Edexcel

Pure Mathematics

Year 1

Integration 1

Past paper questions from Core Maths 1

Edited by: K V Kumaran

Past paper questions from

Edexcel Core Maths 1.

From Jan 2005 to May 2019.

Integration 01

This Section 1 has 45 Questions on application on integration.

Please check the Edexcel website for the solutions.

1. (i) Given that $y = 5x^3 + 7x + 3$, find

(*a*) $\dfrac{dy}{dx}$,

(3)

(*b*) $\dfrac{d^2y}{dx^2}$.

(1)

(ii) Find $\int \left(1 + 3\sqrt{x} - \dfrac{1}{x^2}\right) dx$.

(4)

(C1 Jan 2005, Q2)

2. The gradient of the curve C is given by

$$\frac{dy}{dx} = (3x - 1)^2.$$

The point $P(1, 4)$ lies on C.

(*a*) Find an equation of the normal to C at P.

(4)

(*b*) Find an equation for the curve C in the form $y = f(x)$.

(5)

(*c*) Using $\dfrac{dy}{dx} = (3x - 1)^2$, show that there is no point on C at which the tangent is parallel to the line $y = 1 - 2x$.

(2)

(C1 Jan 2005, Q9)

3. Given that $y = 6x - \dfrac{4}{x^2}$, $x \neq 0$,

(*a*) find $\dfrac{dy}{dx}$,

(2)

(*b*) find $\int y \, dx$.

(3)

(C1 May 2005, Q2)

4. Given that $y = 2x^2 - \dfrac{6}{x^3}$, $x \neq 0$,

(*a*) find $\dfrac{dy}{dx}$,

(2)

(*b*) find $\int y \, dx$.

(3)

(C1 Jan 2006, Q4)

5. The curve with equation $y = \mathrm{f}(x)$ passes through the point (1, 6). Given that

$$\mathrm{f}'(x) = 3 + \frac{5x^2 + 2}{x^{\frac{1}{2}}}, \quad x > 0,$$

find f(x) and simplify your answer. **(7)**

(C1 Jan 2006, Q8)

6. Find $\int (6x^2 + 2 + x^{-\frac{1}{2}})\ \mathrm{d}x$, giving each term in its simplest form.

(4)

(C1 May 2006, Q1)

7. The curve C with equation $y = \mathrm{f}(x)$, $x \neq 0$, passes through the point $(3, 7\frac{1}{2})$.

Given that $\mathrm{f}'(x) = 2x + \dfrac{3}{x^2}$,

(a) find f(x).

(5)

(b) Verify that f(–2) = 5.

(1)

(c) Find an equation for the tangent to C at the point (–2, 5), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

(C1 May 2006, Q10)

8. (a) Show that $(4 + 3\sqrt{x})^2$ can be written as $16 + k\sqrt{x} + 9x$, where k is a constant to be found.

(2)

(b) Find $\int (4 + 3\sqrt{x})^2\ \mathrm{d}x$.

(3)

(C1 Jan 2007, Q6)

9. The curve C has equation $y = \mathrm{f}(x)$, $x \neq 0$, and the point $P(2, 1)$ lies on C. Given that

$$\mathrm{f}'(x) = 3x^2 - 6 - \frac{8}{x^2},$$

(a) find f(x).

(5)

(b) Find an equation for the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are integers.

(4)

(C1 Jan 2007, Q7)

10. Given that $y = 3x^2 + 4\sqrt{x}$, $x > 0$, find

(a) $\dfrac{dy}{dx}$,

(2)

(b) $\dfrac{d^2y}{dx^2}$,

(2)

(c) $\int y \ dx$.

(3)

(C1 May 2007, Q3)

11. The curve C with equation $y = f(x)$ passes through the point (5, 65).

Given that $f'(x) = 6x^2 - 10x - 12$,

(a) use integration to find $f(x)$.

(4)

(b) Hence show that $f(x) = x(2x + 3)(x - 4)$.

(2)

(c) Sketch C, showing the coordinates of the points where C crosses the x-axis. **(3)**

(C1 May 2007, Q9)

12. Find $\int (3x^2 + 4x^5 - 7) \ dx$.

(4)

(C1 Jan 2008, Q1)

13. The curve C has equation $y = f(x)$, $x > 0$, and $f'(x) = 4x - 6\sqrt{x} + \dfrac{8}{x^2}$.

Given that the point $P(4, 1)$ lies on C,

(a) find $f(x)$ and simplify your answer.

(6)

(b) Find an equation of the normal to C at the point $P(4, 1)$.

