Edexcel

Pure Mathematics

Year 2

Geometric Sequences.

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1. The second and fourth terms of a geometric series are 7.2 and 5.832 respectively.

The common ratio of the series is positive.

For this series, find

(*a*) the common ratio,
(2)

(*b*) the first term,
(2)

(*c*) the sum of the first 50 terms, giving your answer to 3 decimal places,
(2)

(*d*) the difference between the sum to infinity and the sum of the first 50 terms, giving your answer to 3 decimal places.
(2)

(C2, Jan 2005 Q6)

2. (*a*) A geometric series has first term *a* and common ratio *r*. Prove that the sum of the first *n* terms of the series is

$$\frac{a(1-r^n)}{1-r}.$$
(4)

Mr King will be paid a salary of £35 000 in the year 2005. Mr King's contract promises a 4% increase in salary every year, the first increase being given in 2006, so that his annual salaries form a geometric sequence.

(*b*) Find, to the nearest £100, Mr King's salary in the year 2008.
(2)

Mr King will receive a salary each year from 2005 until he retires at the end of 2024.

(*c*) Find, to the nearest £1000, the total amount of salary he will receive in the period from 2005 until he retires at the end of 2024.
(4)

(C2, Jan 2005 Q9)

3. The first term of a geometric series is 120. The sum to infinity of the series is 480.

(*a*) Show that the common ratio, r, is $\frac{3}{4}$.

(3)

(*b*) Find, to 2 decimal places, the difference between the 5th and 6th terms.

(2)

(*c*) Calculate the sum of the first 7 terms.

(2)

The sum of the first n terms of the series is greater than 300.

(*d*) Calculate the smallest possible value of n.

(4)
(C2, Jan 2006 Q4)

4. A geometric series has first term a and common ratio r. The second term of the series is 4 and the sum to infinity of the series is 25.

(*a*) Show that $25r^2 - 25r + 4 = 0$.

(4)

(*b*) Find the two possible values of r.

(2)

(*c*) Find the corresponding two possible values of a.

(2)

(*d*) Show that the sum, S_n, of the first n terms of the series is given by

$$S_n = 25(1 - r^n).$$

(1)

Given that r takes the larger of its two possible values,

(*e*) find the smallest value of n for which S_n exceeds 24.

(2)
(C2, May2006 Q9)

5. A geometric series is $a + ar + ar^2 + \ldots$

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

(4)

(b) Find

$$\sum_{k=1}^{10} 100(2^k).$$

(3)

(c) Find the sum to infinity of the geometric series

$$\frac{5}{6} + \frac{5}{18} + \frac{5}{54} + \ldots$$

(3)

(d) State the condition for an infinite geometric series with common ratio r to be convergent.

(1)

(C2, Jan 2007 Q10)

6. A trading company made a profit of £50 000 in 2006 (Year 1).

A model for future trading predicts that profits will increase year by year in a geometric sequence with common ratio r, $r > 1$.

The model therefore predicts that in 2007 (Year 2) a profit of £50 000r will be made.

(a) Write down an expression for the predicted profit in Year n.

(1)

The model predicts that in Year n, the profit made will exceed £200 000.

(b) Show that $n > \dfrac{\log 4}{\log r} + 1$.

(3)

Using the model with $r = 1.09$,

(c) find the year in which the profit made will first exceed £200 000,

(2)

(d) find the total of the profits that will be made by the company over the 10 years from 2006 to 2015 inclusive, giving your answer to the nearest £10 000. **(3)**

(C2, May 2007 Q8)

7. The fourth term of a geometric series is 10 and the seventh term of the series is 80.

For this series, find

(*a*) the common ratio, **(2)**

(*b*) the first term, **(2)**

(*c*) the sum of the first 20 terms, giving your answer to the nearest whole number. **(2)**

(C2, Jan 2008 Q2)

8. A geometric series has first term 5 and common ratio $\frac{4}{5}$.

Calculate

(*a*) the 20th term of the series, to 3 decimal places, **(2)**

(*b*) the sum to infinity of the series. **(2)**

Given that the sum to k terms of the series is greater than 24.95,

(*c*) show that $k > \frac{\log 0.002}{\log 0.8}$, **(4)**

(*d*) find the smallest possible value of k. **(1)**

(C2, Jan 2008 Q6)

9. The first three terms of a geometric series are $(k + 4)$, k and $(2k - 15)$ respectively, where k is a positive constant.

(*a*) Show that $k^2 - 7k - 60 = 0$. **(4)**

(*b*) Hence show that $k = 12$. **(2)**

(*c*) Find the common ratio of this series. **(2)**

(*d*) Find the sum to infinity of this series. **(2)**

(C2, Jan 2009 Q9)

10. The third term of a geometric sequence is 324 and the sixth term is 96.

(*a*) Show that the common ratio of the sequence is $\frac{2}{3}$.

