OCR Core Maths 4

Past paper questions
Implicit Functions

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Implicit Functions

- Given a function in the form $y = f(x)$ we can differentiate it. Implicit differentiation allows us to differentiate a function without making y the subject first. It uses the chain rule that
$$\frac{d\,f(y)}{dx} = \frac{d\,f(y)}{dy} \times \frac{dy}{dx}.$$
So all you do is differentiate the y bits with respect to y and then multiply by $\frac{dy}{dx}$. For example differentiate $y^4 + x^4 = \sin y$ with respect to x. This gives
$$4y^3\frac{dy}{dx} + 4x^3 = \cos y\frac{dy}{dx} \qquad \Rightarrow \qquad \frac{dy}{dx} = \frac{4x^3}{\cos y - 4y^3}.$$
You must be on the lookout for products in terms of x and y; for example $2xy = e^{2y}$ would differentiate to
$$2x\frac{dy}{dx} + 2y = 2e^{2y}\frac{dy}{dx} \quad \text{so} \quad \frac{dy}{dx} = \frac{2y}{2e^{2y} - 2x} = \frac{y}{e^{2y} - x}.$$

- Another example; find all the stationary points on the curve $x^2 + y^2 + xy = 3$. Differentiating w.r.t. x we find
$$2x + 2y\frac{dy}{dx} + y + x\frac{dy}{dx} = 0 \qquad \Rightarrow \qquad \frac{dy}{dx} = -\frac{2x + y}{2y + x}.$$
Stationary points are where $\frac{dy}{dx} = 0$ so solve
$$0 = -\frac{2x + y}{2y + x} \qquad \Rightarrow \qquad y = -2x.$$
Substituting this *back into the original equation* we find
$$x^2 + (-2x)^2 + x(-2x) = 3 \qquad \Rightarrow \qquad x = \pm 1 \qquad \Rightarrow \qquad \text{Points are } (1, -2) \text{ and } (-1, 2).$$

- If you discover $\dfrac{dy}{dx} = \dfrac{f(x, y)}{g(x, y)}$ and are asked to find where the tangents to a curve are parallel to the y-axis (i.e. vertical) then you need to solve where the bottom line is zero, i.e. solve $g(x, y) = 0$.

1.

The equation of a curve is $xy^2 = 2x + 3y$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2 - y^2}{2xy - 3}$. [5]

(ii) Show that the curve has no tangents which are parallel to the y-axis. [3]

Q6 June 2005

2.

Given that $\sin y = xy + x^2$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [5]

Q2 Jan 2006

3.

Find the gradient of the curve $4x^2 + 2xy + y^2 = 12$ at the point $(1, 2)$. [4]

Q1 June 2006

4.

The equation of a curve is $2x^2 + xy + y^2 = 14$. Show that there are two stationary points on the curve and find their coordinates. [8]

Q7 Jan 2007

5.

The equation of a curve is $x^2 + 3xy + 4y^2 = 58$. Find the equation of the normal at the point $(2, 3)$ on the curve, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [8]

Q6 June 2007

6.

Find the equation of the normal to the curve

$$x^3 + 4x^2y + y^3 = 6$$

at the point $(1, 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

Q4 Jan 2008

7.

The equation of a curve is $x^2y - xy^2 = 2$.

(i) Show that $\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$. [3]

(ii) **(a)** Show that, if $\frac{dy}{dx} = 0$, then $y = 2x$. [2]

(b) Hence find the coordinates of the point on the curve where the tangent is parallel to the x-axis. [3]

Q3 June 2008

8.

The equation of a curve is $x^3 + y^3 = 6xy$.

(i) Find $\frac{dy}{dx}$ in terms of x and y. **[4]**

(ii) Show that the point $(2^{\frac{4}{3}}, 2^{\frac{5}{3}})$ lies on the curve and that $\frac{dy}{dx} = 0$ at this point. **[4]**

(iii) The point (a, a), where $a > 0$, lies on the curve. Find the value of a and the gradient of the curve at this point. **[4]**

Q8 Jan 2009

9.

