Edexcel

Pure Mathematics

Year 2

Implicit Differentiation

Past paper questions from Core Maths 4 and IAL C34

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1. A curve has equation

$$x^2 + 2xy - 3y^2 + 16 = 0.$$

Find the coordinates of the points on the curve where $\frac{dy}{dx} = 0$. **(7)**

(C4 June 2005, Q2)

2. A curve C is described by the equation

$$3x^2 + 4y^2 - 2x + 6xy - 5 = 0.$$

Find an equation of the tangent to C at the point (1, –2), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(7)**

(C4 Jan 2006, Q1)

3. A curve C is described by the equation

$$3x^2 - 2y^2 + 2x - 3y + 5 = 0.$$

Find an equation of the normal to C at the point (0, 1), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(7)**

(C4 June 2006, Q1)

4. A set of curves is given by the equation $\sin x + \cos y = 0.5$.

(*a*) Use implicit differentiation to find an expression for $\frac{dy}{dx}$. **(2)**

For $-\pi < x < \pi$ and $-\pi < y < \pi$,

(*b*) find the coordinates of the points where $\frac{dy}{dx} = 0$. **(5)**

(C4 Jan 2007, Q5)

5. A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(*a*) Find the coordinates of the two points on the curve where $x = -8$. **(3)**

(*b*) Find the gradient of the curve at each of these points. **(6)**

(C4 Jan 2008, Q5)

6. A curve has equation $3x^2 - y^2 + xy = 4$. The points P and Q lie on the curve. The gradient of the tangent to the curve is $\frac{8}{3}$ at P and at Q.

(*a*) Use implicit differentiation to show that $y - 2x = 0$ at P and at Q. **(6)**

(*b*) Find the coordinates of P and Q. **(3)**

(C4 June 2008, Q4)

7. A curve C has the equation $y^2 - 3y = x^3 + 8$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. **(4)**

(b) Hence find the gradient of C at the point where $y = 3$. **(3)**

(C4 Jan 2009, Q1)

8. The curve C has the equation $y\mathrm{e}^{-2x} = 2x + y^2$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. **(5)**

The point P on C has coordinates (0, 1).

(b) Find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

(C4 June 2009, Q4)

9. A curve C has equation

$$2^x + y^2 = 2xy.$$

Find the exact value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at the point on C with coordinates (3, 2). **(7)**

(C4 June 2010, Q3)

10. Find the gradient of the curve with equation

$$\ln y = 2x \ln x, \qquad x > 0, \quad y > 0,$$

at the point on the curve where $x = 2$. Give your answer as an exact value.

(7)

(C4 June 2011, Q5)

11. The curve C has the equation $2x + 3y^2 + 3x^2y = 4x^2$.

The point P on the curve has coordinates (–1, 1).

(a) Find the gradient of the curve at P. **(5)**

(b) Hence find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

(C4 June 2012, Q1)

12. A curve is described by the equation

$$x^2 + 4xy + y^2 + 27 = 0$$

(*a*) Find $\frac{dy}{dx}$ in terms of *x* and *y*.

(5)

A point *Q* lies on the curve.

The tangent to the curve at *Q* is parallel to the *y*-axis.

Given that the *x*-coordinate of *Q* is negative,

(*b*) use your answer to part (*a*) to find the coordinates of *Q*.

(7)

(C4 June 2013, Q7)

13. The curve *C* has equation

$$3^{x-1} + xy - y^2 + 5 = 0$$

Show that $\frac{dy}{dx}$ at the point (1, 3) on the curve *C* can be written in the form $\frac{1}{\lambda}\ln(\mu e^3)$, where λ and μ are integers to be found.

