Edexcel

Pure Mathematics
Year 2
Functions.

Past paper questions from Core Maths 3 and IAL C34

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1. The function f is defined by

$$\mathrm{f}: x \mapsto \frac{5x+1}{x^2+x-2} - \frac{3}{x+2}, \quad x > 1.$$

(*a*) Show that $\mathrm{f}(x) = \dfrac{2}{x-1}, \quad x > 1.$

(4)

(*b*) Find $\mathrm{f}^{-1}(x)$.

(3)

The function g is defined by

$$\mathrm{g}: x \mapsto x^2 + 5, \quad x \in \mathbb{R}.$$

(*c*) Solve $\mathrm{fg}(x) = \frac{1}{4}$.

(3)

(Q3, June 2005)

2. The functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x + \ln 2, \qquad x \in \mathbb{R},$$

$$\mathrm{g} : x \mapsto \mathrm{e}^{2x}, \qquad x \in \mathbb{R}.$$

(*a*) Prove that the composite function gf is

$$\mathrm{gf} : x \mapsto 4\mathrm{e}^{4x}, \qquad x \in \mathbb{R}.$$ **(4)**

(*b*) Sketch the curve with equation $y = \mathrm{gf}(x)$, and show the coordinates of the point where the curve cuts the y-axis.

(1)

(*c*) Write down the range of gf.

(1)

(*d*) Find the value of x for which $\dfrac{\mathrm{d}}{\mathrm{d}x}[\mathrm{gf}(x)] = 3$, giving your answer to 3 significant figures.

(4)

(Q8, Jan 2006)

3. For the constant k, where $k > 1$, the functions f and g are defined by

$$\mathrm{f}: x \mapsto \ln(x + k), \quad x > -k,$$

$$\mathrm{g}: x \mapsto |2x - k|, \quad x \in \mathbb{R}.$$

(a) On separate axes, sketch the graph of f and the graph of g.

On each sketch state, in terms of k, the coordinates of points where the graph meets the coordinate axes.

(5)

(*b*) Write down the range of f.

(1)

(*c*) Find $\mathrm{fg}\left(\dfrac{k}{4}\right)$ in terms of k, giving your answer in its simplest form. **(2)**

The curve C has equation $y = \mathrm{f}(x)$. The tangent to C at the point with x-coordinate 3 is parallel

to the line with equation $9y = 2x + 1$.

(*d*) Find the value of k. **(4)**

(Q7, June 2006)

4.
The function f is defined by

$$f : x \mapsto \ln(4 - 2x), \quad x < 2 \text{ and } x \in \mathbb{R}.$$

(*a*) Show that the inverse function of f is defined by

$$f^{-1} : x \mapsto 2 - \tfrac{1}{2}e^{x}$$

and write down the domain of f^{-1}.

(4)

(*b*) Write down the range of f^{-1}.

(1)

(*c*) Sketch the graph of $y = f^{-1}(x)$. State the coordinates of the points of intersection with the x and y axes.

(4)

(Q6, Jan 2007)

5. The functions f and g are defined by

$$f : \mapsto \ln(2x - 1), \qquad x \in \mathbb{R}, \; x > \tfrac{1}{2},$$

$$g : \mapsto \frac{2}{x-3}, \qquad x \in \mathbb{R}, \; x \neq 3.$$

(*a*) Find the exact value of fg(4).

(2)

(*b*) Find the inverse function $f^{-1}(x)$, stating its domain.

(4)

(*c*) Sketch the graph of $y = |g(x)|$. Indicate clearly the equation of the vertical asymptote and the coordinates of the point at which the graph crosses the y-axis.

(3)

(*d*) Find the exact values of x for which $\left|\frac{2}{x-3}\right| = 3$.

(3)

(Q5, June 2007)

6. The functions f and g are defined by

$$f : x \mapsto 1 - 2x^3, \quad x \in \mathbb{R}.$$

$$g : x \mapsto \frac{3}{x} - 4, \quad x > 0, \; x \in \mathbb{R}.$$

(*a*) Find the inverse function f^{-1}.

(2)

(*b*) Show that the composite function gf is

$$gf : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

(*c*) Solve gf $(x) = 0$.

(2)

(*d*) Use calculus to find the coordinates of the stationary point on the graph of $y = gf(x)$.

(5)

(Q8, Jan 2008)

7. The function f is defined by

$$\text{f}: x \mapsto \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}, \quad x > 3.$$

(*a*) Show that $\text{f}(x) = \dfrac{1}{x+1}, \quad x > 3.$

(4)

(*b*) Find the range of f.

(2)

(*c*) Find $\text{f}^{-1}(x)$. State the domain of this inverse function.

(3)

The function g is defined by

$$\text{g}: x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(*d*) Solve $\text{fg}(x) = \dfrac{1}{8}$.

(3)

(Q4, June 2008)

8. The functions f and g are defined by

$$\text{f} : x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R},$$

$$\text{g} : x \mapsto \text{e}^{x^2}, \quad x \in \mathbb{R}.$$

(*a*) Write down the range of g.

(1)

(*b*) Show that the composite function fg is defined by

$$\text{fg} : x \mapsto x^2 + 3\text{e}^{x^2}, \quad x \in \mathbb{R}.$$

(2)

(*c*) Write down the range of fg.

(1)

(*d*) Solve the equation $\dfrac{\text{d}}{\text{d}x}[\text{fg}(x)] = x(x\text{e}^{x^2} + 2)$.

(6)

(Q5, Jan 2009)

9.

(i) Find the exact solutions to the equations

(*a*) $\ln(3x - 7) = 5$,

(3)

(*b*) $3^x \text{e}^{7x+2} = 15$.

(5)

(ii) The functions f and g are defined by

$$\text{f}(x) = \text{e}^{2x} + 3, \qquad x \in \mathbb{R},$$

$$\text{g}(x) = \ln(x - 1), \quad x \in \mathbb{R}, \quad x > 1.$$

(*a*) Find f^{-1} and state its domain.

(4)

(*b*) Find fg and state its range.

(3)

(Q9, Jan 2010)

10. The function f is defined by

$$f : x \mapsto |2x - 5|, \quad x \in \mathbb{R}.$$

(*a*) Sketch the graph with equation $y = f(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(2)

(*b*) Solve $f(x) = 15 + x$.

(3)

The function g is defined by

$$g : x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \le x \le 5.$$

(*c*) Find fg(2).

(2)

(*d*) Find the range of g.

