OCR Core Maths 2

Past paper questions Integration

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Integration

- *Calculus* is the combined study of differentiation *and* integration (and their relationship). A good description is that calculus is the study of change in the same way that geometry is the study of shapes.
- Integration is the reverse of differentiation. That is if $\frac{dy}{dx} = f(x)$ then $y = \int f(x)\, dx$. For example if $\frac{dy}{dx} = 3x^3$ then $y = \int 3x^3\, dx = \frac{3}{4}x^4 + c$.
- The general rule is therefore $\displaystyle\int ax^n\, dx = \frac{ax^{n+1}}{n+1} + c$.
- $\int y\, dx$ is an *indefinite* integral because there are no limits on the integral sign. When evaluating these integrals *never* forget an *arbitrary constant* added on at the end. For example $\int 6x^2\, dx = 2x^3 + c$.

- $\int_a^b y\, dx$ is a *definite integral* and is the area between the curve and the x-axis from $x = a$ to $x = b$. Areas under the x-axis are negative. (For areas between the curve and the y-axis switch the x and the y and use $\int_p^q x\, dy$ between $y = p$ and $y = q$.)
- To find the area *between* two curves between $x = a$ and $x = b$ evaluate

$$\int_a^b (\text{top} - \text{bottom})\, dx.$$

- For example, given that $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, find the area under the curve from $x = 1$ and $x = 2$.

$$\begin{aligned}\int_1^2 \sqrt{x} + \frac{1}{\sqrt{x}}\, dx &= \int_1^2 x^{\frac{1}{2}} + x^{-\frac{1}{2}}\, dx \\ &= \left[\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}}\right]_1^2 \\ &= \left(\frac{2}{3} \times 2^{\frac{3}{2}} + 2 \times 2^{\frac{1}{2}}\right) - \left(\frac{2}{3} + 2\right) \\ &= \frac{10\sqrt{2}}{3} - \frac{8}{3}.\end{aligned}$$

- To calculate integrals where one of the limits is infinite (∞ or $-\infty$), proceed as normal until you input the ∞ or $-\infty$ into the integral. Then you must **not** write such things as

$$\frac{1}{\infty}, \quad \frac{1}{3^\infty}, \quad \frac{2\infty + 1}{3\infty - 2}, \quad 2^{\frac{1}{\infty}}, \qquad \text{and the like.}$$

You must think what these things would equal and just write down the number; in the previous four cases you would get 0, 0, $\frac{2}{3}$ and 1. (If, in the C2 exam, you think that when you put in ∞ you get an infinite answer, chances are you've made a mistake somewhere.)

For example:

$$\begin{aligned}
\int_2^\infty \frac{8}{5x^3}dx &= \int_2^\infty \frac{8}{5}x^{-3}dx \\
&= \left[-\frac{4}{5}x^{-2}\right]_2^\infty \\
&= (0) - \left(-\frac{4}{5} \times \frac{1}{2^2}\right) = \frac{1}{5}.
\end{aligned}$$

1.

(i) Find $\int (2x+1)(x+3)\,\mathrm{d}x$. [4]

(ii) Evaluate $\int_0^9 \frac{1}{\sqrt{x}}\,\mathrm{d}x$. [3]

Q3 June 2005

2.

(i) Find the binomial expansion of $\left(x^2 + \frac{1}{x}\right)^3$, simplifying the terms. [4]

(ii) Hence find $\int \left(x^2 + \frac{1}{x}\right)^3 \mathrm{d}x$. [4]

Q6 June 2005

3.

(a) Find $\int \left(x^{\frac{1}{2}} + 4\right)\mathrm{d}x$. [4]

(b) **(i)** Find the value, in terms of a, of $\int_1^a 4x^{-2}\,\mathrm{d}x$, where a is a constant greater than 1. [3]

(ii) Deduce the value of $\int_1^\infty 4x^{-2}\,\mathrm{d}x$. [1]

Q6 Jan 2006

4.

The cubic polynomial $2x^3 + kx^2 - x + 6$ is denoted by $\mathrm{f}(x)$. It is given that $(x+1)$ is a factor of $\mathrm{f}(x)$.

