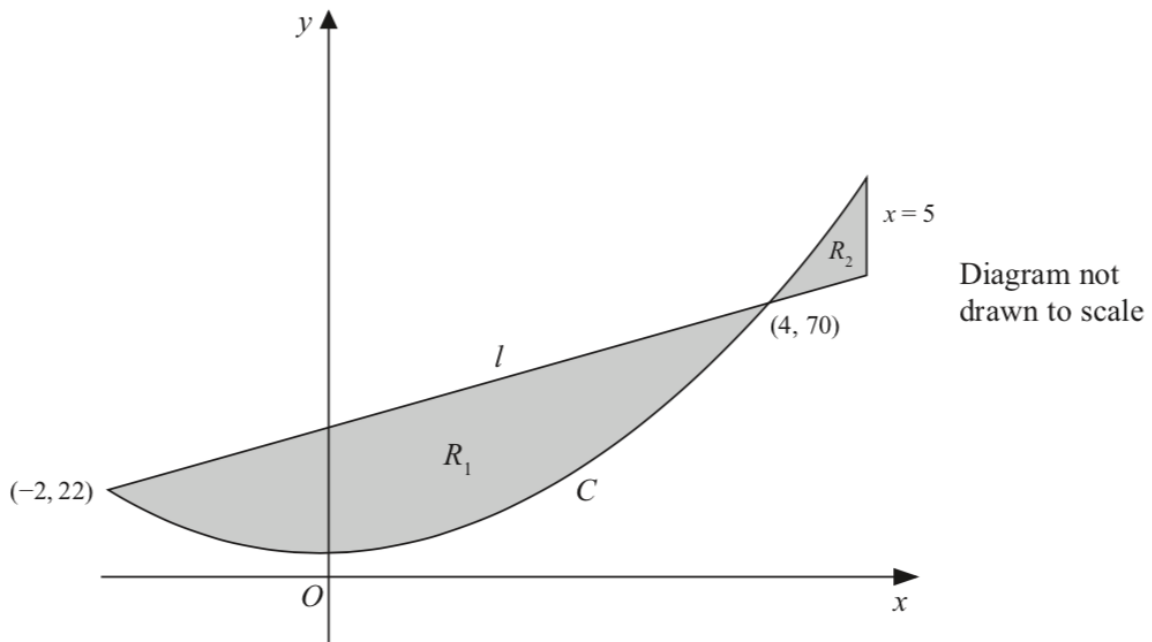


Figure 3

A design for a logo consists of two finite regions  $R_1$  and  $R_2$ , shown shaded in Figure 3.

The region  $R_1$  is bounded by the straight line  $l$  and the curve  $C$ .

The region  $R_2$  is bounded by the straight line  $l$ , the curve  $C$  and the line with equation  $x = 5$

The line  $l$  has equation  $y = 8x + 38$

The curve  $C$  has equation  $y = 4x^2 + 6$

Given that the line  $l$  meets the curve  $C$  at the points  $(-2, 22)$  and  $(4, 70)$ , use integration to find

(a) the area of the larger lower region, labelled  $R_1$  (6)

(b) the exact value of the total area of the two shaded regions. (3)

Given that

$$\frac{\text{Area of } R_1}{\text{Area of } R_2} = k$$

(c) find the value of  $k$ . (1)

(IAL C12 May 2018, Q15)

52.