(3)

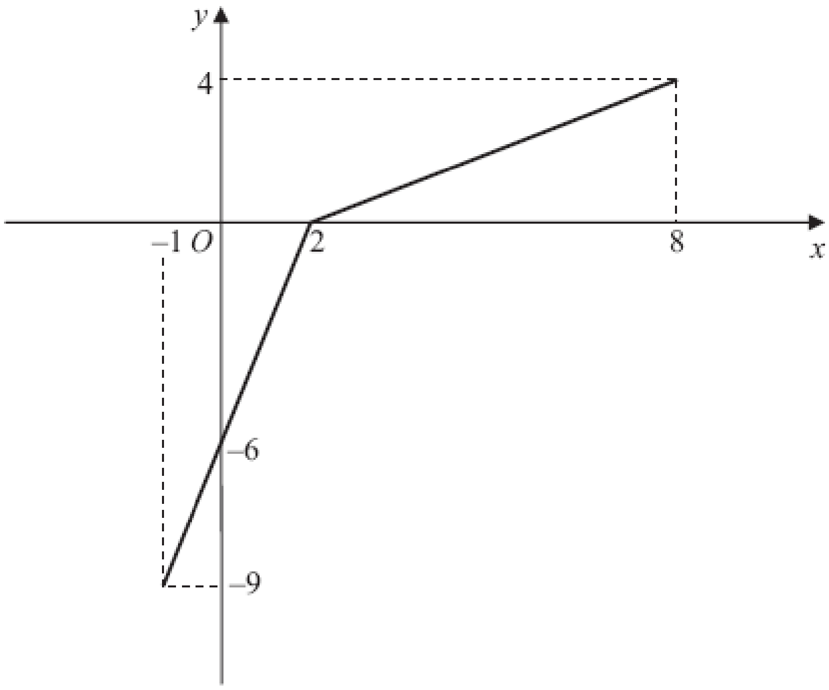
(Q4, June 2010)

11. The function f is defined by

$$f: x \mapsto \frac{3-2x}{x-5}, \quad x \in \mathbb{R}, \quad x \neq 5.$$

(*a*) Find $f^{-1}(x)$.

(3)



The function g has domain $-1 \le x \le 8$, and is linear from $(-1, -9)$ to $(2, 0)$ and from $(2, 0)$ to $(8, 4)$. Figure 2 shows a sketch of the graph of $y = g(x)$

(*b*) Write down the range of g.

(1)

(*c*) Find gg(2).

(2)

(*d*) Find fg(8).

(2)

(*e*) On separate diagrams, sketch the graph with equation

(i) $y = |\, g(x) |$,

(ii) $y = g^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes. **(4)**

(*f*) State the domain of the inverse function g^{-1}.

(1)

(Q6, Jan 2011)

12. The function f is defined by

$$f : x \mapsto 4 - \ln (x + 2), \quad x \in \mathbb{R}, \quad x \neq -1.$$

(*a*) Find $f^{-1}(x)$.

(3)

(*b*) Find the domain of f^{-1}.

(1)

The function g is defined by

$$g : x \mapsto e^{x^2} - 2, \quad x \in \mathbb{R}.$$

(*c*) Find fg(x), giving your answer in its simplest form.

(3)

(*d*) Find the range of fg.

(1)

(Q4, June 2011)

13. The function f is defined by

$$f : x \mapsto \frac{3(x+1)}{2x^2+7x-4} - \frac{1}{x+4}, \quad x \in \mathbb{R},\ x > \frac{1}{2}.$$

(*a*) Show that f(x) = $\dfrac{1}{2x-1}$.

(4)

(*b*) Find $f^{-1}(x)$.

(3)

(*c*) Find the domain of f^{-1}.

(1)

$$g(x) = \ln (x + 1).$$

(*d*) Find the solution of fg(x) = $\dfrac{1}{7}$, giving your answer in terms of e. **(4)**

(Q7, Jan 2012)

14. The functions f and g are defined by

$$f : x \mapsto e^x + 2, \qquad x \in \mathbb{R},$$

$$g : x \mapsto \ln x, \qquad x > 0.$$

(*a*) State the range of f.

(1)

(*b*) Find fg(x), giving your answer in its simplest form.

(2)

(*c*) Find the exact value of x for which f($2x$ + 3) = 6.

(4)

(*d*) Find f^{-1}, the inverse function of f, stating its domain.

(3)

(*e*) On the same axes sketch the curves with equation y = f(x) and $y = f^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes. **(4)**

(Q6, June 2012)

15. $$\text{h}(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \quad x \geq 0.$$

(*a*) Show that $\text{h}(x) = \dfrac{2x}{x^2+5}$.

(4)

(*b*) Hence, or otherwise, find $\text{h}'(x)$ in its simplest form.

(3)

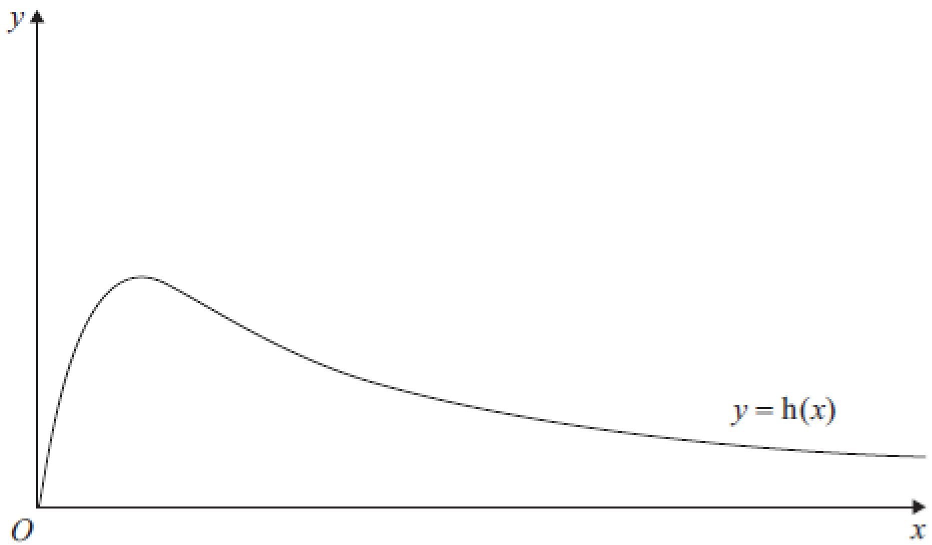


Figure 2

Figure 2 shows a graph of the curve with equation $y = \text{h}(x)$.

(*c*) Calculate the range of $\text{h}(x)$.

(5)

(Q7, Jan 2013)

16. The functions f and g are defined by

$$\text{f}: x \mapsto 2|x| + 3, \qquad x \in \mathbb{R}$$

$$\text{g}: x \mapsto 3 - 4x, \qquad x \in \mathbb{R}$$

(*a*) State the range of f.

(2)

(*b*) Find fg(1).

(2)

(*c*) Find g^{-1}, the inverse function of g.

(2)

(*d*) Solve the equation

$$\text{gg}(x) + [\text{g}(x)]^2 = 0$$

(5)

(Q7, June 2013_R)

17. $$g(x) = \frac{x}{x+3} + \frac{3(2x+1)}{x^2+x-6}, \qquad x > 3$$

(*a*) Show that $g(x) = \frac{x+1}{x-2}, \quad x > 3$

(4)

(*b*) Find the range of g.

(2)

(*c*) Find the exact value of *a* for which $g(a) = g^{-1}(a)$.