(i) Show that $k = -5$, and factorise $\mathrm{f}(x)$ completely. [6]

(ii) Find $\int_{-1}^2 \mathrm{f}(x)\,\mathrm{d}x$. [4]

(iii) Explain with the aid of a sketch why the answer to part **(ii)** does not give the area of the region between the curve $y = \mathrm{f}(x)$ and the x-axis for $-1 \leqslant x \leqslant 2$. [2]

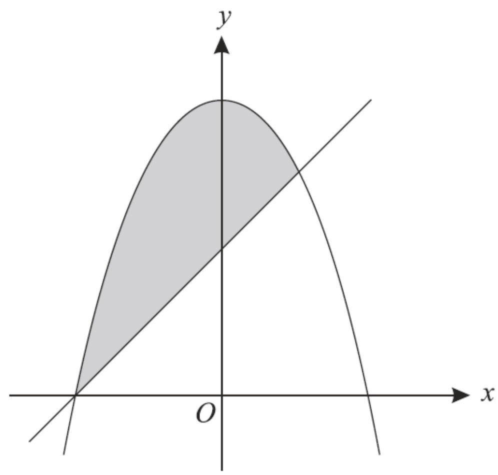
Q8 Jan 2006

5.

The gradient of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x^{-\frac{1}{2}}$, and the curve passes through the point $(4, 5)$. Find the equation of the curve. [6]

Q3 June 2006

6.



The diagram shows the curve $y = 4 - x^2$ and the line $y = x + 2$.

(i) Find the x-coordinates of the points of intersection of the curve and the line. [2]

(ii) Use integration to find the area of the shaded region bounded by the line and the curve. [6]

Q4 June 2006

7.

(i) Find $\int (4x - 5)\,\mathrm{d}x$. [2]

(ii) The gradient of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = 4x - 5$. The curve passes through the point $(3, 7)$. Find the equation of the curve. [3]

Q3 Jan 2007

8.

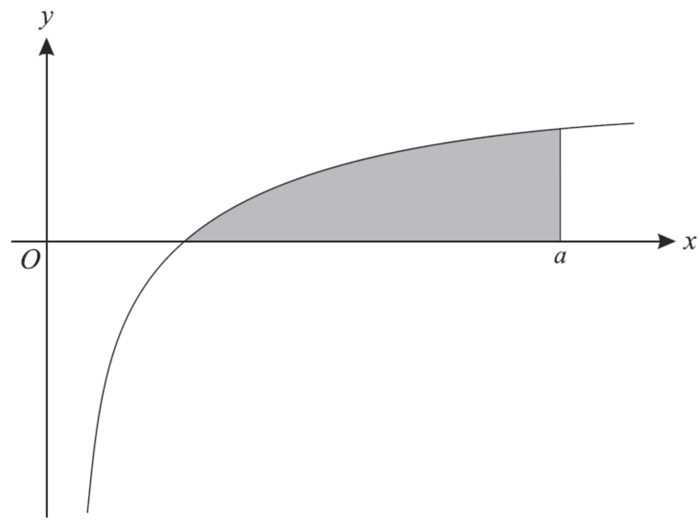
(a) **(i)** Find $\int x(x^2 - 4)\,\mathrm{d}x$. [3]

(ii) Hence evaluate $\int_1^6 x(x^2 - 4)\,\mathrm{d}x$. [2]

(b) Find $\int \frac{6}{x^3}\,\mathrm{d}x$. [3]

Q6 June 2007

9.



The diagram shows the graph of $y = 1 - 3x^{-\frac{1}{2}}$.

(i) Verify that the curve intersects the x-axis at (9, 0). [1]

(ii) The shaded region is enclosed by the curve, the x-axis and the line $x = a$ (where $a > 9$). Given that the area of the shaded region is 4 square units, find the value of a. [9]

Q10 Jan 2007

10.

The gradient of a curve is given by $\frac{dy}{dx} = 12\sqrt{x}$. The curve passes through the point (4, 50). Find the equation of the curve. [6]

Q5 Jan 2008

11.

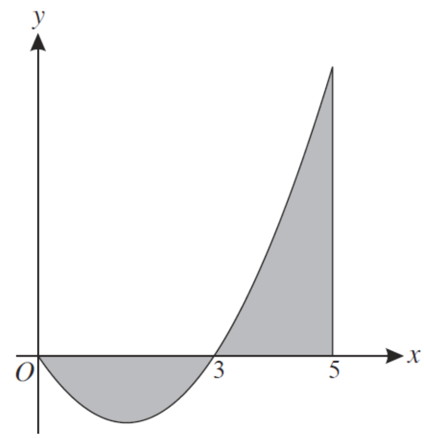
(a) Find $\int x^3(x^2 - x + 5)\,dx$. [4]

(b) **(i)** Find $\int 18x^{-4}\,dx$. [2]

(ii) Hence evaluate $\int_2^{\infty} 18x^{-4}\,dx$. [2]

Q7 June 2008

12.