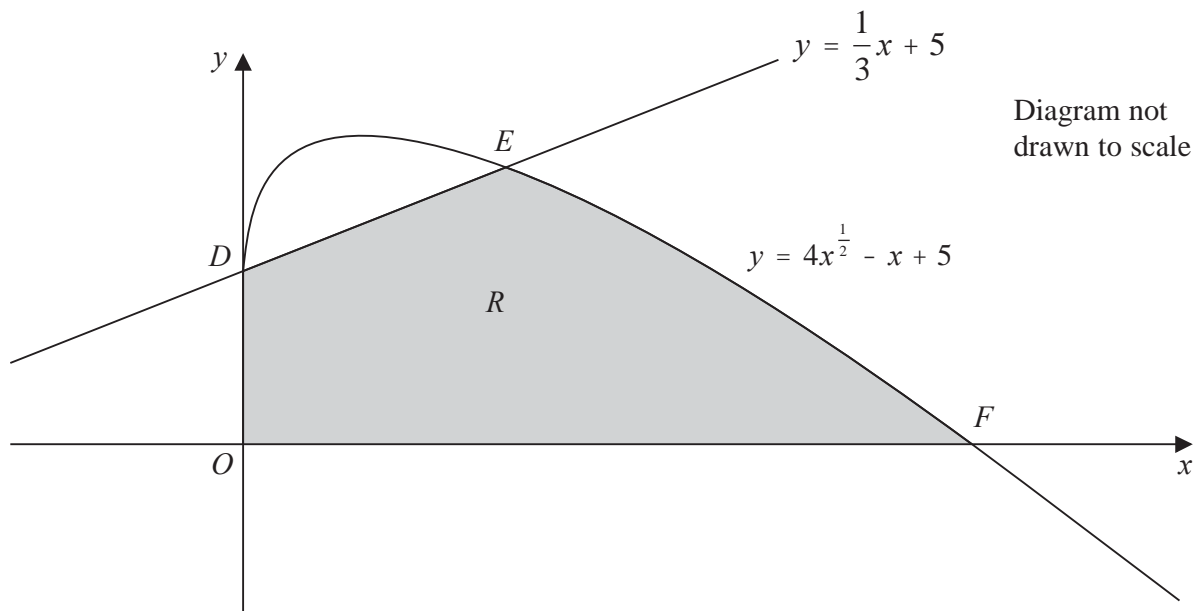


Figure 1

The finite region  $R$ , which is shown shaded in Figure 1, is bounded by the coordinate axes, the straight line  $l$  with equation  $y = \frac{1}{3}x + 5$  and the curve  $C$

with equation  $y = 4x^{\frac{1}{2}} - x + 5$ ,  $x \geq 0$

The line  $l$  meets the curve  $C$  at the point  $D$  on the  $y$ -axis and at the point  $E$ , as shown in Figure 1.

(a) Use algebra to find the coordinates of the points  $D$  and  $E$ . (4)

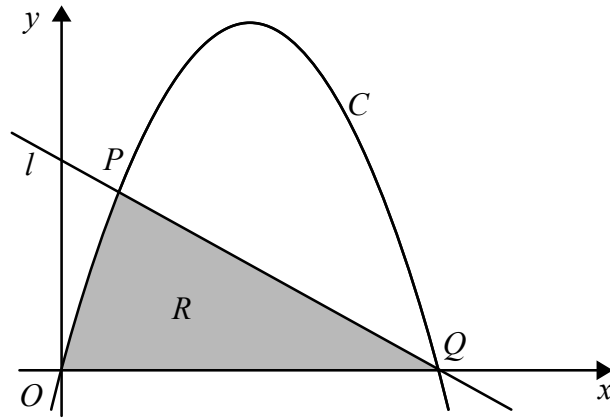
The curve  $C$  crosses the  $x$ -axis at the point  $F$ .

(b) Verify that the  $x$  coordinate of  $F$  is 25 (1)

(c) Use algebraic integration to find the exact area of the shaded region  $R$ . (6)

(IAL C12 Oct 2018, Q10)

53.



**Figure 3**

The straight line  $l$  with equation  $y = 5 - 3x$  cuts the curve  $C$ , with equation  $y = 20x - 12x^2$ , at the points  $P$  and  $Q$ , as shown in Figure 3.

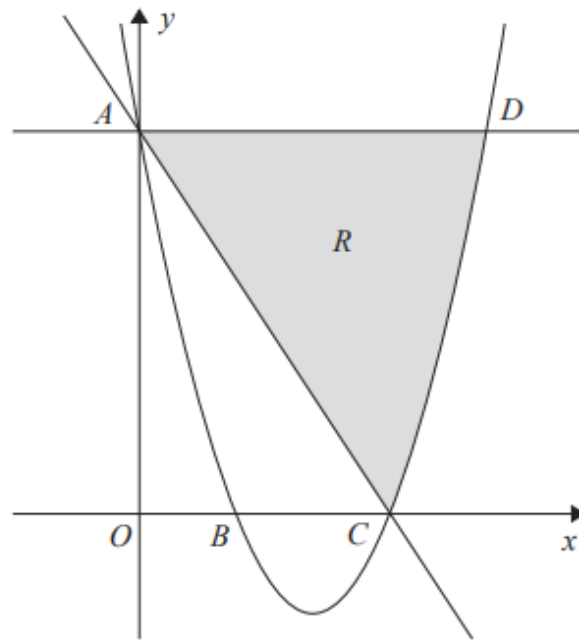
(a) Use algebra to find the exact coordinates of the points  $P$  and  $Q$ . (5)

The finite region  $R$ , shown shaded in Figure 3, is bounded by the line  $l$ , the  $x$ -axis and the curve  $C$ .

(b) Use calculus to find the exact area of  $R$ . (6)

(IAL C12 Jan 2019, Q15)

54.



**Figure 4**

Figure 4 shows a sketch of the curve with equation  $y = 2x^2 - 11x + 12$ . The curve crosses the  $y$ -axis at the point  $A$  and crosses the  $x$ -axis at the points  $B$  and  $C$ .

(a) Find the coordinates of the points  $A$ ,  $B$  and  $C$ .

**(3)**

The point  $D$  lies on the curve such that the line  $AD$  is parallel to the  $x$ -axis.

The finite region  $R$ , shown shaded in Figure 4, is bounded by the curve, the line  $AC$  and the line  $AD$ .

(b) Use algebraic integration to find the exact area of  $R$ .

**(7)**

**(IAL C12 May 2019, Q16)**