(4)

(Q5, June 2013)

18. The function f has domain $-2 \leq x \leq 6$ and is linear from (–2, 10) to (2, 0) and from (2, 0) to (6, 4). A sketch of the graph of $y = f(x)$ is shown in Figure 1.

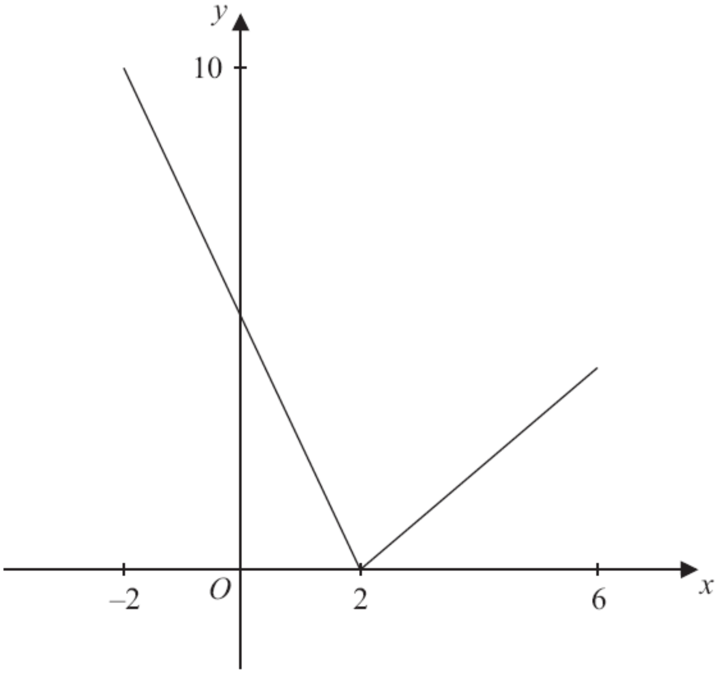


Figure 1

(*a*) Write down the range of f.

(1)

(*b*) Find ff(0).

(2)

The function g is defined by

$$g : x \rightarrow \frac{4+3x}{5-x}, \qquad x \in \mathbb{R}, \qquad x \neq 5$$

(*c*) Find $g^{-1}(x)$

(3)

(*d*) Solve the equation $gf(x) = 16$

(5)

(Q7, June 2013)

19. The function f is defined by

$$\mathrm{f}: x \rightarrow \mathrm{e}^{2x} + k^2, \qquad x \in \mathbb{R}, \qquad k \text{ is a positive constant.}$$

(*a*) State the range of f.

(1)

(*b*) Find f^{-1} and state its domain.

(3)

The function g is defined by

$$\mathrm{g}: x \rightarrow \ln(2x), \qquad x > 0$$

(*c*) Solve the equation

$$\mathrm{g}(x) + \mathrm{g}(x^2) + \mathrm{g}(x^3) = 6$$

giving your answer in its simplest form.

(4)

(*d*) Find fg(x), giving your answer in its simplest form.

(2)

(*e*) Find, in terms of the constant k, the solution of the equation

$$\mathrm{fg}(x) = 2k^2$$

(2)

(Q6, June 2014_R)

20.

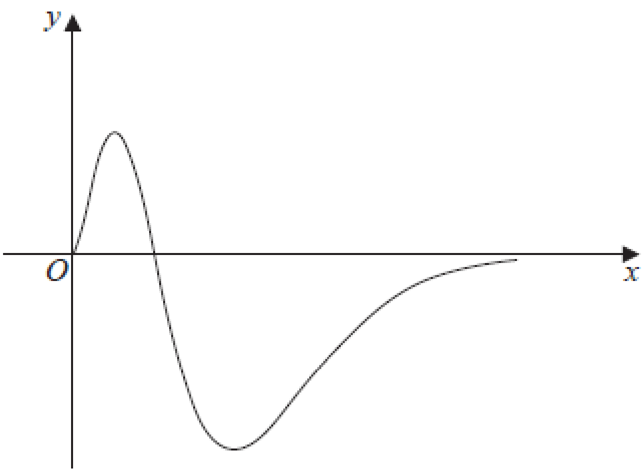


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$\mathrm{g}(x) = x^2(1 - x)\mathrm{e}^{-2x}, \qquad x \geq 0.$$

(*a*) Show that $\mathrm{g}'(x) = \mathrm{f}(x)\mathrm{e}^{-2x}$, where f($x$) is a cubic function to be found.

(3)

(*b*) Hence find the range of g.

(6)

(*c*) State a reason why the function $\mathrm{g}^{-1}(x)$ does not exist.

(1)

(Q7, June 2015)

21. The functions f and g are defined by

$$\mathrm{f} : x \rightarrow 7x - 1, \qquad x \in \mathbb{R},$$

$$\mathrm{g} : x \rightarrow \frac{4}{x-2}, \qquad x \neq 2, x \in \mathbb{R},$$

(*a*) Solve the equation $\mathrm{fg}(x) = x$.

(4)

(*b*) Hence, or otherwise, find the largest value of a such that $\mathrm{g}(a) = \mathrm{f}^{-1}(a)$.

(1)

(Q1, June 2016)

22.

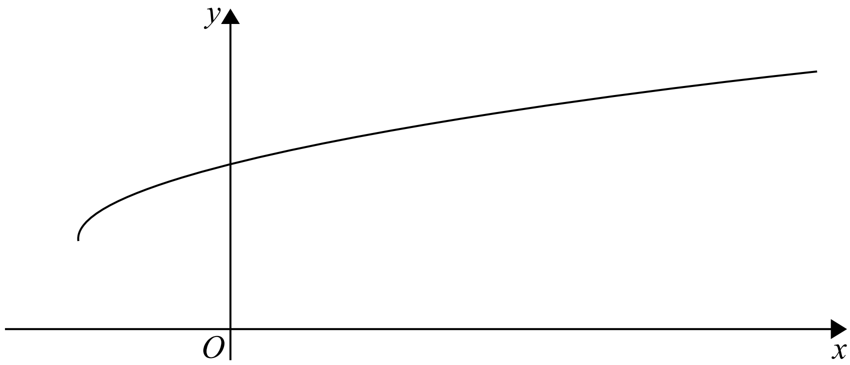


Figure 1

Figure 1 shows a sketch of part of the graph of $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = 3 + \sqrt{x+2}\,, \qquad x \geqslant -2$$

(*a*) State the range of g.

(1)

(*b*) Find $\mathrm{g}^{-1}(x)$ and state its domain.

(3)

(*c*) Find the exact value of x for which

$$\mathrm{g}(x) = x$$

(4)

(*d*) Hence state the value of a for which

$$\mathrm{g}(a) = \mathrm{g}^{-1}(a)$$

(1)

(Q3, June 2017)

23. The function g is defined by

$$\mathrm{g}\,(x) = \frac{6x}{2x+3} \qquad x > 0$$

(*a*) Find the range of g.