(2)

(*b*) Find the first term of the sequence.

(2)

(*c*) Find the sum of the first 15 terms of the sequence.

(3)

(*d*) Find the sum to infinity of the sequence.

(2)

(C2, June 2009 Q5)

11. A car was purchased for £18 000 on 1st January.

On 1st January each following year, the value of the car is 80% of its value on 1st January in the previous year.

(*a*) Show that the value of the car exactly 3 years after it was purchased is £9216.

(1)

The value of the car falls below £1000 for the first time *n* years after it was purchased.

(*b*) Find the value of *n*.

(3)

An insurance company has a scheme to cover the cost of maintenance of the car. The cost is £200 for the first year, and for every following year the cost increases by 12% so that for the 3rd year the cost of the scheme is £250.88.

(*c*) Find the cost of the scheme for the 5th year, giving your answer to the nearest penny.

(2)

(*d*) Find the total cost of the insurance scheme for the first 15 years.

(3)

(C2, Jan 2010 Q6)

12. The adult population of a town is 25 000 at the end of Year 1.

A model predicts that the adult population of the town will increase by 3% each year, forming a geometric sequence.

(*a*) Show that the predicted adult population at the end of Year 2 is 25 750.

(1)

(*b*) Write down the common ratio of the geometric sequence.

(1)

The model predicts that Year N will be the first year in which the adult population of the town exceeds 40 000.

(*c*) Show that

$$(N - 1) \log 1.03 > \log 1.6$$

(3)

(*d*) Find the value of N.

(2)

At the end of each year, each member of the adult population of the town will give £1 to a charity fund.

Assuming the population model,

(*e*) find the total amount that will be given to the charity fund for the 10 years from the end of Year 1 to the end of Year 10, giving your answer to the nearest £1000.

(3)

(C2, June 2010 Q9)

13. The second and fifth terms of a geometric series are 750 and –6 respectively.

Find

(*a*) the common ratio of the series,

(3)

(*b*) the first term of the series,

(2)

(*c*) the sum to infinity of the series.

(2)

(C2, Jan 2011 Q3)

14. The second and third terms of a geometric series are 192 and 144 respectively.

For this series, find

(*a*) the common ratio,

(2)

(*b*) the first term,

(2)

(*c*) the sum to infinity,

(2)

(*d*) the smallest value of n for which the sum of the first n terms of the series exceeds 1000.

(4)

(C2, May 2011 Q6)

15. A geometric series has first term $a = 360$ and common ratio $r = \frac{7}{8}$.

Giving your answers to 3 significant figures where appropriate, find

(*a*) the 20th term of the series,

(2)

(*b*) the sum of the first 20 terms of the series,

(2)

(*c*) the sum to infinity of the series.

(2)

(C2, May 2012 Q1)

16. A geometric series is $a + ar + ar^2 + \ldots$

(*a*) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

The third and fifth terms of a geometric series are 5.4 and 1.944 respectively and all the terms in the series are positive.

For this series find,

(*b*) the common ratio,

(2)

(*c*) the first term,

(2)

(*d*) the sum to infinity.

(3)

(C2, May 2012 Q9)

17. A company predicts a yearly profit of £120 000 in the year 2013. The company predicts that the yearly profit will rise each year by 5%. The predicted yearly profit forms a geometric sequence with common ratio 1.05.

(*a*) Show that the predicted profit in the year 2016 is £138 915.

(1)

(*b*) Find the first year in which the yearly predicted profit exceeds £200 000.

(5)

(*c*) Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound.

(3)

(C2, Jan 2013 Q3)

18. The first three terms of a geometric series are

$$18, 12 \text{ and } p$$

respectively, where p is a constant.

Find

(*a*) the value of the common ratio of the series,

(1)

(*b*) the value of p,

(1)

(*c*) the sum of the first 15 terms of the series, giving your answer to 3 decimal places.

(2)

(C2, May 2013 Q1)

19. The first three terms of a geometric series are $4p$, $(3p + 15)$ and $(5p + 20)$ respectively, where p is a **positive** constant.

(*a*) Show that $11p^2 - 10p - 225 = 0$.

(4)

(*b*) Hence show that $p = 5$.

(2)

(*c*) Find the common ratio of this series.

(2)

(*d*) Find the sum of the first ten terms of the series, giving your answer to the nearest integer.

(3)

(C2, May 2013_R Q5)

20. The first term of a geometric series is 20 and the common ratio is $\frac{7}{8}$. The sum to infinity of the series is S_∞.

(a) Find the value of S_∞.

(2)

The sum to N terms of the series is S_N.

(b) Find, to 1 decimal place, the value of S_{12}.

(2)

(c) Find the smallest value of N, for which $S_\infty - S_N < 0.5$.