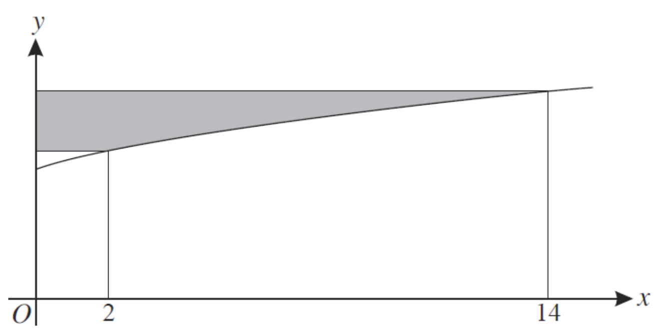
The diagram shows part of the curve $y = x^2 - 3x$ and the line $x = 5$.

(i) Explain why $\int_0^5 (x^2 - 3x)\,dx$ does not give the total area of the regions shaded in the diagram. [1]

(ii) Use integration to find the exact total area of the shaded regions. [7]

Q7 Jan 2008

13.



The diagram shows the curve $y = 3 + \sqrt{x + 2}$.

The shaded region is bounded by the curve, the y-axis, and two lines parallel to the x-axis which meet the curve where $x = 2$ and $x = 14$.

(i) Show that the area of the shaded region is given by

$$\int_5^7 (y^2 - 6y + 7)\,dy. \qquad [3]$$

(ii) Hence find the exact area of the shaded region. [4]

Q5 June 2008

14.

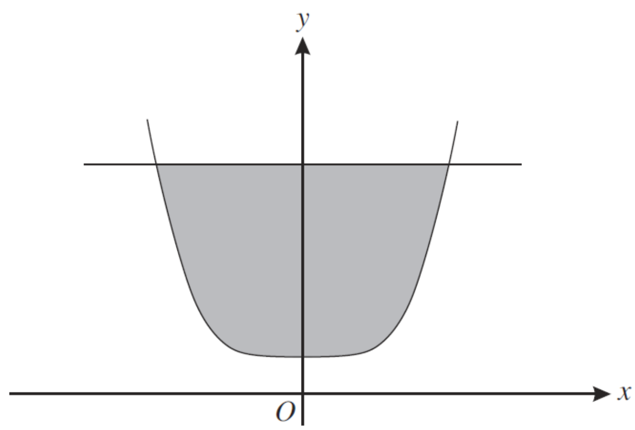
Find

(i) $\int (x^3 + 8x - 5)\,dx$, **[3]**

(ii) $\int 12\sqrt{x}\,dx$. **[3]**

Q1 Jan 2009

15.



The diagram shows the curve $y = x^4 + 3$ and the line $y = 19$ which intersect at $(-2, 19)$ and $(2, 19)$. Use integration to find the exact area of the shaded region enclosed by the curve and the line. **[7]**

Q4 Jan 2009

16.

(i) Find the binomial expansion of $(x^2 - 5)^3$, simplifying the terms. **[4]**

(ii) Hence find $\int (x^2 - 5)^3\,dx$. **[4]**

Q4 June 2009

17.

The gradient of a curve is given by $\frac{dy}{dx} = 3x^2 + a$, where a is a constant. The curve passes through the points $(-1, 2)$ and $(2, 17)$. Find the equation of the curve. **[8]**

Q4 June 2009

18.

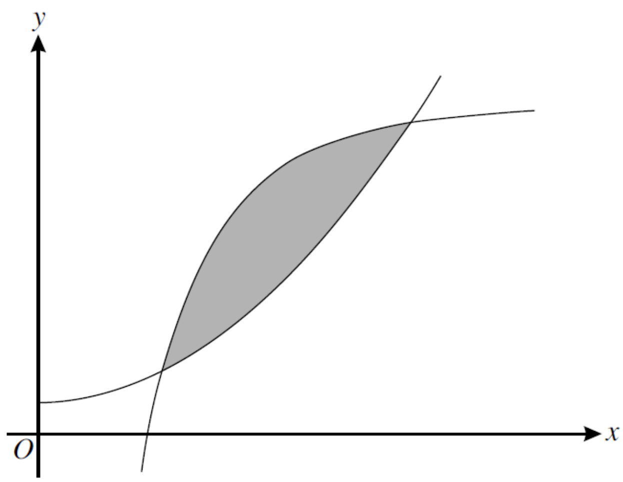
The gradient of a curve is given by $\frac{dy}{dx} = 6x - 4$. The curve passes through the distinct points (2, 5) and (p, 5).