(2)

(*b*) Find $\mathrm{g}^{-1}(x)$ and state its domain.

(3)

(*c*) Find the function $\mathrm{gg}(x)$, writing your answer as a single fraction in its simplest form.

(3)

(Q3, IAL, June 2017)

24. The function f is defined by

$$\mathrm{f}: x \mapsto \frac{3x-5}{x+1}, \qquad x \in \mathbb{R}, x \neq -1$$

(*a*) Find an expression for $\mathrm{f}^{-1}(x)$.

(3)

(*b*) Show that

$$\mathrm{ff}(x) = \frac{x+a}{x-1}, \qquad x \in \mathbb{R}, x \neq -1, x \neq 1$$

where a is an integer to be determined.

(4)

The function g is defined by

$$\mathrm{g}: x \mapsto x^2 - 3x, \qquad x \in \mathbb{R}, 0 \leq x \leq 5$$

(*c*) Find the value of fg(2).

(2)

(*d*) Find the range of g.

(3)

(Q7, IAL, June 2014)

25. The function g is defined by

$$\mathrm{g}: x \mapsto |8 - 2x|, \qquad x \in \mathbb{R}, \qquad x \geq 0$$

(*a*) Sketch the graph with equation $y = \mathrm{g}(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(3)

(*b*) Solve the equation

$$|8 - 2x| = x + 5$$

(3)

The function f is defined by

$$\mathrm{f}: x \mapsto x^2 - 3x + 1, \qquad x \in \mathbb{R}, \qquad 0 \leq x \leq 4$$

(*c*) Find fg(5). **(2)**

(*d*) Find the range of f. You must make your method clear. **(4)**

(Q7, IAL, Jan 2015)

26. Given that

$$\mathrm{f}(x) = \frac{4}{3x+5}, \qquad x > 0$$

$$\mathrm{g}(x) = \frac{1}{x}, \qquad x > 0$$

(*a*) state the range of f, **(2)**

(*b*) find $\mathrm{f}^{-1}(x)$, **(3)**

(*c*) find fg(x). **(1)**

(*d*) Show that the equation fg(x) = gf(x) has no real solutions.

(4)

(Q4, IAL, Jan 2017)

27.

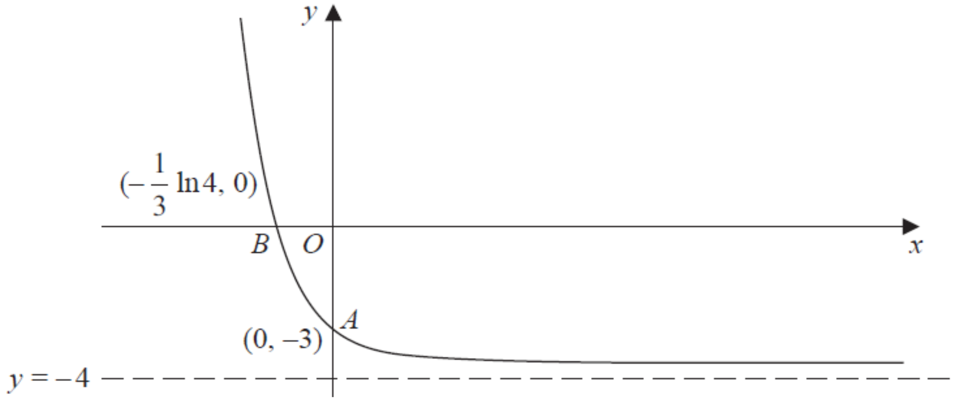


Figure 4

Figure 4 shows a sketch of part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$

The curve meets the coordinate axes at the points $A(0, -3)$ and $B(-\frac{1}{3}\ln 4, 0)$ and the curve
has an asymptote with equation $y = -4$

In separate diagrams, sketch the graph with equation

(*a*) $y = |f(x)|$

(4)

(*b*) $y = 2f(x) + 6$

(3)

On each sketch, give the exact coordinates of the points where the curve crosses or meets
the coordinate axes and the equation of any asymptote.

Given that

$$f(x) = e^{-3x} - 4, \qquad x \in \mathbb{R}$$

$$g(x) = \ln\left(\frac{1}{x+2}\right), \qquad x > -2$$

(*c*) state the range of f,

(1)

(*d*) find $f^{-1}(x)$,

(3)

(*e*) express $fg(x)$ as a polynomial in x.

(3)

(Q11, IAL, Jan 2016)

28. **Figure 1**

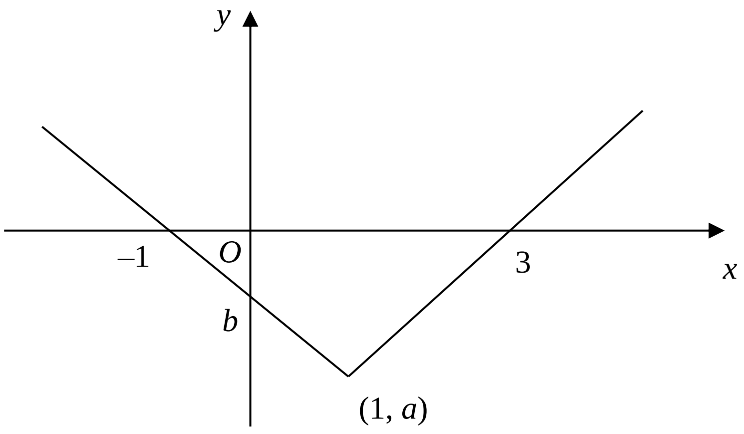


Figure 1 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the x-axis at $(3, 0)$. The other line meets the x-axis at $(–1, 0)$ and the y-axis at $(0, b)$, $b < 0$.

In separate diagrams, sketch the graph with equation

(*a*) $y = f(x + 1)$,

(2)

(*b*) $y = f(|x|)$.

(3)

Indicate clearly on each sketch the coordinates of any points of intersection with the axes.

Given that $f(x) = |x - 1| - 2$, find

(*c*) the value of *a* and the value of *b*,

(2)

(*d*) the value of *x* for which $f(x) = 5x$.

(4)

(Q6, June 2005)

29. **Figure 1**

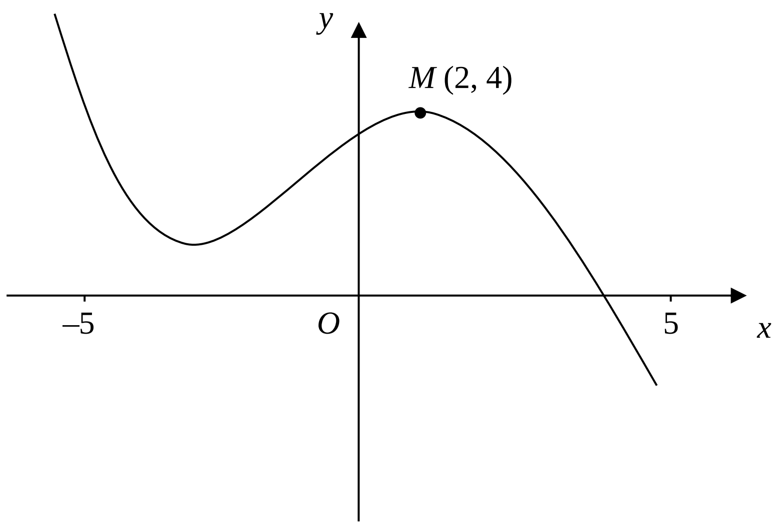


Figure 1 shows the graph of $y = \mathrm{f}(x)$, $-5 \leq x \leq 5$.