(4)

(C2, May 2014 Q6)

21. A geometric series has first term a, where $a \neq 0$, and common ratio r.
The sum to infinity of this series is 6 times the first term of the series.

(a) Show that $r = \frac{5}{6}$.

(2)

Given that the fourth term of this series is 62.5,

(b) find the value of a,

(2)

(c) find the difference between the sum to infinity and the sum of the first 30 terms, giving your answer to 3 significant figures.

(4)

(C2, May 2014_R Q2)

22. (i) All the terms of a geometric series are positive. The sum of the first two terms is 34 and the sum to infinity is 162.

Find

(a) the common ratio,

(4)

(b) the first term.

(2)

(ii) A different geometric series has a first term of 42 and a common ratio of $\frac{6}{7}$.

Find the smallest value of n for which the sum of the first n terms of the series exceeds 290.

(4)

(C2, May 2015 Q5)

23. A geometric series has first term a and common ratio $r = \frac{3}{4}$.

The sum of the first 4 terms of this series is 175.

(a) Show that $a = 64$.

(2)

(b) Find the sum to infinity of the series.

(2)

(c) Find the difference between the 9th and 10th terms of the series.
Give your answer to 3 decimal places.

(3)

(C2, May 2016 Q1)

24. The first three terms of a geometric sequence are

$$7k - 5, \qquad 5k - 7, \qquad 2k + 10$$

where k is a constant.

(a) Show that $11k^2 - 130k + 99 = 0$

(4)

Given that k is not an integer,

(b) show that $k = \frac{9}{11}$

(2)

For this value of k,

(c) (i) evaluate the fourth term of the sequence, giving your answer as an exact fraction,

(ii) evaluate the sum of the first ten terms of the sequence.

(6)

(C2, May 2017 Q9)

25. A geometric series with common ratio $r = -0.9$ has sum to infinity 10 000.
For this series,

(a) find the first term,

(2)

(b) find the fifth term,

(2)

(c) find the sum of the first twelve terms, giving this answer to the nearest integer.

(3)

(C2, May 2018 Q6)

26. A business plan predicts a yearly profit of £250 000 for the year 2019.
The business plan also predicts that the yearly profit will increase by 10% each year forming a geometric sequence.

(a) Find the first year in which the business plan predicts that the yearly profit will exceed £1 000 000

(4)

(b) Find the total predicted profits for the years 2019 to 2030 inclusive, giving your answer to the nearest pound.

(3)

(C2, May 2019 Q4)

27. In the first month after opening, a mobile phone shop sold 300 phones. A model for future sales assumes that the number of phones sold will increase by 5% per month, so that 300×1.05 will be sold in the second month, 300×1.052 in the third month, and so on.

Using this model, calculate

(*a*) the number of phones sold in the 24th month,

(2)

(*b*) the total number of phones sold over the whole 24 months.

(2)

This model predicts that, in the *N*th month, the number of phones sold in that month exceeds 3000 for the first time.

(*c*) Find the value of *N*.

(3)

(IAL C12, Jan 2014 Q9)

28. (i) Find the value of $\sum_{r=1}^{20}(3+5r)$.

(3)

(ii) Given that $\sum_{r=0}^{\infty}\frac{a}{4^r}=16$, find the value of the constant a.

(4)

(IAL C12, May 2014 Q9)

29. A business is expected to have a yearly profit of £275 000 for the year 2016. The profit is expected to increase by 10% per year, so that the expected yearly profits form a geometric sequence with common ratio 1.1.

(*a*) Show that the difference between the expected profit for the year 2020 and the expected profit for the year 2021 is £40 300 to the nearest hundred pounds.

(3)

(*b*) Find the first year for which the expected yearly profit is more than one million pounds.

(4)

(*c*) Find the total expected profits for the years 2016 to 2026 inclusive, giving your answer to the nearest hundred pounds.

(3)

(IAL C12, Jan 2015 Q12)

30. The first term of a geometric series is 6 and the common ratio is 0.92.

For this series, find

(*a*) (i) the 25th term, giving your answer to 2 significant figures,

(ii) the sum to infinity. **(4)**

The sum to *n* terms of this series is greater than 72.

(*b*) Calculate the smallest possible value of *n*. **(4)**

(IAL C12, May 2016 Q9)

31. The resident population of a city is 130 000 at the end of Year 1

A model predicts that the resident population of the city will increase by 2% each year, with the populations at the end of each year forming a geometric sequence.

(*a*) Show that the predicted resident population at the end of Year 2 is 132 600.

(1)

(*b*) Write down the value of the common ratio of the geometric sequence.

(1)

The model predicts that Year N will be the first year which will end with the resident population of the city exceeding 260 000.

(*c*) Show that

$$N > \frac{\log_{10} 2}{\log_{10} 1.02} + 1$$

(4)

(*d*) Find the value of N.