(i) Find the equation of the curve. **[4]**

(ii) Find the value of p. **[3]**

Q2 Jan 2010

19.



The diagram shows parts of the curves $y = x^2 + 1$ and $y = 11 - \frac{9}{x^2}$, which intersect at (1, 2) and (3, 10). Use integration to find the exact area of the shaded region enclosed between the two curves. **[7]**

Q5 Jan 2010

20.

(a) Use integration to find the exact area of the region enclosed by the curve $y = x^2 + 4x$, the x-axis and the lines $x = 3$ and $x = 5$. **[4]**

(b) Find $\int (2 - 6\sqrt{y})\, dy$. **[3]**

(c) Evaluate $\int_1^{\infty} \frac{8}{x^3}\, dx$. **[4]**

Q6 June 2010

21.

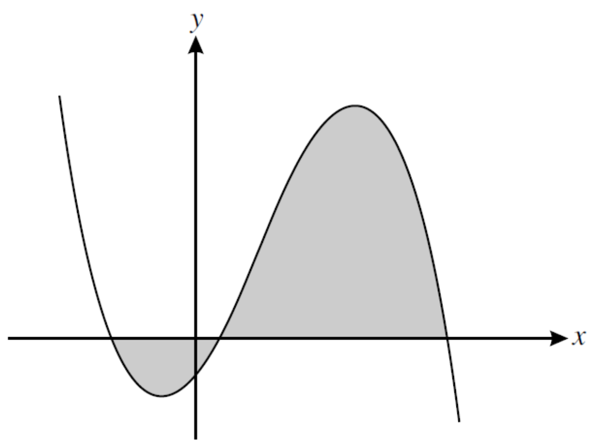
(a) Find $\int \frac{x^3 + 3x^{\frac{1}{2}}}{x}\,dx$. **[4]**

(b) **(i)** Find, in terms of a, the value of $\int_2^a 6x^{-4}\,dx$, where a is a constant greater than 2. **[3]**

(ii) Deduce the value of $\int_2^{\infty} 6x^{-4}\,dx$. **[1]**

Q6 Jan 2011

22.



The diagram shows the curve $y = f(x)$, where $f(x) = -4x^3 + 9x^2 + 10x - 3$.

(i) Verify that the curve crosses the x-axis at $(3, 0)$ and hence state a factor of $f(x)$. **[2]**

(ii) Express $f(x)$ as the product of a linear factor and a quadratic factor. **[3]**

(iii) Hence find the other two points of intersection of the curve with the x-axis. **[2]**

(iv) The region enclosed by the curve and the x-axis is shaded in the diagram. Use integration to find the total area of this region. **[5]**

Q9 Jan 2011

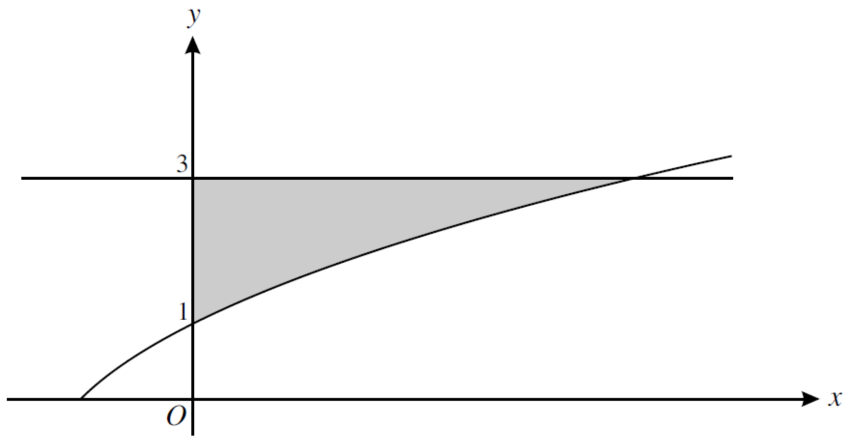
23.

(i) Find $\int \left(6x^{\frac{1}{2}} - 1\right) dx$. **[3]**

(ii) Hence find the equation of the curve for which $\frac{dy}{dx} = 6x^{\frac{1}{2}} - 1$ and which passes through the point $(4, 17)$. **[3]**

Q2 June 2011

24.