The point M (2, 4) is the maximum turning point of the graph.

Sketch, on separate diagrams, the graphs of

(*a*) $y = \mathrm{f}(x) + 3$,

(2)

(*b*) $y = |\mathrm{f}(x)|$,

(2)

(*c*) $y = \mathrm{f}(|x|)$.

(3)

Show on each graph the coordinates of any maximum turning points.

(Q1, Jan 2006)

30. Given that k is a positive constant,

(*a*) sketch the graph with equation

$$y = 2\,|x| - k$$

Show on your sketch the coordinates of each point at which the graph crosses the x-axis
and the y-axis.

(2)

(*b*) Find, in terms of k, the values of x for which

$$2\,|x| - k = \frac{1}{2}x + \frac{1}{4}k$$

(3)

(Q12, IAL C34 Jan 2019)

31. **Figure 1**

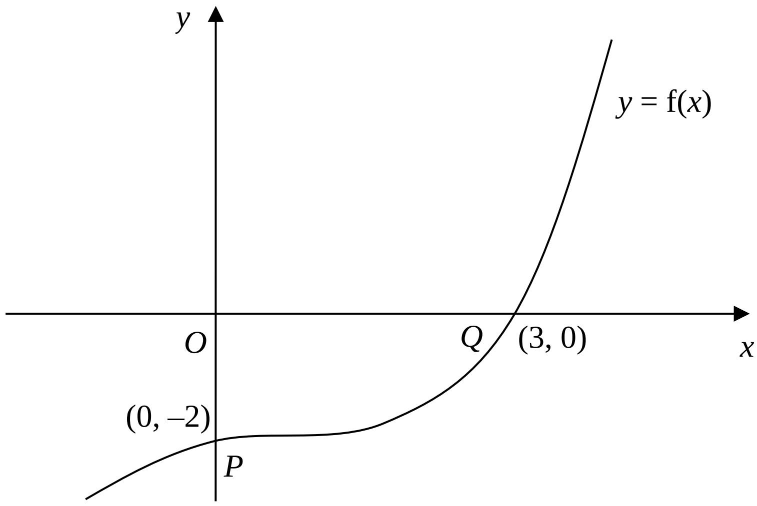


Figure 1 shows part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$, where f is an increasing function of x. The curve passes through the points $P(0, -2)$ and $Q(3, 0)$ as shown.

In separate diagrams, sketch the curve with equation

(*a*) $y = |\mathrm{f}(x)|$,

(3)

(*b*) $y = \mathrm{f}^{-1}(x)$,

(3)

(*c*) $y = \frac{1}{2}\mathrm{f}(3x)$.

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

(Q3, June 2006)

32. The function f is defined by

$$\mathrm{f} : x \mapsto 2x^2 + 3kx + k^2 \qquad x \in \mathbb{R}, -4k \leqslant x \leqslant 0$$

where k is a positive constant.

(*a*) Find, in terms of k, the range of f.

(4)

The function g is defined by

$$\mathrm{g} : x \mapsto 2k - 3x \qquad x \in \mathbb{R}$$

Given that $\mathrm{gf}(-2) = -12$

(*b*) find the possible values of k.

(4)

(Q3, IAL C34 Jan 2019)

33.

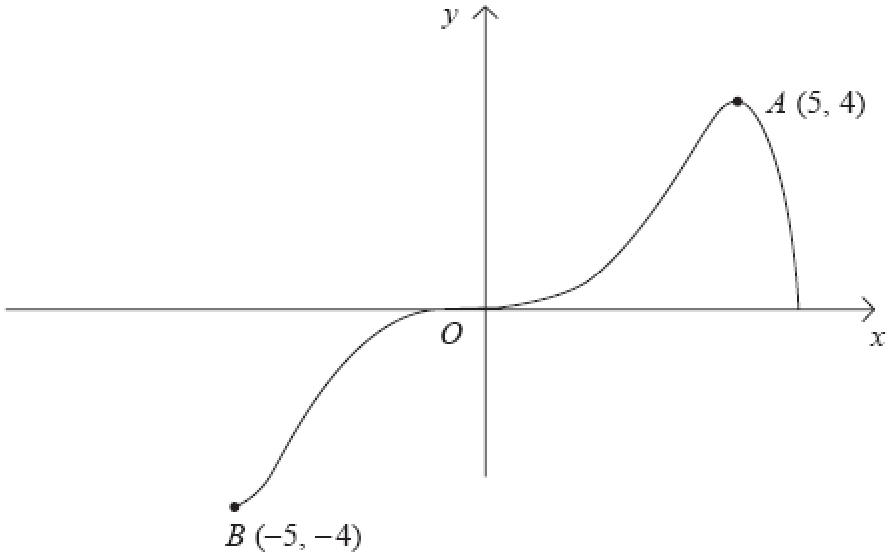


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$.

The curve passes through the origin O and the points $A(5, 4)$ and $B(-5, -4)$.

In separate diagrams, sketch the graph with equation

(*a*) $y = |\mathrm{f}(x)|$,

(3)

(*b*) $y = \mathrm{f}(|x|)$,

(3)

(*c*) $y = 2\mathrm{f}(x + 1)$.

(4)

On each sketch, show the coordinates of the points corresponding to A and B.

(Q4, Jan 2008)

34. Given that a is a positive constant,

(*a*) on separate diagrams, sketch the graph with equation

(i) $y = a - |x|$

(ii) $y = |3x - 2a|$

Show on each sketch the coordinates, in terms of a, of each point at which the graph crosses or meets the axes.

(4)

(*b*) Find, in terms of a, the values of x for which

$$a - |x| = |3x - 2a|$$

(4)

(Q9, IAL C34 Oct 2019)

35.