(4)

(C1 Jan 2008, Q9)

14. Find $\int (2 + 5x^2) \ dx$.

(3)

(C1 June 2008, Q1)

15. The gradient of a curve C is given by $\frac{dy}{dx} = \frac{(x^2+3)^2}{x^2}$, $x \neq 0$.

(a) Show that $\frac{dy}{dx} = x^2 + 6 + 9x^{-2}$.

(2)

The point (3, 20) lies on C.

(b) Find an equation for the curve C in the form $y = f(x)$.

(6)

(C1 June 2008, Q11)

16. Find $\int (12x^5 - 8x^3 + 3)\ dx$, giving each term in its simplest form.

(4)

(C1 Jan 2009, Q2)

17. A curve has equation $y = f(x)$ and passes through the point (4, 22).

Given that

$$f'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7,$$

use integration to find f(x), giving each term in its simplest form.

(5)

(C1 Jan 2009, Q5)

18. Given that $y = 2x^3 + \frac{3}{x^2}$, $x \neq 0$, find

(a) $\frac{dy}{dx}$,

(3)

(b) $\int y\ dx$, simplifying each term.

(3)

(C1 June 2009, Q3)

19.

$$\frac{dy}{dx} = 5x^{-\frac{1}{2}} + x\sqrt{x}, \qquad x > 0.$$

Given that $y = 35$ at $x = 4$, find y in terms of x, giving each term in its simplest form.

(7)

(C1 Jan 2010, Q4)

20. Find

$$\int (8x^3 + 6x^{\frac{1}{2}} - 5)\ dx,$$

giving each term in its simplest form.

(4)

(C1 May 2010, Q2)

21. The curve C has equation $y = \mathrm{f}(x), \quad x > 0$, where

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x - \frac{5}{\sqrt{x}} - 2.$$

Given that the point P (4, 5) lies on C, find

(*a*) $\mathrm{f}(x)$,

(5)

(*b*) an equation of the tangent to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

(C1 May 2010, Q11)

22. Find

$$\int (12x^5 - 3x^2 + 4x^{\frac{1}{3}}) \; \mathrm{d}x,$$

giving each term in its simplest form.

(5)

(C1 Jan 2011, Q2)

23. The curve with equation $y = \mathrm{f}(x)$ passes through the point $(-1, 0)$.

Given that

$$\mathrm{f}'(x) = 12x^2 - 8x + 1,$$

find $\mathrm{f}(x)$.

(5)

(C1 Jan 2011, Q7)

24. Given that $y = 2x^5 + 7 + \dfrac{1}{x^3}$, $x \neq 0$, find, in their simplest form,

(*a*) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$,

(3)

(*b*) $\displaystyle\int y \; \mathrm{d}x$.

(4)

(C1 May 2011, Q2)

25. Given that $y = x^4 + 6x^{\frac{1}{2}}$, find in their simplest form

(*a*) $\dfrac{dy}{dx}$,

(3)

(*b*) $\int y \, dx$.

(3)

(C1 Jan 2012, Q1)

26. A curve with equation $y = f(x)$ passes through the point (2, 10). Given that

$$f'(x) = 3x^2 - 3x + 5,$$

find the value of f(1).

(5)

(C1 Jan 2012, Q7)

27. Find

$$\int \left(6x^2 + \frac{2}{x^2} + 5 \right) dx,$$

giving each term in its simplest form.

(4)

(C1 May 2012, Q1)

28. The point *P* (4, –1) lies on the curve *C* with equation $y = f(x)$, $x > 0$, and

$$f'(x) = \frac{1}{2}x - \frac{6}{\sqrt{x}} + 3.$$

(*a*) Find the equation of the tangent to *C* at the point *P*, giving your answer in the form $y = mx + c$, where *m* and *c* are integers.

(4)

(*b*) Find f(*x*).

(4)

(C1 May 2012, Q7)

29. $$\frac{dy}{dx} = -x^3 + \frac{4x - 5}{2x^3}, \quad x \neq 0.$$

Given that $y = 7$ at $x = 1$, find y in terms of x, giving each term in its simplest form.

(6)

(C1 Jan 2013, Q8)

30. Find

$$\int \left(3x^2 - \frac{4}{x^2}\right) dx,$$

giving each term in its simplest form.

(4)

(C1 May 2013R, Q3)

31. A curve has equation $y = \text{f}(x)$. The point P with coordinates (9, 0) lies on the curve.

Given that

$$\text{f}'(x) = \frac{x+9}{\sqrt{x}}, \qquad x > 0,$$

(*a*) find $\text{f}(x)$.