(7)

(C4 June 2013_R, Q2)

14. A curve *C* has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(*a*) Find $\frac{dy}{dx}$ in terms of *x* and *y*. **(5)**

(*b*) Find an equation of the tangent to *C* at the point (3, –2), giving your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers. **(2)**

(C4 June 2014, Q1)

15.

$$x^2 + y^2 + 10x + 2y - 4xy = 10$$

(*a*) Find $\frac{dy}{dx}$ in terms of *x* and *y*, fully simplifying your answer. **(5)**

(*b*) Find the values of *y* for which $\frac{dy}{dx} = 0$. **(5)**

(C4 June 2014_R, Q3)

16. The curve *C* has equation

$$x^2 - 3xy - 4y^2 + 64 = 0.$$

(*a*) Find $\frac{dy}{dx}$ in terms of *x* and *y*. **(5)**

(*b*) Find the coordinates of the points on *C* where $\frac{dy}{dx} = 0$. **(6)**

(C4 June 2015, Q2)

17. The curve C has equation

$$2x^2y + 2x + 4y - \cos(\pi y) = 17.$$

(*a*) Use implicit differentiation to find $\dfrac{dy}{dx}$ in terms of x and y.

(5)

The point P with coordinates $\left(3, \dfrac{1}{2}\right)$ lies on C.

The normal to C at P meets the x-axis at the point A.

(*b*) Find the x coordinate of A, giving your answer in the form $\dfrac{a\pi + b}{c\pi + d}$, where a, b, c and d are integers to be determined.

(4)

(C4 June 2016, Q3)

18. The curve C has equation

$$4x^2 - y^3 - 4xy + 2^y = 0$$

The point P with coordinates $(-2, 4)$ lies on C.

(*a*) Find the exact value of $\dfrac{dy}{dx}$ at the point P.

(6)

The normal to C at P meets the y-axis at the point A.

(*b*) Find the y coordinate of A, giving your answer in the form $p + q\ln 2$, where p and q are constants to be determined.

(3)

(C4 June 2017, Q4)

19.

The curve C has equation

$$x^2 + xy + y^2 - 4x - 5y + 1 = 0$$

(a) Use implicit differentiation to find $\dfrac{dy}{dx}$ in terms of x and y.

(5)

(b) Find the x coordinates of the two points on C where $\dfrac{dy}{dx} = 0$

Give exact answers in their simplest form.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

(C4 June 2018, Q2)

20.

The curve C has equation

$$x^2 - y^3 - x - x\sin(\pi y) = -2$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The point P with coordinates (3, 2) lies on C.

The tangent to C at P meets the y-axis at the point Q.

(b) Find the y coordinate of Q, giving your answer in the form $\frac{a\pi + b}{\pi + c}$ where a, b and c are integers to be found.

(3)

(C4 June 2019, Q3)

21. (*a*) Prove, by using logarithms, that

$$\frac{d}{dx}(2^x) = 2^x \ln 2$$

(3)

The curve C has the equation

$$2x + 3y^2 + 3x^2y + 12 = 4 \times 2^x$$

The point P, with coordinates (2, 0), lies on C.

(*b*) Find an equation of the tangent to C at P.

(6)

(IAL C34 Jan 2014, Q5)

22. A curve C has the equation

$$x^3 - 3xy - x + y^3 - 11 = 0$$

Find an equation of the tangent to C at the point (2, –1), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

(IAL C34 June 2014, Q2)

23. Given $x = \tan^2 4y$, $0 < y < \frac{\pi}{8}$, find $\frac{dy}{dx}$ as a function of x.

Write your answer in the form $\frac{1}{A\left(x^p + x^q\right)}$, where A, p and q are constants to be found.

(5)

(IAL C34 Jan 2015, Q6)

24. A curve has equation

$$4x^2 - y^2 + 2xy + 5 = 0$$

The points P and Q lie on the curve.

Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$ at P and at Q,

(*a*) use implicit differentiation to show that $y - 6x = 0$ at P and at Q.

(6)

(*b*) Hence find the coordinates of P and Q.