(i) Given that $14x^2 - 7xy + y^2 = 2$, show that $\frac{dy}{dx} = \frac{28x - 7y}{7x - 2y}$. **[4]**

(ii) The points L and M on the curve $14x^2 - 7xy + y^2 = 2$ each have x-coordinate 1. The tangents to the curve at L and M meet at N. Find the coordinates of N. **[6]**

Q8 June 2009

10.

Find the equation of the normal to the curve $x^3 + 2x^2y = y^3 + 15$ at the point $(2, 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[8]**

Q7 Jan 2010

11.

Find the coordinates of the two stationary points on the curve with equation

$$x^2 + 4xy + 2y^2 + 18 = 0.$$ **[7]**

Q5 June 2010

12.

The points P and Q lie on the curve with equation

$$2x^2 - 5xy + y^2 + 9 = 0.$$

The tangents to the curve at P and Q are parallel, each having gradient $\frac{3}{8}$.

(i) Show that the x- and y-coordinates of P and Q are such that $x = 2y$. **[5]**

(ii) Hence find the coordinates of P and Q. **[3]**

Q8 Jan 2011

13.

The equation of a curve C is $(x + 3)(y + 4) = x^2 + y^2$.

(i) Find $\frac{dy}{dx}$ in terms of x and y. **[4]**

(ii) The line $2y = x + 3$ meets C at two points. What can be said about the tangents to C at these points? Justify your answer. **[2]**

(iii) Find the equation of the tangent at the point (6, 0), giving your answer in the form $ax + by = c$, where a, b and c are integers. **[2]**

Q3 Jan 2012

14.

(a) Find the gradient of the curve $x^2 + xy + y^2 = 3$ at the point $(-1, -1)$. **[4]**

(b) A curve C has parametric equations

$$x = 2t^2 - 1, \quad y = t^3 + t.$$

(i) Find the coordinates of the point on C at which the tangent is parallel to the y-axis. **[3]**

(ii) Find the values of t for which x and y have the same rate of change with respect to t. **[3]**

Q8 June 2012

15.

The equation of a curve is $xy^2 = x^2 + 1$. Find $\frac{dy}{dx}$ in terms of x and y, and hence find the coordinates of the stationary points on the curve. **[7]**

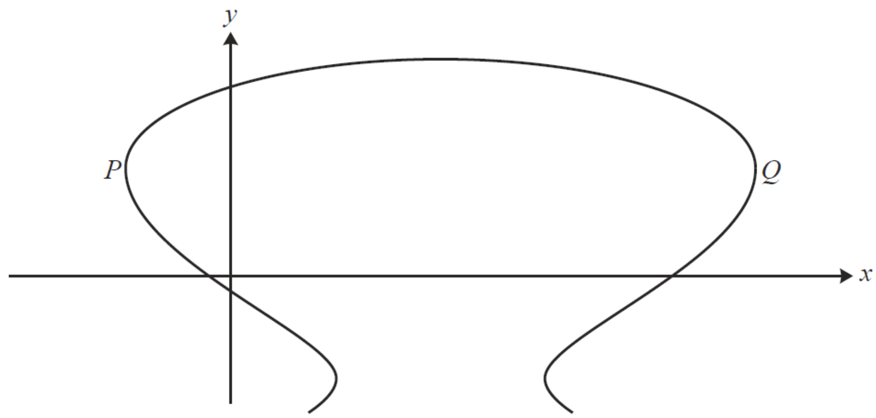
Q3 Jan 2013

16.

The equation of a curve is $y = \cos 2x + 2\sin x$. Find $\frac{dy}{dx}$ and hence find the coordinates of the stationary points on the curve for $0 < x < \pi$. **[6]**

Q4 June 2013

17.



The diagram shows the curve with equation $x^2 + y^3 - 8x - 12y = 4$. At each of the points P and Q the tangent to the curve is parallel to the y-axis. Find the coordinates of P and Q. **[8]**

Q6 June 2014

18.

A curve has equation $(x+y)^2 = xy^2$. Find the gradient of the curve at the point where $x = 1$. **[7]**

Q7 June 2015