OCR Core Maths 2

Past paper questions Graphs & Transformations

Edited by K V Kumaran

Email: kvkumaran@gmail.com

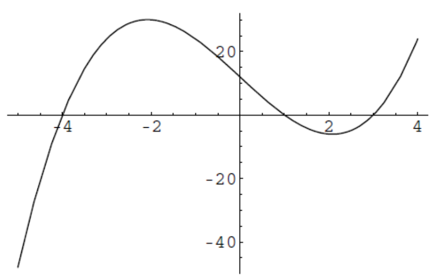
Phone: 07961319548

Some Important Graphs

- Know the shape of the graph $y = x^n$ for $n = \{1, 2, 3, 4 \ldots\}$.
- If the power is even, then the graph will be $\bigcup$-shaped. They all pass through the points $(-1, 1)$, $(0, 0)$ and $(1, 1)$. The bigger the power, the faster it goes to infinity. Slightly more subtle is the point that in the range $-1 < x < 1$ then the higher the power, the *smaller* y-value (because $0.2 \times 0.2 \times 0.2 < 0.2 \times 0.2$). They are all 'even functions' with the y-axis as a line of symmetry.
- If the power is odd then they will (with the exception of $y = x^1 = x$, which is a straight line) be shaped like $y = x^3$. They all pass through $(-1, -1)$, $(0, 0)$ and $(1, 1)$. Similar arguments as for even powers exist here. They are all 'odd functions' with the origin being a point of rotational symmetry.
- The family of curves $y = ax^2 + bx + c$ are parabolas. If a is positive then you get a "happy" $\bigcup$-type curve. If a is negative then you get a "sad" $\bigcap$-type curve. They have a line of symmetry and a vertex (turning point) that you can discover by completing the square (see later).
- If you have a curve that is factorised then you can sketch it easily. For example

$$y = (x - 1)(x + 4)(x - 3)$$

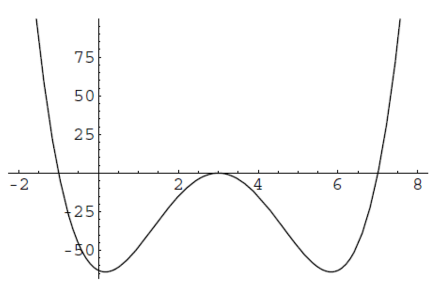
is a cubic curve that crosses the x-axis at $(1, 0)$, $(-4, 0)$ and $(3, 0)$. It crosses the y-axis when $x = 0$, which gives $(0, 12)$. If x is huge, y is huge and positive and if x is massively negative, then so is y. So



- If a factor is repeated, then it merely touches the x-axis at that point. So

$$y = (x - 3)^2(x + 1)(x - 7)$$

is a quartic curve that crosses the x-axis at $(-1, 0)$ and $(7, 0)$, but only touches at $(3, 0)$.



Transforming Graphs

- Given $y = f(x)$ then:

Replacement	Graph Shape
None	Normal Graph
x by $x - a$	Graph translated by a vector $\binom{a}{0}$
x by $-x$	Graph reflected in the y-axis
x by $\frac{x}{2}$	Graph stretched by a factor of 2 parallel to the x-axis $\longleftrightarrow$
x by $2x$	Graph stretched by a factor of $\frac{1}{2}$ parallel to the x-axis $\rightarrow\leftarrow$
y by $y - b$	Graph translated by a vector $\binom{0}{b}$
y by $-y$	Graph reflected in the x-axis
y by $\frac{y}{2}$	Graph stretched by a factor of 2 parallel to the y-axis $\updownarrow$
y by $2y$	Graph stretched by a factor of $\frac{1}{2}$ parallel to the y-axis $\downarrow\uparrow$

- For example: Find the equation of $y^2 + 2x^2 = 2x + 1$ after the translation $\binom{1}{-1}$. So we replace x by $x - 1$ and y by $y + 1$. Therefore

$$(y+1)^2 + 2(x-1)^2 = 2(x-1) + 1 \quad \Rightarrow \quad y^2 + 2y + 2x^2 = 6x - 4.$$

- For example: Explain the transformation that maps

$$y = \frac{2}{\sqrt{1+x}} \text{ onto } y = \frac{1}{\sqrt{3+x}}.$$

Rewriting the second equation as $2y = \frac{2}{\sqrt{1+(x+2)}}$ we can see y has been replaced by $2y$ and x has been replaced by $x + 2$. Therefore the curve has been translated by $\binom{-2}{0}$ and also stretched by a factor of $\frac{1}{2}$ parallel to the y-axis; i.e. every y-value has halved.