(3)

(IAL C34 June 2015, Q1)

25. A curve C has equation

$$3^x + 6y = \frac{3}{2}xy^2$$

Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point on C with coordinates (2, 3). Give your answer in the form $\frac{a + \ln b}{8}$, where a and b are integers. **(7)**

(IAL C34 Jan 2016, Q3)

26. A curve C has equation

$$3x^2 + 2xy - 2y^2 + 4 = 0$$

Find an equation for the tangent to C at the point (2, 4), giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

(IAL C34 June 2016, Q1)

27. Find an equation of the tangent to the curve

$$x^3 + 3x^2y + y^3 = 37$$

at the point (1, 3). Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

(IAL C34 Jan 2017, Q1)

28. A curve C has equation

$$3x^2 + 2xy - 2y^2 + 4 = 0$$

Find an equation for the tangent to C at the point (2, 4), giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

(IAL C34 June 2017, Q1)

29. The curve C has equation

$$y^3 + x^2y - 6x = 0$$

(*a*) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(*b*) Hence find the exact coordinates of the points on C for which $\frac{dy}{dx} = 0$

(6)

(IAL C34 Oct 2017, Q2)

30. A curve C has equation

$$3^x + xy = x + y^2, \qquad y > 1$$

The point P with coordinates (4, 11) lies on C.

Find the exact value of $\frac{dy}{dx}$ at the point P.

Give your answer in the form $a + b\ln 3$, where a and b are rational numbers.

(6)

(IAL C34 Jan 2018, Q1)

31. The curve C satisfies the equation

$$xe^{5-2y} - y = 0 \qquad x > 0, \qquad y > 0$$

The point P with coordinates $(2e^{-1}, 2)$ lies on C.

The tangent to C at P cuts the x-axis at the point A and cuts the y-axis at the point B.

Given that O is the origin, find the exact area of triangle OAB, giving your answer in its simplest form.

(7)

(IAL C34 June 2018, Q10)

32. A curve C has equation

$$x^3 - 4xy + 2x + 3y^2 - 3 = 0$$

Find an equation of the normal to C at the point $(-3, 2)$, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(7)

(IAL C34 Oct 2018, Q10)

33.

The curve C has equation

$$3ye^{-2x} = 4x^2 + y^2 + 2$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The point P on C has coordinates $(0, 2)$.

(b) Find the equation of the normal to C at P giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

(IAL C34 Jun 2019, Q4)

34.

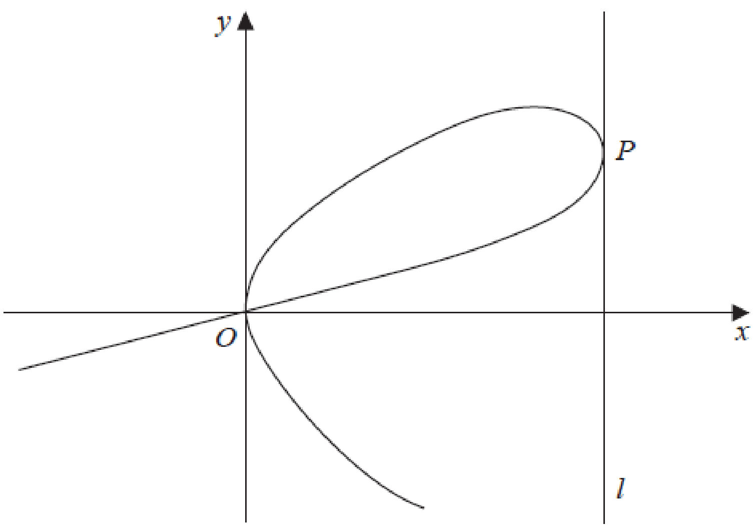

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$2x^2 + y^3 = kxy$$

where k is a positive constant.

(*a*) Find $\frac{dy}{dx}$ in terms of x, y and k.

(4)

The line l is parallel to the y-axis and touches the curve at the point P, as shown in Figure 1.

(*b*) Find, in terms of k, the coordinates of the point P.

(5)

(IAL C34 Oct 2019, Q11)

35. The curve C has equation

$$81y^3 + 64x^2y + 256x = 0$$

(*a*) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(*b*) Hence find the coordinates of the points on C where $\frac{dy}{dx} = 0$

(6)

(IAL C34 Jan 2019, Q4)