OCR Core Maths 4

Past paper questions Integrations

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Differentiation & Integration

- Know the contents of the formula booklet well. Very well! Lots of problems can be solved simply by looking at the table of differentials and integrals and knowing that integration 'undoes' a differentiation. Some questions get you to differentiate something and *then* get you to integrate something similar. *Always view the question as a whole!*

- When using radians we can differentiate the trigonometric functions. The results are as follows:

$$y = \sin x \qquad y = \cos x \qquad y = \tan x$$
$$\frac{dy}{dx} = \cos x, \qquad \frac{dy}{dx} = -\sin x, \qquad \frac{dy}{dx} = \sec^2 x.$$

  One can derive the third result from the other two using the quotient rule and that $\tan x \equiv \frac{\sin x}{\cos x}$.

- You can also use these results along with the chain rule to differentiate functions like the following; $y = \sin(x^2+1)$ by letting $u = x^2+1$ and $y = (\tan x)^{10}$ by letting $u = \tan x$.

$$y = \sin(x^2+1) \qquad y = (\tan x)^{10}$$
$$\frac{dy}{dx} = 2x\cos(x^2+1), \qquad \frac{dy}{dx} = 10\sec^2 x(\tan x)^9.$$

- Integration by substitution is a way of integrating by replacing the variable given to you (usually x) and replacing it by another (usually u). These days the substitution you are to use is given to you in the exam, but practice will get you better at spotting what to substitute (usually the most complicated term in the integration or the denominator of a fraction). For example $\int x^3(x^4+1)^7\,dx$ we should use $u = x^4+1$.

$$\begin{aligned}
&\int x^3(x^4+1)^7\,dx && u = x^4+1\\
=&\int x^3u^7\,dx && \frac{du}{dx} = 4x^3\\
=&\int x^3u^7\,\frac{du}{4x^3} && \frac{du}{4x^3} = dx\\
=&\frac{1}{4}\int u^7\,du\\
=&\frac{u^8}{32}+c = \frac{(x^4+1)^8}{32}+c.
\end{aligned}$$

  We have effectively *"used and abused"* u to help us to get the answer. (NOTE: I have been *very* sloppy in the above integration because I have mixed my x and u variables; you shouldn't really do this, but it makes the process of conversion clearer.)

- When dealing with definite integrals we need to also convert the limits of the integration and there is no need to convert back to x at the end since all definite integrals are merely numbers. For example

$$\begin{aligned} &\int_3^4 2x\sqrt{x^2-4}\,dx & u &= x^2-4 & x=3 &\Rightarrow u=5 \\ &= \int_5^{12} 2xu^{1/2}\frac{du}{2x} & \frac{du}{dx} &= 2x & x=4 &\Rightarrow u=12 \\ &= \int_5^{12} u^{1/2}\,du & \frac{du}{2x} &= dx \\ &= \left[\frac{2}{3}u^{3/2}\right]_5^{12} \\ &= 20.3 \text{ (3sf)}. \end{aligned}$$

- Know the result $\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$.

- We know that if $y = e^{f(x)}$ then $\frac{dy}{dx} = f'(x)e^{f(x)}$. Therefore by reversal we find

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c.$$

For example[2]

$$\int x^3e^{x^4}\,dx = \frac{1}{4}\int 4x^3e^{x^4}\,dx = \frac{1}{4}e^{x^4} + c.$$

- Know that $\int \frac{1}{x}\,dx = \ln x + c$.