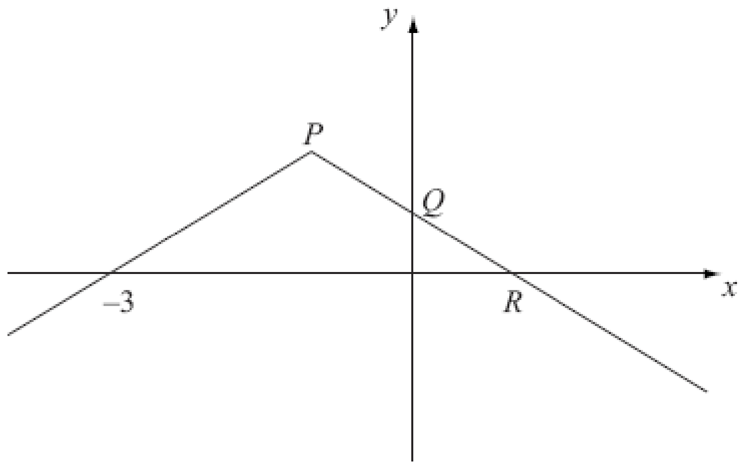


Figure 1

Figure 1 shows the graph of $y = f(x)$, $x \in \mathbb{R}$,
The graph consists of two line segments that meet at the point P.
The graph cuts the y-axis at the point Q and the x-axis at the points $(-3, 0)$ and R.
Sketch, on separate diagrams, the graphs of

(*a*) $y = |f(x)|$,

(2)

(*b*) $y = f(-x)$.

(2)

Given that $f(x) = 2 - |x + 1|$,

(*c*) find the coordinates of the points P, Q and R,

(3)

(*d*) solve $f(x) = \frac{1}{2}x$.

(5)

(Q3, Jan 2008)

36.

$$f(x) = \frac{5x+2}{x-3} \qquad x \in \mathbb{R},\ x \neq 3$$

$$g(x) = 2x^2 - 1 \qquad x \in \mathbb{R}$$

(*a*) Write down the range of g.

(1)

(*b*) Find fg(x), simplifying your answer.

(2)

(*c*) Find $f^{-1}(x)$.

(3)

(*d*) Find the exact values of x for which

$$f^{-1}(x) = f(x)$$

giving your answers as fully simplified surds.

(4)

(Q3, IAL C34 Oct 2019

37.

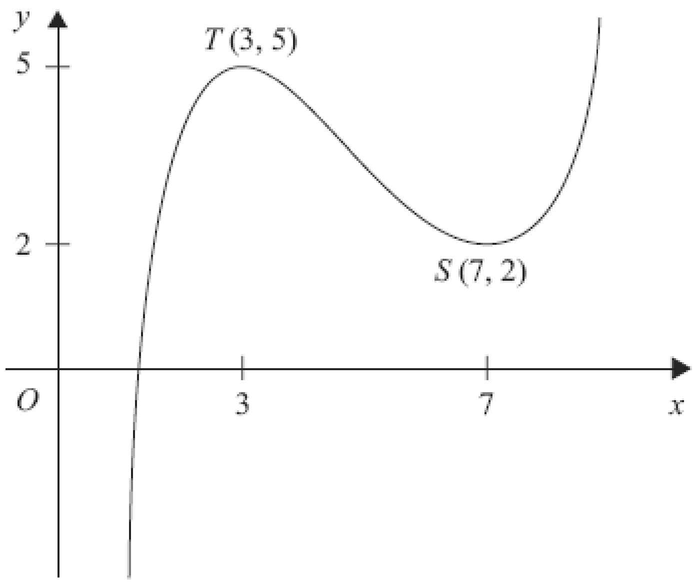


Figure 1

Figure 1 shows the graph of $y = \mathrm{f}(x)$, $1 < x < 9$.

The points $T(3, 5)$ and $S(7, 2)$ are turning points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 2\mathrm{f}(x) - 4$,

(3)

(b) $y = |\mathrm{f}(x)|$.

(3)

Indicate on each diagram the coordinates of any turning points on your sketch.

(Q3, Jan 2009)

38. It is given that

$$\mathrm{f}(x) = e^{-2x} \qquad x \in \mathbb{R}$$

$$\mathrm{g}(x) = \frac{x}{x-3} \qquad x > 3$$

(a) Sketch the graph of $y = \mathrm{f}(x)$, showing the coordinates of any points where the graph crosses the axes.

(2)

(b) Find the range of g

(2)

(c) Find $\mathrm{g}^{-1}(x)$, stating the domain of g^{-1}

(4)

(d) Using algebra, find the exact value of x for which $\mathrm{fg}(x) = 3$

(4)

(Q10, IAL C34 Jan 2018)

39.

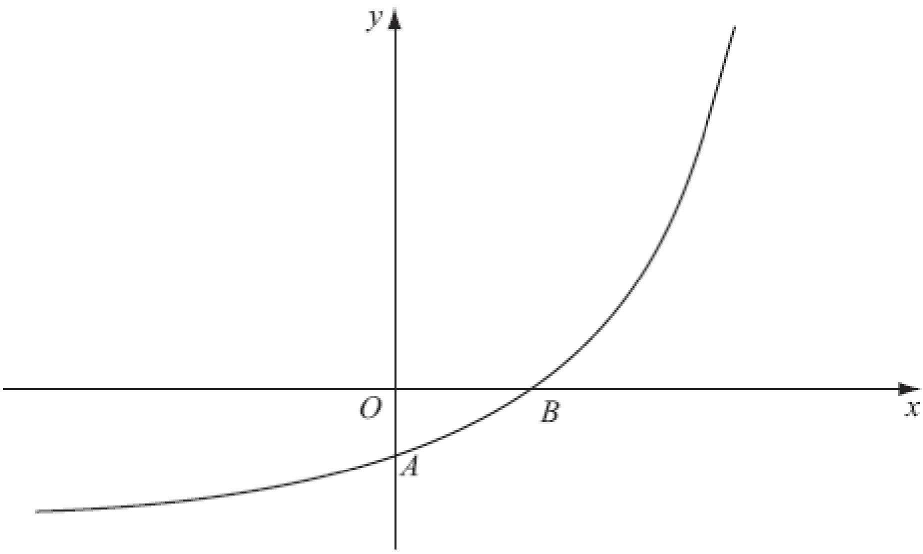


Figure 2 shows a sketch of part of the curve with equation $y = \text{f}(x)$, $x \in \mathbb{R}$.

The curve meets the coordinate axes at the points $A(0, 1 - k)$ and $B(\frac{1}{2} \ln k, 0)$, where k is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(*a*) $y = |\text{f}(x)|$,

(3)

(*b*) $y = \text{f}^{-1}(x)$.

(2)

Show on each sketch the coordinates, in terms of k, of each point at which the curve meets or cuts the axes.

Given that $\text{f}(x) = \text{e}^{2x} - k$,

(*c*) state the range of f,

(1)

(*d*) find $\text{f}^{-1}(x)$,

(3)

(*e*) write down the domain of f^{-1}.

(1)

(Q5, June 2009)

40.

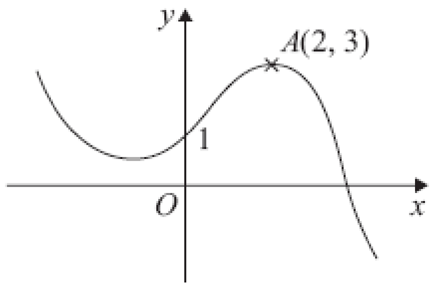


Figure 1

Figure 1 shows a sketch of the graph of $y = \mathrm{f}(x)$.

