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Pearson Edexcel

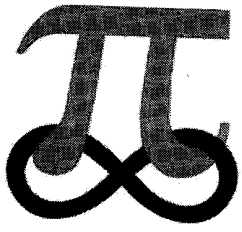
GCSE Maths 9 -1

Past Exam Questions

by Topics.

Algebra: Functions

Solutions



1. f is the function $f(x) = 2x + 5$.

(*a*) Find f(3). $f(3) = 2\times3+5 = 11$

11

(1)

(*b*) Express the inverse function f^{-1} in the form $f^{-1}(x) =$

$y = 2x+5$

$x = 2y+5$

$y = \frac{x-5}{2}$

$f^{-1}(x) = \frac{x-5}{2}$

(2)

g is the function $g(x) = x^2 - 25$.

(*c*) Find g(–3). $g(-3) = (-3)^2 - 25 = 9 - 25 = -16$

-16

(1)

(*d*) (i) Find gf(x).
Give your answer as simply as possible.

$gf(x) = (2x+5)^2 - 25$

$= 4x^2 + 20x + 25 - 25$

$= 4x^2 + 20x$

$gf(x) = 4x^2 + 20x$

(ii) Solve $gf(x) = 0$.

$4x^2 + 20x = 0$

$4x(x+5) = 0$

$x = 0, x = -5$

$x = 0, -5$

(5)

(Total 9 marks)

2. $g(x) = \dfrac{4x}{3-x}$ $\quad f(x) = 2x - 5$

Given that $x > 3$, find the exact value of x such that $g^{-1}(x) = f(x)$.

$g^{-1}(x)$, $y = \dfrac{4x}{3-x}$ $\quad 3x - xy = 4y$ $\quad xy + 4y = 3x$ $\quad y(x+4) = 3x$ $\quad y = \dfrac{3x}{x+4}$

$x = \dfrac{4y}{3-y}$

$\dfrac{3x}{x+4} = 2x - 5$

$(2x-5)(x+4) = 3x$

$2x^2 + 3x - 20 = 3x$

$x^2 = 10$, $x = \sqrt{10}$ as $x > 3$

(5 marks)

~~4.~~ f is the function $f(x) = 2x + 5$. Same as Q1

(*a*) Find f(3).

.....

(1)

(*b*) Express the inverse function f^{-1} in the form $f^{-1}(x) =$

$f^{-1}(x) =$ .....

(2)

g is the function $g(x) = x^2 - 25$.

(*c*) Find g(–3).

.....

(1)

(*d*) (i) Find gf(*x*).
Give your answer as simply as possible.

$gf(x) =$ .....

(ii) Solve $gf(x) = 0$.

.....

(5)

(9 marks)

4. f is a function such that

$f(x) = \frac{1}{x^2+1}$

$f(1/2) = \frac{1}{(1/2)^2+1} = \frac{1}{1/4+1} = \frac{1}{5/4} = 4/5$

(*a*) Find $f(\frac{1}{2})$

4/5

(1)

g is a function such that

$g(x) = \sqrt{x-1} \quad x \geq 1$

$fg = \frac{1}{(\sqrt{x-1})^2+1} = \frac{1}{x-1+1} = 1/x$

(*b*) Find fg(*x*)
Give your answer as simply as possible.

fg(*x*) = 1/x

(2)

(3 marks)

5. The function f is defined as

$f(x) = \frac{x-6}{2}$

(*a*) Find f(8).

$f(8) = \frac{8-6}{2} = \frac{2}{2} = 1$

1

(1)

(*b*) Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

$y = \frac{x-6}{2}$

$x = \frac{y-6}{2}$

$y = 2x+6$

$f^{-1}(x) =$ 2x+6

(2)

The function g is defined as

$$g(x) = \sqrt{x-4}$$

(*c*) Express the function gf in the form gf(x) = …
Give your answer as simply as possible.

$gf = \sqrt{\frac{x-6}{2} - 4} = \sqrt{\frac{x-6-8}{2}} = \sqrt{\frac{x-14}{2}}$

gf(x) = $\sqrt{\frac{x-14}{2}}$

(2)

(5 marks)

6. $f(x) = \frac{4}{x-3}$ $g(x) = \frac{x-2}{x}$

(*a*) Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

$y = \frac{4}{x-3}$

$x = \frac{4}{y-3}$

$y-3 = \frac{4}{x}$

$y = \frac{4}{x} + 3$

$f^{-1}(x) = \frac{4}{x} + 3$

(*b*) Solve $fg(a) = 1$

(3)

You must show your working.

$fg(x) = \frac{4}{\frac{x-2}{x} - 3} = \frac{4}{\frac{x-2-3x}{x}} = \frac{4x}{-2-2x}$

$fg(a) \quad \frac{4a}{-2-2a} = 1$

$4a = -2-2a$

$6a = -2$

$a = -\frac{1}{3}$

$a = -\frac{1}{3}$

(3)

(6 marks)

7. For all values of x

$$f(x) = (x + 1)^2 \quad \text{and} \quad g(x) = 2(x - 1)$$

(*a*) Show that $gf(x) = 2x(x + 2)$

$gf(x) = 2((x+1)^2 - 1) = 2(x^2 + 2x + 1 - 1)$

$= 2(x^2 + 2x)$

$= 2x(x + 2)$

(2)

(*b*) Find $g^{-1}(7)$

way (1) $g(x) = 2x - 2$

$y = 2x - 2$

$x = \frac{y+2}{2}$

$g^{-1}(x) = \frac{x+2}{2}$

$g^{-1}(7) = 9/2$

way (2)

$2(x - 1) = 7$

$x = 7/2 + 1$

$= 9/2$

$9/2$

(2)

(4 marks)

8. $f(x) = 4\sin x°$

(*a*) Find f(23)

Give your answer correct to 3 significant figures.