- We know (by the chain rule) that if $y = \ln(f(x))$ then $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$. Therefore by reversal we find

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c.$$

Be on the lookout for expressions where the top line is almost the derivative of the bottom line. For example[3]

$$\int \frac{x^3}{x^4+1}\,dx = \frac{1}{4}\int \frac{4x^3}{x^4+1}\,dx = \frac{1}{4}\ln|x^4+1| + c.$$

- Know the results

$$\int \cos ax\,dx = \frac{1}{a}\sin ax + c \qquad \text{and} \qquad \int \sin ax\,dx = -\frac{1}{a}\cos ax + c.$$

- Always be on the look out for integrals involving a mixture of trigonometric functions. These are usually handled by means of a substitution. For example $\int \cos x(\sin x)^7\,dx$ is best handled by $u = \sin x$ to give $\frac{1}{8}(\sin x)^8 + c$.

- Also know the useful results (all derived from reverse chain rule)

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c \qquad \text{and} \qquad \int f'(x)\sin f(x)\,dx = -\cos f(x) + c.$$

  For example $\int x^3\cos(x^4)\,dx = \frac{1}{4}\sin(x^4) + c$.

- When an integral is made up of two 'bits' then we can sometimes use *integration by parts*. It states

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx.$$

  So you will need to decide which 'bit' of the integral you will need to differentiate and which 'part' to integrate. For example in $\int x\sin x\,dx$ it is quite clear that we will need to differentiate the x 'part' and integrate the $\sin x$ 'part'.

$$\begin{aligned}\int x\sin x\,dx &= -x\cos x - \int -\cos x\,dx\\ &= -x\cos x + \sin x + c.\end{aligned}$$

- Another example (this time a definite integral)

$$\begin{aligned}\int_0^2 xe^{2x}\,dx &= \left[\tfrac{1}{2}xe^{2x}\right]_0^2 - \int_0^2 \tfrac{1}{2}e^{2x}\,dx\\ &= \left[\tfrac{1}{2}xe^{2x}\right]_0^2 - \left[\tfrac{1}{4}e^{2x}\right]_0^2\\ &= (e^4 - 0) - \left(\tfrac{e^4}{4} - \tfrac{1}{4}\right) = \tfrac{3e^4}{4} + \tfrac{1}{4}.\end{aligned}$$

- Initially $\int \ln x\,dx$ looks nothing like it has anything to do with integration by parts because it only has one 'part'. However if we write $\ln x$ as $1 \times \ln x$ we can integrate the 1 and differentiate the $\ln x$:

$$\int \ln x\,dx = \int 1\ln x\,dx = x\ln x - \int x.\tfrac{1}{x}\,dx = x\ln x - x + c.$$

  This principle can be extended to integrals of the type $\int x^n\ln x\,dx$:

$$\int x^n\ln x\,dx = \tfrac{x^{n+1}}{n+1}\ln x - \int \tfrac{x^{n+1}}{n+1}.\tfrac{1}{x}\,dx = \tfrac{x^{n+1}}{n+1}\ln x - \tfrac{x^{n+1}}{(n+1)^2} + c.$$

- Very occasionally you will need to integrate by parts *twice* to get the final answer. This will almost always be of the form $\int kx^2(\text{something})\,dx$. For example find $\int x^2e^{2x}\,dx$:

$$\begin{aligned}\int x^2e^{2x}\,dx &= \frac{x^2}{2}e^{2x} - \left(\int xe^{2x}\,dx\right)\\ &= \frac{x^2}{2}e^{2x} - \left(\frac{x}{2}e^{2x} - \int \frac{1}{2}e^{2x}\,dx\right)\\ &= \frac{x^2}{2}e^{2x} - \left(\frac{x}{2}e^{2x} - \frac{1}{4}e^{2x}\right) + c\\ &= \frac{x^2}{2}e^{2x} - \frac{x}{2}e^{2x} + \frac{1}{4}e^{2x} + c.\end{aligned}$$

- For the cases of $\int \sin^2 x\,dx$ and $\int \cos^2 x\,dx$ we need to recall two forms of the double angle formula for $\cos 2x$: Namely $\cos 2x = 1 - 2\sin^2 x$ (for $\int \sin^2 x\,dx$) and $\cos 2x = 2\cos^2 x - 1$ (for $\int \cos^2 x\,dx$). Re-arranging them both we find:

$$\begin{aligned}\int \sin^2 x\,dx &= \int (\tfrac{1}{2} - \tfrac{1}{2}\cos 2x)\,dx = \tfrac{x}{2} - \tfrac{1}{4}\sin 2x + c,\\ \int \cos^2 x\,dx &= \int (\tfrac{1}{2} + \tfrac{1}{2}\cos 2x)\,dx = \tfrac{x}{2} + \tfrac{1}{4}\sin 2x + c.\end{aligned}$$

  Learn the technique rather than the result!