(1)

(IAL C12, Jan 2016 Q9)

32. Wheat is to be grown on a farm.

A model predicts that the mass of wheat harvested on the farm will increase by 1.5% per year, so that the mass of wheat harvested each year forms a geometric sequence.

Given that the mass of wheat harvested during year one is 6000 tonnes,

(*a*) show that, according to the model, the mass of wheat harvested on the farm during year 4 will be approximately 6274 tonnes.

(2)

During year N, according to the model, there is predicted to be more than 8000 tonnes of wheat harvested on the farm.

(*b*) Find the smallest possible value of N.

(5)

It costs £5 per tonne to harvest the wheat.

(*c*) Assuming the model, find the total amount that it would cost to harvest the wheat from year one to year 10 inclusive. Give your answer to the nearest £1000.

(3)

(IAL C12, May 2017 Q11)

33. A new mineral has been discovered and is going to be mined over a number of years.

A model predicts that the mass of the mineral mined each year will decrease by 15% per year, so that the mass of the mineral mined each year forms a geometric sequence.

Given that the mass of the mineral mined during year 1 is 8000 tonnes,

(*a*) show that, according to the model, the mass of the mineral mined during year 6 will be approximately 3550 tonnes.

(2)

According to the model, there is a limit to the total mass of the mineral that can be mined.

(*b*) With reference to the geometric series, state why this limit exists.

(1)

(*c*) Calculate the value of this limit.

(2)

It is decided that a total mass of 40 000 tonnes of the mineral is required. This is going to be mined from year 1 to year N inclusive.

(*d*) Assuming the model, find the value of N.

(5)

(IAL C12, Oct 2017 Q14)

34. The first term of a geometric series is 20 and the common ratio is 0.9

(*a*) Find the value of the fifth term.

(2)

(*b*) Find the sum of the first 8 terms, giving your answer to one decimal place.

(2)

Given that $S_\infty - S_N < 0.04$, where S_N is the sum of the first N terms of this series,

(*c*) show that $0.9^N < 0.0002$

(4)

(*d*) Hence find the smallest possible value of N.

(2)

(IAL C12, Jan 2018 Q9)

35. A cyclist aims to travel a total of 1200 km over a number of days.
He cycles 12 km on day 1
He increases the distance that he cycles each day by 6% of the distance cycled on the previous day, until he reaches the total of 1200 km.

(*a*) Show that on day 8 he cycles approximately 18 km.

(3)

He reaches his total of 1200 km on day N, where N is a positive integer.

(*b*) Find the value of N.

(4)

The cyclist stops when he reaches 1200 km.

(*c*) Find the distance that he cycles on day N. Give your answer to the nearest km.

(2)

(IAL C12, May 2018 Q9)

36. The first three terms of a geometric series are $(k + 5)$, k and $(2k – 24)$ respectively, where k is a constant.

(a) Show that $k^2 – 14k – 120 = 0$

(3)

(b) Hence find the possible values of k.

(2)

(c) Given that the series is convergent, find

(i) the common ratio,

(ii)the sum to infinity.

(4)

(IAL C12, Oct 2018 Q16)

37. Karen is going to raise money for a charity.

She aims to cycle a **total** distance of 1000 km over a number of days.

On day one she cycles 25 km.

She increases the distance that she cycles each day by 10% of the distance cycled on the previous day, until she reaches the total distance of 1000 km.

She reaches the **total** distance of 1000 km on day N, where N is a positive integer.

(a) Find the value of N.

(4)

On day one, 50 people donated money to the charity. Each day, 20 more people donated to the charity than did so on the previous day, so that 70 people donated money on day two, 90 people donated money on day three, and so on.

(b) Find the number of people who donated to the charity on day fifteen.

(2)

Each day, the donation given by each person was £5

(c) Find the total amount of money donated by the end of day fifteen.

(3)

(IAL C12, Jan 2019 Q12)

38. The 4th term of a geometric series is 125 and the 7th term is 8

(a) Show that the common ratio of this series is $\dfrac{2}{5}$

(2)

(b) Hence find, to 3 decimal places, the difference between the sum to infinity and the sum of the first 10 terms of this series.

(4)

(IAL C12, May 2019 Q1)

39. A colony of ants is being studied.

The number of ants in the colony at the start of the study was 140 000

Two years after the start of the study the number of ants in the colony is 150 000

A model predicts that the number of ants in the colony will increase by p % each year. Hence the number of ants in the colony at the end of each year of study form a geometric sequence.

Assuming the model,

(a) find the value of p, giving your answer to 2 decimal places.

(3)

According to the model, at the end of N years of study the number of ants in the colony exceeds 500 000

(b) Find, showing all steps in your working, the smallest integer value of N.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(5)

(IAL C12, Oct 2019 Q14)