The graph intersects the y-axis at the point (0, 1) and the point A(2, 3) is the maximum turning point.

Sketch, on separate axes, the graphs of

(i) $y = \mathrm{f}(-x) + 1$,

(ii) $y = \mathrm{f}(x + 2) + 3$,

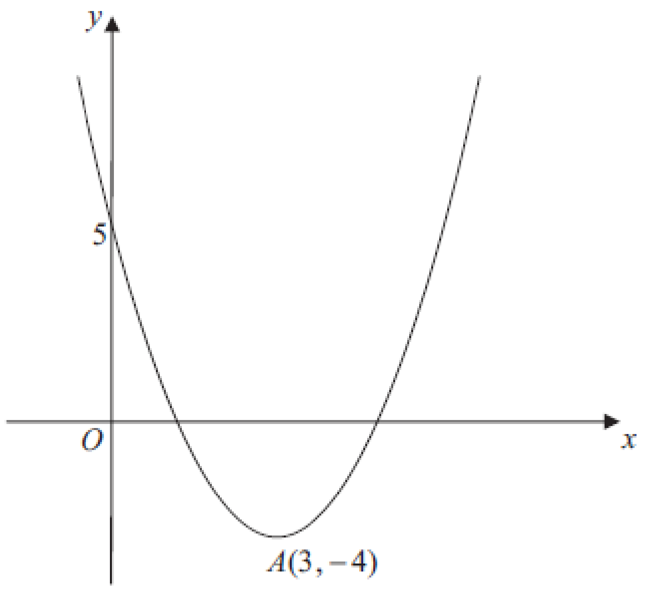
(iii) $y = 2\mathrm{f}(2x)$.

On each sketch, show the coordinates of the point at which your graph intersects the y-axis and the coordinates of the point to which A is transformed.

(9)

(Q6, Jan 2010)

41.



The curve has a turning point at $A(3, -4)$ and also passes through the point $(0, 5)$.

(a) Write down the coordinates of the point to which A is transformed on the curve with equation

(i) $y = |f(x)|$,

(ii) $y = 2f(\frac{1}{2}x)$.

(4)

(b) Sketch the curve with equation $y = f(|x|)$.

On your sketch show the coordinates of all turning points and the coordinates of the point at which the curve cuts the y-axis.

(3)

The curve with equation $y = f(x)$ is a translation of the curve with equation $y = x^2$.

(c) Find $f(x)$.

(2)

(d) Explain why the function f does not have an inverse.

(1)

(Q6, June 2010)

42.

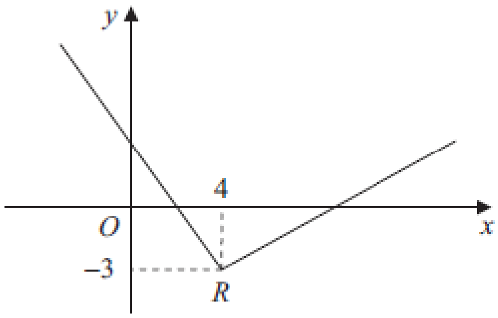


Figure 1

Figure 1 shows part of the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point R (4, – 3), as shown in Figure 1.

Sketch, on separate diagrams, the graphs of

(a) $y = 2\mathrm{f}(x + 4)$,

(3)

(b) $y = |\mathrm{f}(-x)|$.

(3)

On each diagram, show the coordinates of the point corresponding to R.

(Q3, June 2011)

43. Given that a is a positive constant and

$$\mathrm{f}(x) = |3x - a|, \quad x \in \mathbb{R}$$

(a) sketch the graph with equation $y = \mathrm{f}(x)$, showing the coordinates of the points where the graph cuts or meets the coordinate axes.

(2)

Given that $x = 4$ is a solution to the equation $|3x - a| = \frac{1}{2}x + 2$

(b) find the two possible values of a.

(3)

For one of the values of a, $x = 4$ is the smaller of the two solutions. For this value of a,

(c) find the value of the larger solution.

(2)

(Q5, C3 June 2018)

44.

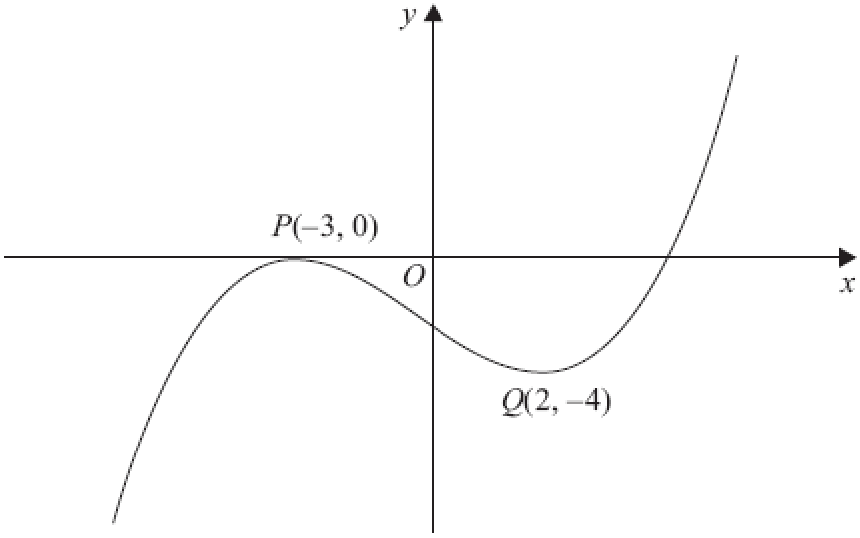


Figure 1

Figure 1 shows the graph of equation $y = \mathrm{f}(x)$.

The points $P(-3, 0)$ and $Q(2, -4)$ are stationary points on the graph.

Sketch, on separate diagrams, the graphs of

(*a*) $y = 3\mathrm{f}(x + 2)$,

(3)

(*b*) $y = |\mathrm{f}(x)|$.

(3)

On each diagram, show the coordinates of any stationary points.

(Q2, Jan 2012)

45. (*a*) Sketch the graph with equation

$$y = |4x - 3|$$

stating the coordinates of any points where the graph cuts or meets the axes.

(2)

Find the complete set of values of x for which

(*b*)

$$|4x - 3| > 2 - 2x$$

(4)

(*c*)

$$|4x - 3| > \frac{3}{2} - 2x$$

(2)

(Q5, June 2014_R)

46.

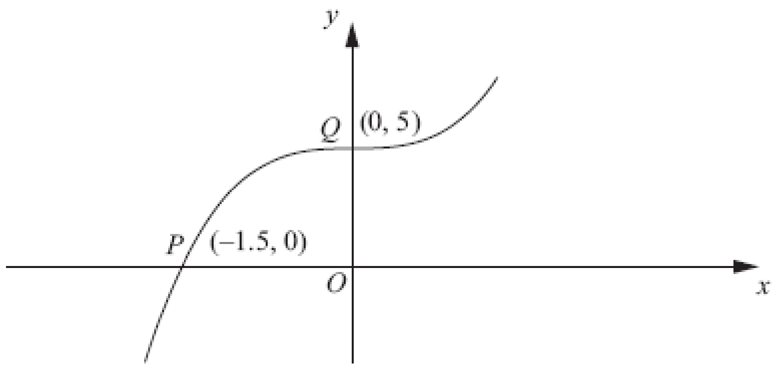


Figure 2

Figure 2 shows part of the curve with equation $y = \mathrm{f}(x)$.