1.

Evaluate $\int_0^{\frac{1}{2}\pi} x\cos x\,\mathrm{d}x$, giving your answer in an exact form. [5]

Q2 June 2005

2.

(i) Show that the substitution $x = \tan\theta$ transforms $\int \frac{1}{(1+x^2)^2}\,\mathrm{d}x$ to $\int \cos^2\theta\,\mathrm{d}\theta$. [3]

(ii) Hence find the exact value of $\int_0^1 \frac{1}{(1+x^2)^2}\,\mathrm{d}x$. [4]

Q4 June 2005

3.

(i) Use integration by parts to find $\int x\sec^2 x\,\mathrm{d}x$. [4]

(ii) Hence find $\int x\tan^2 x\,\mathrm{d}x$. [3]

Q4 Jan 2006

4.

(i) Show that the substitution $x = \sin^2\theta$ transforms $\int \sqrt{\frac{x}{1-x}}\,\mathrm{d}x$ to $\int 2\sin^2\theta\,\mathrm{d}\theta$. [4]

(ii) Hence find $\int_0^1 \sqrt{\frac{x}{1-x}}\,\mathrm{d}x$. [5]

Q6 Jan 2006

5.

(i) Express $\frac{3-2x}{x(3-x)}$ in partial fractions. [3]

(ii) Show that $\int_1^2 \frac{3-2x}{x(3-x)}\,\mathrm{d}x = 0$. [4]

(iii) What does the result of part **(ii)** indicate about the graph of $y = \frac{3-2x}{x(3-x)}$ between $x = 1$ and $x = 2$? [1]

Q3 June 2006

6.

(i) Show that the substitution $u = e^x + 1$ transforms $\int \frac{e^{2x}}{e^x + 1}\,dx$ to $\int \frac{u-1}{u}\,du$. [3]

(ii) Hence show that $\int_0^1 \frac{e^{2x}}{e^x + 1}\,dx = e - 1 - \ln\left(\frac{e+1}{2}\right)$. [5]

Q6 June 2006

7.

(i) Show that $\int \cos^2 6x\,dx = \frac{1}{2}x + \frac{1}{24}\sin 12x + c$. [3]

(ii) Hence find the exact value of $\int_0^{\frac{1}{12}\pi} x\cos^2 6x\,dx$. [6]

Q8 June 2006

8.

Find the exact value of $\int_1^2 x\ln x\,dx$. [5]

Q2 Jan 2007

9.

Use the substitution $u = 2x - 5$ to show that $\int_{\frac{5}{2}}^{3} (4x-8)(2x-5)^7\,dx = \frac{17}{72}$. [5]

Q4 Jan 2007

10.

(i) Express $\frac{2x+1}{(x-3)^2}$ in the form $\frac{A}{x-3} + \frac{B}{(x-3)^2}$, where A and B are constants. [3]

(ii) Hence find the exact value of $\int_4^{10} \frac{2x+1}{(x-3)^2}\,dx$, giving your answer in the form $a + b\ln c$, where a, b and c are integers. [4]

Q6 Jan 2007

11.

Find the exact value of $\int_0^1 x^2 e^x\,dx$. [6]

Q2 June 2007

12.

Find the exact volume generated when the region enclosed between the x-axis and the portion of the curve $y = \sin x$ between $x = 0$ and $x = \pi$ is rotated completely about the x-axis. [6]

Q3 June 2007

13.