1.

(i) Sketch the curve $y = x^3$. [1]

(ii) Describe a transformation that transforms the curve $y = x^3$ to the curve $y = -x^3$. [2]

(iii) The curve $y = x^3$ is translated by p units, parallel to the x-axis. State the equation of the curve after it has been transformed. [2]

Q3 June 2005

2.

(i) Sketch the curve $y = \frac{1}{x^2}$. [2]

(ii) Hence sketch the curve $y = \frac{1}{(x-3)^2}$. [2]

(iii) Describe fully a transformation that transforms the curve $y = \frac{1}{x^2}$ to the curve $y = \frac{2}{x^2}$. [3]

Q4 Jan 2006

3.

(i) By expanding the brackets, show that

$$(x-4)(x-3)(x+1) = x^3 - 6x^2 + 5x + 12.$$ [3]

(ii) Sketch the curve

$$y = x^3 - 6x^2 + 5x + 12,$$

giving the coordinates of the points where the curve meets the axes. Label the curve C_1. [3]

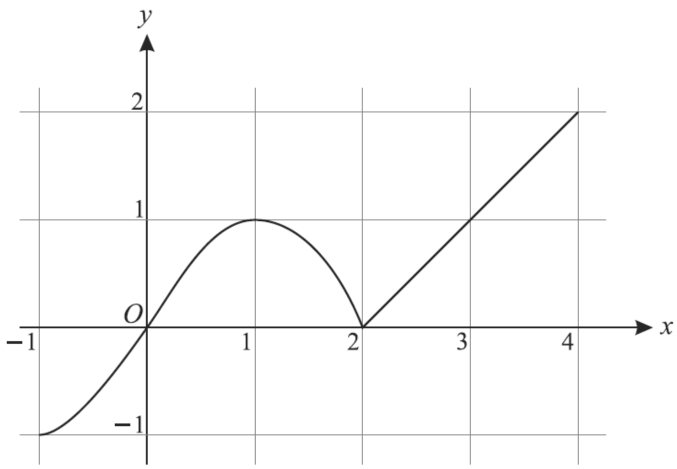
(iii) On the same diagram as in part **(ii)**, sketch the curve

$$y = -x^3 + 6x^2 - 5x - 12.$$

Label this curve C_2. [2]

Q4 June 2006

4.



The graph of $y = \mathrm{f}(x)$ for $-1 \leqslant x \leqslant 4$ is shown above.

(i) Sketch the graph of $y = -\mathrm{f}(x)$ for $-1 \leqslant x \leqslant 4$. [2]

(ii) The point $P\,(1,\ 1)$ on $y = \mathrm{f}(x)$ is transformed to the point Q on $y = 3\mathrm{f}(x)$. State the coordinates of Q. [2]

(iii) Describe the transformation which transforms the graph of $y = \mathrm{f}(x)$ to the graph of $y = \mathrm{f}(x+2)$. [2]

Q5 Jan 2007

5.

(a) On separate diagrams, sketch the graphs of

(i) $y = \frac{1}{x}$, [2]

(ii) $y = x^4$. [1]

(b) Describe a transformation that transforms the curve $y = x^3$ to the curve $y = 8x^3$. [2]

Q2 June 2007

6.

(i) Sketch the curve $y = x^3 + 2$. [2]

(ii) Sketch the curve $y = 2\sqrt{x}$. [2]

(iii) Describe a transformation that transforms the curve $y = 2\sqrt{x}$ to the curve $y = 3\sqrt{x}$. [3]

Q5 Jan 2008

7.

(i) The curve $y = x^2$ is translated 2 units in the positive x-direction. Find the equation of the curve after it has been translated. [2]

(ii) The curve $y = x^3 - 4$ is reflected in the x-axis. Find the equation of the curve after it has been reflected. [1]

Q2 June 2008

8.

(i) Expand and simplify $(x - 5)(x + 2)(x + 5)$. [3]

(ii) Sketch the curve $y = (x - 5)(x + 2)(x + 5)$, giving the coordinates of the points where the curve crosses the axes. [3]

Q6 June 2008

9.

(i) Sketch the curve $y = \frac{1}{x^2}$. **[2]**

(ii) The curve $y = \frac{1}{x^2}$ is translated by 3 units in the negative x-direction. State the equation of the curve after it has been translated. **[2]**

(iii) The curve $y = \frac{1}{x^2}$ is stretched parallel to the y-axis with scale factor 4 and, as a result, the point $P(1, 1)$ is transformed to the point Q. State the coordinates of Q. **[2]**

Q4 Jan 2009

10.