The diagram shows the curve $y = -1 + \sqrt{x+4}$ and the line $y = 3$.

(i) Show that $y = -1 + \sqrt{x+4}$ can be rearranged as $x = y^2 + 2y - 3$. **[2]**

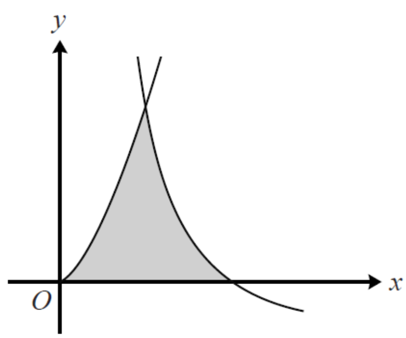
(ii) Hence find by integration the exact area of the shaded region enclosed between the curve, the y-axis and the line $y = 3$. **[5]**

Q4 June 2011

25.

(a) Find $\int (x^2 + 4)(x - 6)\, dx$. **[3]**

(b)



The diagram shows the curve $y = 6x^{\frac{3}{2}}$ and part of the curve $y = \frac{8}{x^2} - 2$, which intersect at the point (1, 6). Use integration to find the area of the shaded region enclosed by the two curves and the x-axis. **[8]**

Q7 Jan 2012

26.

(i) Find $\int (x^2 - 2x + 5)\, dx$. **[3]**

(ii) Hence find the equation of the curve for which $\frac{dy}{dx} = x^2 - 2x + 5$ and which passes through the point (3, 11). **[3]**

Q2 June 2012

27.

A curve has an equation which satisfies $\frac{dy}{dx} = kx(2x - 1)$ for all values of x. The point P (2, 7) lies on the curve and the gradient of the curve at P is 9.

(i) Find the value of the constant k. **[2]**

(ii) Find the equation of the curve. **[5]**

Q3 Jan 2013

28.

The positive constant a is such that $\int_a^{2a} \frac{2x^3 - 5x^2 + 4}{x^2}\,dx = 0$.

(i) Show that $3a^3 - 5a^2 + 2 = 0$. **[6]**

(ii) Show that $a = 1$ is a root of $3a^3 - 5a^2 + 2 = 0$, and hence find the other possible value of a, giving your answer in simplified surd form. **[6]**

Q9 Jan 2013

29.

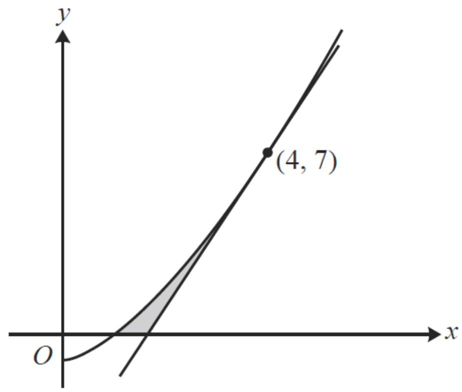
(a) Find $\int (5x^3 - 6x + 1)\,dx$. **[3]**

(b) **(i)** Find $\int 24x^{-3}\,dx$. **[2]**

(ii) Given that $\int_a^{\infty} 24x^{-3}\,dx = 3$, find the value of the positive constant a. **[3]**

Q4 June 2013

30.



The diagram shows the curve $y = x^{\frac{3}{2}} - 1$, which crosses the x-axis at (1, 0), and the tangent to the curve at the point (4, 7).

(i) Show that $\int_1^4 (x^{\frac{3}{2}} - 1)\,dx = 9\frac{2}{5}$. **[4]**

(ii) Hence find the exact area of the shaded region enclosed by the curve, the tangent and the x-axis. **[5]**

Q7 June 2013

31.

A curve has an equation which satisfies $\frac{d^2y}{dx^2} = 3x^{-\frac{1}{2}}$ for all positive values of x. The point $P(4, 1)$ lies on the curve, and the gradient of the curve at P is 5. Find the equation of the curve. **[7]**

Q5 June 2015

32.

The cubic polynomial f(x) is defined by $f(x) = x^3 - 19x + 30$.

(i) Given that $x = 2$ is a root of the equation f(x) = 0, express f(x) as the product of 3 linear factors. **[4]**

(ii) Use integration to find the exact value of $\int_{-5}^{3} f(x)\,dx$. **[4]**

(iii) Explain with the aid of a sketch why the answer to part **(ii)** does not give the area enclosed by the curve $y = f(x)$ and the x-axis for $-5 \leqslant x \leqslant 3$. **[2]**

Q6 June 2015