(i) Find the quotient and the remainder when $2x^3 + 3x^2 + 9x + 12$ is divided by $x^2 + 4$. [4]

(ii) Hence express $\dfrac{2x^3 + 3x^2 + 9x + 12}{x^2 + 4}$ in the form $Ax + B + \dfrac{Cx + D}{x^2 + 4}$, where the values of the constants A, B, C and D are to be stated. [1]

(iii) Use the result of part **(ii)** to find the exact value of $\displaystyle\int_1^3 \frac{2x^3 + 3x^2 + 9x + 12}{x^2 + 4}\,dx$. [5]

Q7 June 2007

14.

(i) Express $\dfrac{x}{(x+1)(x+2)}$ in partial fractions. [3]

(ii) Hence find $\displaystyle\int \frac{x}{(x+1)(x+2)}\,dx$. [2]

Q2 Jan 2008

15.

(i) Given that

$$A(\sin\theta + \cos\theta) + B(\cos\theta - \sin\theta) \equiv 4\sin\theta,$$

find the values of the constants A and B. [3]

(ii) Hence find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{4\sin\theta}{\sin\theta + \cos\theta}\,d\theta,$$

giving your answer in the form $a\pi - \ln b$. [5]

Q7 Jan 2008

16.

(i) Use the substitution $x = \sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}}\,\mathrm{d}x.$$ [6]

(ii) Find the exact value of

$$\int_1^3 \frac{\ln x}{x^2}\,\mathrm{d}x.$$ [5]

Q10 Jan 2008

17.

Find the exact value of $\int_1^{\mathrm{e}} x^4 \ln x\,\mathrm{d}x$. [5]

Q2 June 2008

18.

(i) Given that $\frac{2t}{(t+1)^2}$ can be expressed in the form $\frac{A}{t+1} + \frac{B}{(t+1)^2}$, find the values of the constants A and B. [3]

(ii) Show that the substitution $t = \sqrt{2x-1}$ transforms $\int \frac{1}{x+\sqrt{2x-1}}\,\mathrm{d}x$ to $\int \frac{2t}{(t+1)^2}\,\mathrm{d}t$. [4]

(iii) Hence find the exact value of $\int_1^5 \frac{1}{x+\sqrt{2x-1}}\,\mathrm{d}x$. [4]

Q8 June 2008

19.

Find $\int x\sec^2 x\,\mathrm{d}x$. **[4]**

Q2 Jan 2009

20.

Find the exact value of $\int_0^{\frac{1}{4}\pi} (1+\sin x)^2\,\mathrm{d}x$. **[6]**

Q4 Jan 2009

21.

(i) Show that the substitution $u = \sqrt{x}$ transforms $\int \frac{1}{x(1+\sqrt{x})}\,dx$ to $\int \frac{2}{u(1+u)}\,du$. **[3]**

(ii) Hence find the exact value of $\int_1^9 \frac{1}{x(1+\sqrt{x})}\,dx$. **[5]**

Q5 Jan 2009

22.

Use the substitution $x = \tan\theta$ to find the exact value of

$$\int_1^{\sqrt{3}} \frac{1-x^2}{1+x^2}\,dx.$$ [7]

Q2 June 2009

23.

(i) Differentiate $e^x(\sin 2x - 2\cos 2x)$, simplifying your answer. **[4]**

(ii) Hence find the exact value of $\int_0^{\frac{1}{4}\pi} e^x \sin 2x\,dx$. **[3]**

Q4 June 2009

24.

The expression $\frac{4x}{(x-5)(x-3)^2}$ is denoted by $f(x)$.

(i) Express $f(x)$ in the form $\frac{A}{x-5} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$, where A, B and C are constants. **[4]**

(ii) Hence find the exact value of $\int_1^2 f(x)\,dx$. **[5]**

Q6 June 2009

25.