$f(23) = 4 \sin 23 = 1.56$

1.56

$g(x) = 2x - 3$ **(1)**

(*b*) Find fg(34)

Give your answer correct to 3 significant figures.

$g(34) = 2 \times 34 - 3 = 65$

$f(65) = 4 \sin 65$

$= 3.63$

3.63

(2)

$h(x) = (x + 4)^2$

Ivan needs to solve the following equation $h(x) = 25$

He writes

$(x + 4)^2 = 25$
$x + 4 = 5$
$x = 1$

This is not fully correct.

(*c*) Explain why.

$(x+4)^2 = 25$, $x+4 = \pm 5$, $x+4=5$ or $x+4=-5$

.....

(1)

9. f and g are functions such that **(4 marks)**

$$f(x) = \frac{2}{x^2} \quad \text{and} \quad g(x) = 4x^3$$

(*a*) Find $f(-5) = \frac{2}{25}$

.....

(1)

(*b*) Find $fg(1)$

$g(1) = 4$, $fg(1) = \frac{2}{4^2} = \frac{1}{8}$

.....

(2)

10. The function f and g are such that **(3 marks)**

$f(x) = 5x + 3$ $g(x) = ax + b$ where a and b are constants.

$g(3) = 20$ and $f^{-1}(33) = g(1)$

Find the value of a and the value of b.

$g(3) = 20$, $3a + b = 20$ —— (1)

$f^{-1}(33)$, $5x + 3 = 33$, $x = 6$, $f^{-1}(33) = 6$

$g(1) = a + b$

$\therefore a + b = 6$ —— (2)

(1), (2) $\Rightarrow$ $2a = 14$, $a = 7$

$b = -1$

$a =$ 7

$b =$ -1

(5 marks)

11. Two functions, f and g are defined as

$$f: x \mapsto 1 + \frac{1}{x} \qquad \text{for } x > 0$$

$$g: x \mapsto \frac{x+1}{2} \qquad \text{for } x > 0$$

Given that h = fg

express the inverse function h^{-1} in the form $h^{-1}: x \mapsto \ldots$

$h = fg = 1 + \frac{1}{\frac{x+1}{2}} = 1 + \frac{2}{x+1} = \frac{x+3}{x+1}$

$y = \frac{x+3}{x+1}$, $x = \frac{y+3}{y+1}$, $xy - y = 3 - x$

$y(x-1) = 3-x$, $y = \frac{3-x}{x-1}$

$h^{-1}: x \mapsto \frac{3-x}{x-1}$

12. The function f is such that **(4 marks)**

$$f(x) = \frac{3x-5}{4}$$

(*a*) Find f(–7) $= \frac{-21-5}{4} = \frac{-26}{4} = -13/2$

(1)

(*b*) Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

$y = \frac{3x-5}{4}$

$x = \frac{3y-5}{4}$

$y = \frac{4x+5}{3}$

$f^{-1}(x) = \frac{4x+5}{3}$

(2)

The function g is such that

$$g(x) = \sqrt{19 - x}$$

(*c*) Find fg (3)

$g(3) = \sqrt{19-3} = 4$

$fg(3) = \frac{3\times 4 - 5}{4} = 7/4$

.....

(2)

(*d*) Which values of x cannot be included in any domain of g?

$x > 19$

.....

(2)

(7 marks)

13. f is the function such that $f(x) = \sqrt{4-x}$ and $f(x) \geqslant 0$

(*a*) State which values of x must be excluded from any domain of f

$x > 4$

.....

(1)

The inverse function f^{-1} has domain $x \geqslant 0$

(*b*) Find $f^{-1}(x)$

$y = \sqrt{4-x}$, $x = \sqrt{4-y}$, $x^2 = 4 - y$, $y = 4 - x^2$

$f^{-1}(x) =$ $4 - x^2$

(2)

g is the function such that $g(x) = (5-x)(x-1)$
The composite function fg has domain $x \geqslant 3$

(*c*) Find $fg(x)$
Give your answer in its simplest form.

$= 5x - 5 - x^2 + x$

$= 6x - 5 - x^2$

$fg = \sqrt{4 - (6x - 5 - x^2)} = \sqrt{x^2 - 6x + 9}$

$= \sqrt{(x-3)^2}$

$= (x-3)$

$fg(x) =$ .....

(4)

(7 marks)

14. f is the function such that $f(x) = 3 - 2x$

(*a*) Find f (–4) $= 3-2(-4) = 11$

.....

(1)

(*b*) Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

$y = 3 - 2x, \quad x = 3 - 2y, \quad y = \frac{3-x}{2}$

$f^{-1}(x) = \frac{3-x}{2}$

(2)

g is the function such that $g(x) = x^2 - 5$

(*c*) Solve the equation $gf(x) = ff(x)$
Show clear algebraic working.

$gf(x) = (3-2x)^2 - 5 = 9 - 12x + 4x^2 - 5 = 4 - 12x + 4x^2$

$ff(x) = 3 - 2(3-2x) = 3 - 6 + 4x = 4x - 3$

$4 - 12x + 4x^2 = 4x - 3$

$4x^2 - 16x + 7 = 0$

$(2x - 1)(2x - 7)$

$x = 1/2$ or $x = 7/2$

.....

(5)

(8 marks)