(6)

(*b*) Find the x-coordinates of the two points on $y = \text{f}(x)$ where the gradient of the curve is equal to 10.

(4)

(C1 May 2013R, Q10)

32. Find

$$\int \left(10x^4 - 4x - \frac{3}{\sqrt{x}}\right) dx,$$

giving each term in its simplest form.

(4)

(C1 May 2013, Q2)

33.

$$\text{f}'(x) = \frac{(3 - x^2)^2}{x^2}, \qquad x \neq 0.$$

(*a*) Show that $\text{f}'(x) = 9x^{-2} + A + Bx^2$, where A and B are constants to be found.

(3)

(*b*) Find $\text{f}''(x)$.

(2)

Given that the point (–3, 10) lies on the curve with equation $y = \text{f}(x)$,

(*c*) find $\text{f}(x)$.

(5)

(C1 May 2013, Q9)

34. Given that $y = 2x^5 + \frac{6}{\sqrt{x}}$, $x > 0$, find in their simplest form

(*a*) $\frac{dy}{dx}$

(3)

(*b*) $\int y dx$

(3)
(C1 May 2014R, Q4)

35. $$\frac{dy}{dx} = 6x^{-\frac{1}{2}} + x\sqrt{x}, \qquad x > 0$$

Given that $y = 37$ at $x = 4$, find y in terms of x, giving each term in its simplest form.

(7)
(C1 May 2014R, Q8)

36. Find $\int (8x^3 + 4) \; dx$, giving each term in its simplest form.

(3)
(C1 May 2014, Q1)

37. A curve with equation $y = f(x)$ passes through the point (4, 25).

Given that $f'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1, \quad x > 0,$

(*a*) find $f(x)$, simplifying each term.

(5)

(*b*) Find an equation of the normal to the curve at the point (4, 25). Give your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)
(C1 May 2014, Q10)

38. Given that $y = 4x^3 - \frac{5}{x^2}, \quad x \neq 0,$ find in their simplest form

(*a*) $\frac{dy}{dx}$,

(3)

(*b*) $\int y \; dx$.

(3)
(C1 May 2015, Q3)

39. A curve with equation $y = \mathrm{f}(x)$ passes through the point (4, 9).

Given that

$$\mathrm{f}'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2, \quad x > 0,$$

(*a*) find f(*x*), giving each term in its simplest form.

(5)

Point *P* lies on the curve.

The normal to the curve at *P* is parallel to the line $2y + x = 0$.

(*b*) Find the *x*-coordinate of *P*.

(5)

(C1 May 2015, Q10)

40. Find

$$\int \left(2x^4 - \frac{4}{\sqrt{x}} + 3 \right) \mathrm{d}x$$

giving each term in its simplest form.

(4)

(C1 May 2016, Q1)

41. Find

$$\int \left(2x^5 - \frac{1}{4x^3} - 5 \right) \mathrm{d}x$$

giving each term in its simplest form.

(4)

(C1 May 2017, Q1)

42. The curve *C* has equation $y = \mathrm{f}(x)$, $x > 0$, where

$$\mathrm{f}'(x) = 30 + \frac{6 - 5x^2}{\sqrt{x}}$$

Given that the point *P*(4, –8) lies on *C*,

(*a*) find the equation of the tangent to *C* at *P*, giving your answer in the form $y = mx + c$, where *m* and *c* are constants.

(4)

(*b*) Find f(*x*), giving each term in its simplest form.

(5)

(C1 May 2017, Q7)

43. Given

$$y = 3\sqrt{x} - 6x + 4, \qquad x > 0$$

(*a*) find $\int y \, \mathrm{d}x$, simplifying each term.

(3)

(*b*) (i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$

(ii) Hence find the value of x such that $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

(4)

(C1 May 2018, Q2)

44. The curve C has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}'(x) = (x - 3)(3x + 5)$$

Given that the point P (1, 20) lies on C,

(*a*) find $\mathrm{f}(x)$, simplifying each term.

(5)

(*b*) Show that

$$\mathrm{f}(x) = (x - 3)^2 (x + A)$$

where A is a constant to be found.

(3)

(*c*) Sketch the graph of C. Show clearly the coordinates of the points where C cuts or meets the x-axis and where C cuts the y-axis.

(4)

(C1 May 2018, Q9)

45.

Given that $y = 4$ when $x = 1$ and that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 12x^2 + \frac{4x + 2}{3x^4} \qquad x \neq 0$$

find y in terms of x, giving each term in a simplified form.

(8)

(C1 May 2019, Q6)