By expressing $\cos 2x$ in terms of $\cos x$, find the exact value of $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \frac{\cos 2x}{\cos^2 x}\,dx$. **[5]**

Q3 Jan 2010

26.

Use the substitution $u = 2 + \ln t$ to find the exact value of

$$\int_1^{e} \frac{1}{t(2+\ln t)^2}\,dt.$$ **[6]**

Q4 Jan 2010

27.

(i) State the derivative of $e^{\cos x}$. **[1]**

(ii) Hence use integration by parts to find the exact value of

$$\int_0^{\frac{1}{2}\pi} \cos x \sin x\, e^{\cos x}\,dx.$$ **[6]**

Q8 Jan 2010

28.

Use the substitution $u = \sqrt{x+2}$ to find the exact value of

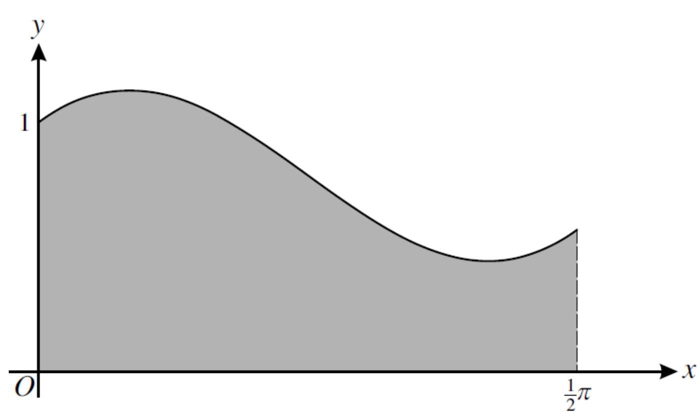
$$\int_{-1}^{7} \frac{x^2}{\sqrt{x+2}}\,dx.$$ **[7]**

Q4 June 2010

29.

(i) Find $\int (x + \cos 2x)^2\,dx$. **[9]**

(ii)



The diagram shows the part of the curve $y = x + \cos 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The shaded region bounded by the curve, the axes and the line $x = \frac{1}{2}\pi$ is rotated completely about the x-axis to form a solid of revolution of volume V. Find V, giving your answer in an exact form. **[4]**

Q9 June 2010

30.

(i) Express $\dfrac{7-2x}{(x-2)^2}$ in the form $\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}$, where A and B are constants. **[3]**

(ii) Hence find the exact value of $\displaystyle\int_4^5 \frac{7-2x}{(x-2)^2}\,dx$. **[4]**

Q2 Jan 2011

31.

(i) Show that the derivative of $\sec x$ can be written as $\sec x \tan x$. **[4]**

(ii) Find $\displaystyle\int \frac{\tan x}{\sqrt{1+\cos 2x}}\,dx$. **[4]**

Q3 Jan 2011

32.

In this question, I denotes the definite integral $\displaystyle\int_2^5 \frac{5-x}{2+\sqrt{x-1}}\,dx$. The value of I is to be found using two different methods.

(i) Show that the substitution $u=\sqrt{x-1}$ transforms I to $\displaystyle\int_1^2 (4u-2u^2)\,du$ and hence find the exact value of I. **[5]**

(ii) **(a)** Simplify $\left(2+\sqrt{x-1}\right)\left(2-\sqrt{x-1}\right)$. **[1]**

(b) By first multiplying the numerator and denominator of $\dfrac{5-x}{2+\sqrt{x-1}}$ by $2-\sqrt{x-1}$, find the exact value of I. **[3]**

Q5 Jan 2011

33.

Show that $\displaystyle\int_0^{\pi} (x^2+5x+7)\sin x\,dx = \pi^2+5\pi+10$. **[7]**

Q7 Jan 2011

34.