(i) Solve the equation $5 - 8x - x^2 = 0$, giving your answers in simplified surd form. **[3]**

(ii) Solve the inequality $5 - 8x - x^2 \leqslant 0$. **[2]**

(iii) Sketch the curve $y = (5 - 8x - x^2)(x + 4)$, giving the coordinates of the points where the curve crosses the coordinate axes. **[5]**

Q5 Jan 2009

11.

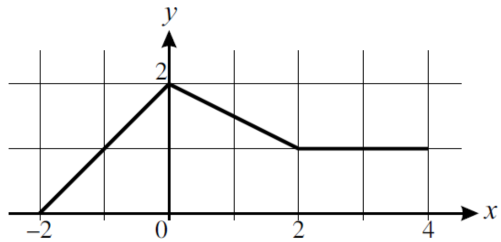
(i) Sketch the curve $y = -\sqrt{x}$. **[2]**

(ii) Describe fully a transformation that transforms the curve $y = -\sqrt{x}$ to the curve $y = 5 - \sqrt{x}$. **[2]**

(iii) The curve $y = -\sqrt{x}$ is stretched by a scale factor of 2 parallel to the x-axis. State the equation of the curve after it has been stretched. **[2]**

Q6 June 2009

12.



The graph of $y = \mathrm{f}(x)$ for $-2 \leqslant x \leqslant 4$ is shown above.

(i) Sketch the graph of $y = 2\mathrm{f}(x)$ for $-2 \leqslant x \leqslant 4$ on the axes provided. **[2]**

(ii) Describe the transformation which transforms the graph of $y = \mathrm{f}(x)$ to the graph of $y = \mathrm{f}(x-1)$. **[2]**

Q2 Jan 2010

13.

(i) Sketch the curve $y = -\frac{1}{x^2}$. **[2]**

(ii) Sketch the curve $y = 3 - \frac{1}{x^2}$. **[2]**

(iii) The curve $y = -\frac{1}{x^2}$ is stretched parallel to the y-axis with scale factor 2. State the equation of the transformed curve. **[1]**

Q2 June 2010

14.

(i) Expand $(x-2)^2(x+1)$, simplifying your answer. **[3]**

(ii) Sketch the curve $y = (x-2)^2(x+1)$, indicating the coordinates of all intercepts with the axes. **[3]**

Q4 June 2010

15.

(i) Sketch the curve $y = -x^3$. **[2]**

(ii) The curve $y = -x^3$ is translated by 3 units in the positive x-direction. Find the equation of the curve after it has been translated. **[2]**

(iii) Describe a transformation that transforms the curve $y = -x^3$ to the curve $y = -5x^3$. **[2]**

Q5 Jan 2011

16.