(i) Find the quotient when $3x^3 - x^2 + 10x - 3$ is divided by $x^2 + 3$, and show that the remainder is x. **[4]**

(ii) Hence find the exact value of

$$\int_0^1 \frac{3x^3 - x^2 + 10x - 3}{x^2 + 3}\, dx.$$ **[4]**

Q3 June 2011

35.

Use the substitution $x = \frac{1}{3}\sin\theta$ to find the exact value of

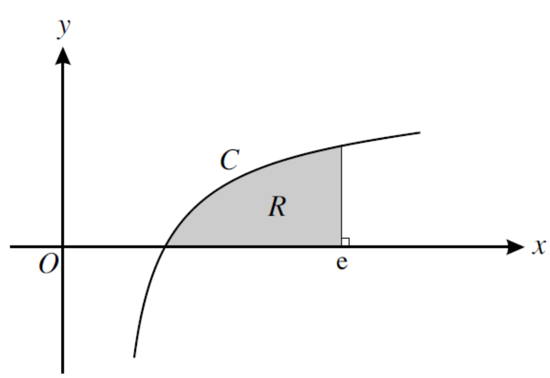
$$\int_0^{\frac{1}{6}} \frac{1}{(1 - 9x^2)^{\frac{3}{2}}}\, dx.$$ **[6]**

Q4 June 2011

36.

(i) Show that $\frac{d}{dx}(x\ln x - x) = \ln x$. **[3]**

(ii)



In the diagram, C is the curve $y = \ln x$. The region R is bounded by C, the x-axis and the line $x = e$.

(a) Find the exact volume of the solid of revolution formed by rotating R completely about the x-axis. **[6]**

(b) The region R is rotated completely about the y-axis. Explain why the volume of the solid of revolution formed is given by

$$\pi e^2 - \pi \int_0^1 e^{2y}\, dy,$$

and find this volume. **[4]**

Q9 June 2011

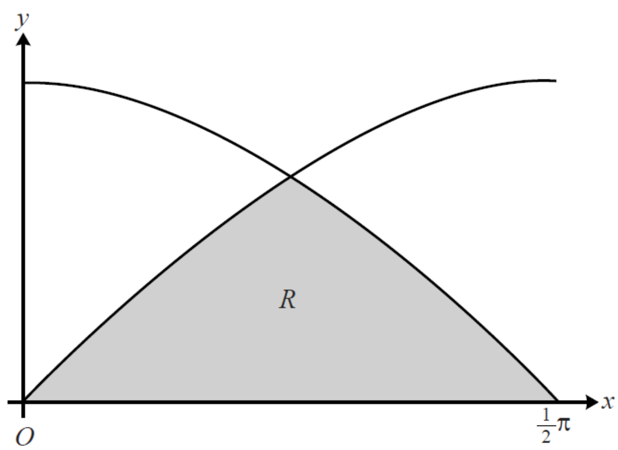
37.

Use the substitution $u = \cos x$ to find the exact value of

$$\int_0^{\frac{1}{3}\pi} \sin^3 x \cos^2 x \, dx \, .$$ **[6]**

Q5 Jan 2012

38.



The diagram shows the curves $y = \cos x$ and $y = \sin x$, for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The region R is bounded by the curves and the x-axis. Find the volume of the solid of revolution formed when R is rotated completely about the x-axis, giving your answer in terms of π. **[7]**

Q6 Jan 2012

39.

Find the exact value of $\int_0^1 (x^2 + 1)e^{2x} \, dx$. **[7]**

Q9 Jan 2012

40.

Use integration by parts to find $\int \ln(x + 2) \, dx$. **[5]**

Q2 June 2012

41.

Use the substitution $u = 1 + \sqrt{x}$ to show that

$$\int_4^9 \frac{1}{1+\sqrt{x}}\,dx = 2 + 2\ln\tfrac{3}{4}.$$ **[7]**

Q6 June 2012

42.

Find the exact value of $\int_0^{\frac{1}{6}\pi} (1 - \sin 3x)^2\,dx$. **[7]**

Q7 June 2012

43.

(i) Express $\frac{x^2 - x - 11}{(x+1)(x-2)^2}$ in partial fractions. **[5]**

(ii) Find the exact value of $\int_3^4 \frac{x^2 - x - 11}{(x+1)(x-2)^2}\,dx$, giving your answer in the form $a + \ln b$, where a and b are rational numbers. **[4]**

Q9 June 2012

44.

Find $\int x \cos 3x\,dx$. **[4]**

Q1 Jan 2013

45.

Use the substitution $u = 2x + 1$ to evaluate $\int_0^{\frac{1}{2}} \frac{4x - 1}{(2x+1)^5}\,dx$. **[7]**

Q6 Jan 2013

46.

(i) Given that $y = \ln(1 + \sin x) - \ln(\cos x)$, show that $\frac{dy}{dx} = \frac{1}{\cos x}$. **[4]**

(ii) Using this result, evaluate $\int_0^{\frac{1}{3}\pi} \sec x\,dx$, giving your answer as a single logarithm. **[3]**

Q7 Jan 2013

47.

(i) Use algebraic division to express $\dfrac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6}$ in the form $Ax + B + \dfrac{Cx + D}{x^2 - x - 6}$, where A, B, C and D are constants. **[4]**

(ii) Hence find $\displaystyle\int_4^6 \frac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6}\,dx$, giving your answer in the form $a + \ln b$. **[7]**

Q10 Jan 2013

48.

Find $\displaystyle\int x^8 \ln(3x)\,dx$. **[5]**

Q2 June 2013

49.

(i) Show that $\dfrac{1}{1 - \tan x} - \dfrac{1}{1 + \tan x} \equiv \tan 2x$. **[2]**

(ii) Hence evaluate $\displaystyle\int_{\frac{1}{12}\pi}^{\frac{1}{6}\pi} \left(\frac{1}{1 - \tan x} - \frac{1}{1 + \tan x}\right) dx$, giving your answer in the form $a \ln b$. **[5]**

Q5 June 2013

50.

Use the substitution $u = 1 + \ln x$ to find $\displaystyle\int \frac{\ln x}{x(1 + \ln x)^2}\,dx$. **[6]**

Q6 June 2013

51.

Show that $\displaystyle\int_0^{\frac{1}{4}\pi} \frac{1 - 2\sin^2 x}{1 + 2\sin x \cos x}\,dx = \tfrac{1}{2}\ln 2$. **[5]**

Q4 June 2014

52.

(i) Use division to show that $\dfrac{t^3}{t+2} \equiv t^2 - 2t + 4 - \dfrac{8}{t+2}$. **[3]**

(ii) Find $\displaystyle\int_1^2 6t^2 \ln(t+2)\,dt$. Give your answer in the form $A + B\ln 3 + C\ln 4$. **[6]**

Q8 June 2014

53.

Express $\dfrac{2+x^2}{(1+2x)(1-x)^2}$ in partial fractions and hence show that $\displaystyle\int_0^{\frac{1}{4}} \frac{2+x^2}{(1+2x)(1-x)^2}\,dx = \tfrac{1}{2}\ln\tfrac{3}{2}+\tfrac{1}{3}$. **[9]**

Q9 June 2014

54.

By first using the substitution $t=\sqrt{x+1}$, find $\displaystyle\int e^{2\sqrt{x+1}}\,dx$. **[6]**

Q5 June 2015

55.

(i) Use the quotient rule to show that the derivative of $\dfrac{\cos x}{\sin x}$ is $\dfrac{-1}{\sin^2 x}$. **[2]**

(ii) Show that $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \frac{\sqrt{1+\cos 2x}}{\sin x\sin 2x}\,dx = \frac{1}{2}(\sqrt{6}-\sqrt{2})$. **[6]**

Q6 June 2015