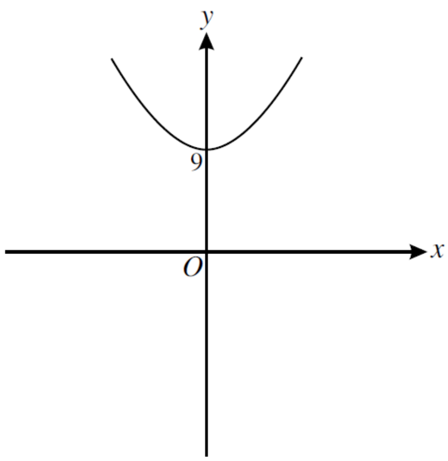


Fig. 1

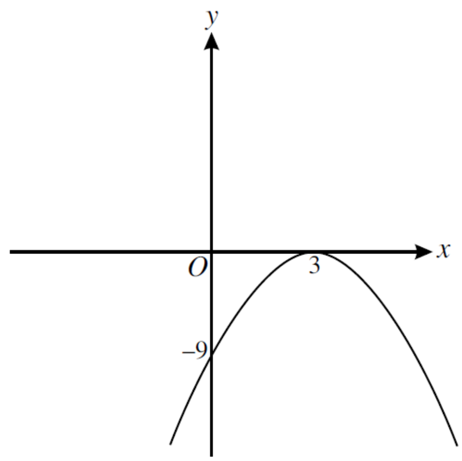


Fig. 2

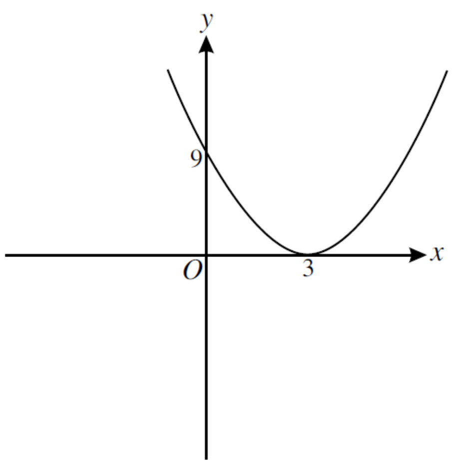


Fig. 3

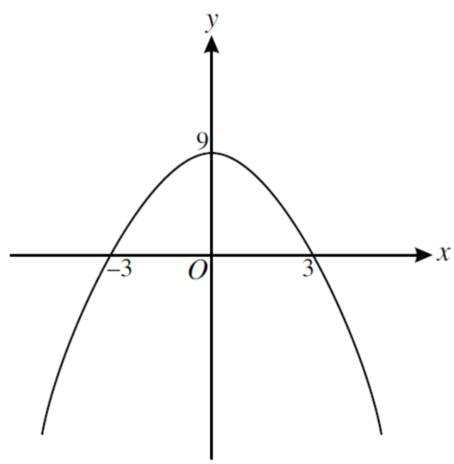


Fig. 4

(i) Each diagram shows a quadratic curve. State which diagram corresponds to the curve

(a) $y = (3 - x)^2$, **[1]**

(b) $y = x^2 + 9$, **[1]**

(c) $y = (3 - x)(x + 3)$. **[1]**

(ii) Give the equation of the curve which does not correspond to any of the equations in part **(i)**. **[2]**

Q7 Jan 2010

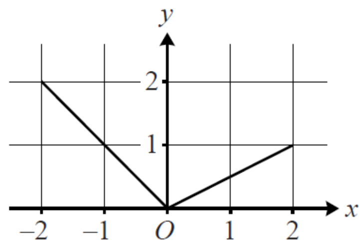
17.

(i) Sketch the curve $y = \frac{1}{x}$. **[2]**

(ii) Describe fully the single transformation that transforms the curve $y = \frac{1}{x}$ to the curve $y = \frac{1}{x} + 4$. **[2]**

Q2 June 2011

18.



The graph of $y = \mathrm{f}(x)$ for $-2 \leqslant x \leqslant 2$ is shown above.

(i) Sketch the graph of $y = \mathrm{f}(-x)$ for $-2 \leqslant x \leqslant 2$. **[2]**

(ii) Sketch the graph of $y = \mathrm{f}(x) + 2$ for $-2 \leqslant x \leqslant 2$. **[2]**

Q2 Jan 2012

19.

(i) Sketch the curve $y = \sqrt{x}$. **[2]**

(ii) Describe the transformation that transforms the curve $y = \sqrt{x}$ to the curve $y = \sqrt{x-4}$. **[2]**

(iii) The curve $y = \sqrt{x}$ is stretched by a scale factor of 5 parallel to the x-axis. State the equation of the transformed curve. **[2]**

Q5 June 2012

20.

(i) Sketch the curve $y = (1+x)(2-x)(3+x)$, giving the coordinates of all points of intersection with the axes. **[3]**

(ii) Describe the transformation that transforms the curve $y = (1+x)(2-x)(3+x)$ to the curve $y = (1-x)(2+x)(3-x)$. **[2]**

Q3 Jan 2013

21.

(i) Sketch the curve $y = \dfrac{2}{x^2}$. **[2]**

(ii) The curve $y = \dfrac{2}{x^2}$ is translated by 5 units in the negative x-direction. Find the equation of the curve after it has been translated. **[2]**

(iii) Describe a transformation that transforms the curve $y = \dfrac{2}{x^2}$ to the curve $y = \dfrac{1}{x^2}$. **[2]**

Q5 June 2013

22.

The curve $y = \mathrm{f}(x)$ passes through the point P with coordinates (2, 5).

(i) State the coordinates of the point corresponding to P on the curve $y = \mathrm{f}(x) + 2$. **[1]**

(ii) State the coordinates of the point corresponding to P on the curve $y = \mathrm{f}(2x)$. **[1]**

(iii) Describe the transformation that transforms the curve $y = \mathrm{f}(x)$ to the curve $y = \mathrm{f}(x+4)$. **[2]**

Q4 June 2014

23.

(i) Sketch the curve $y = -\frac{1}{x}$. **[2]**

(ii) The curve $y = -\frac{1}{x}$ is translated by 2 units parallel to the x-axis in the positive direction. State the equation of the transformed curve. **[2]**

(iii) Describe a transformation that transforms the curve $y = -\frac{1}{x}$ to the curve $y = -\frac{1}{3x}$. **[2]**

Q2 June 2015