The curve passes through the points $P(-1.5, 0)$and $Q(0, 5)$ as shown.

On separate diagrams, sketch the curve with equation

(*a*) $y = |\mathrm{f}(x)|$

(2)

(*b*) $y = \mathrm{f}(|x|)$

(2)

(*c*) $y = 2\mathrm{f}(3x)$

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

(Q4, June 2012)

47. Given that a and b are positive constants,

(*a*) on separate diagrams, sketch the graph with equation

(i) $y = |2x - a|$

(ii) $y = |2x - a| + b$

Show, on each sketch, the coordinates of each point at which the graph crosses or meets the axes.

(4)

Given that the equation

$$|2x - a| + b = \frac{3}{2}x + 8$$

has a solution at $x = 0$ and a solution at $x = c$,

(*b*) find c in terms of a.

(4)

(Q6, June 201

48.

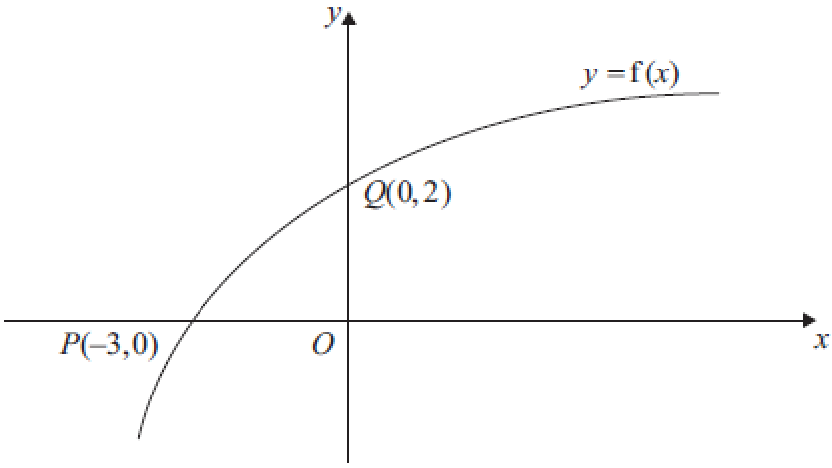


Figure 1

Figure 1 shows part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve passes through the points $Q(0, 2)$ and $P(-3, 0)$ as shown.

(*a*) Find the value of ff (-3).

(2)

On separate diagrams, sketch the curve with equation

(*b*) $y = \mathrm{f}^{-1}(x)$,

(2)

(*c*) $y = \mathrm{f}(|x|) - 2$,

(2)

(*d*) $y = 2\mathrm{f}\left(\frac{1}{2}x\right)$.

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

(Q3, Jan 2013)

49. Given that

$$\mathrm{f}(x) = \ln x, \qquad x > 0$$

sketch on separate axes the graphs of

(i) $y = \mathrm{f}(x)$,

(ii) $y = |\mathrm{f}(x)|$,

(iii) $y = -\mathrm{f}(x - 4)$.

Show, on each diagram, the point where the graph meets or crosses the x-axis.

In each case, state the equation of the asymptote.

(7)

(Q2, June 2013)

50.

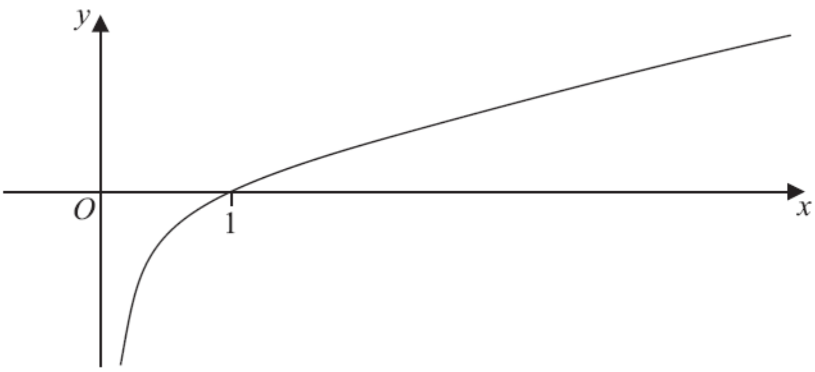


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, $x > 0$, where f is an increasing function of x. The curve crosses the x-axis at the point (1, 0) and the line $x = 0$ is an asymptote to the curve.

On separate diagrams, sketch the curve with equation

(*a*) $y = \mathrm{f}(2x)$, $x > 0$

(2)

(*b*) $y = |\mathrm{f}(x)|$, $x > 0$

(3)

Indicate clearly on each sketch the coordinates of the point at which the curve crosses or meets the x-axis.

(Q2, June 2013_R)

51.

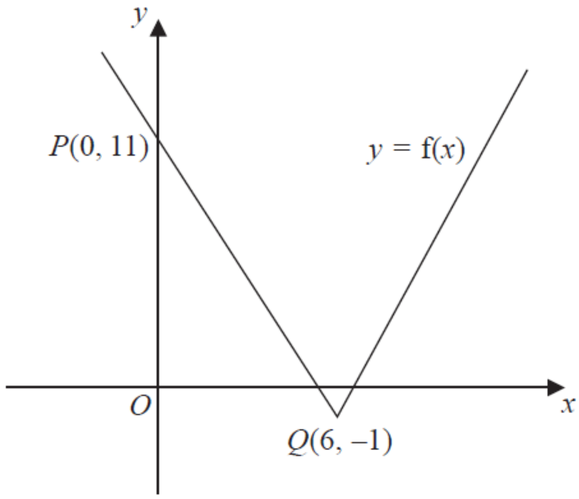


Figure 1

Figure 1 shows part of the graph with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $Q(6, -1)$.

The graph crosses the y-axis at the point $P(0, 11)$.

Sketch, on separate diagrams, the graphs of

(a) $y = |\mathrm{f}(x)|$

(2)

(b) $y = 2\mathrm{f}(-x) + 3$

(3)

On each diagram, show the coordinates of the points corresponding to P and Q.

Given that $\mathrm{f}(x) = a|x - b| - 1$, where a and b are constants,

(c) state the value of a and the value of b. **(2)**

(Q4, June 2014)

52. Given that

$$\mathrm{f}(x) = 2\mathrm{e}^x - 5, \quad x \in \mathbb{R},$$

(a) sketch, on separate diagrams, the curve with equation

(i) $y = \mathrm{f}(x)$,

(ii) $y = |\mathrm{f}(x)|$.

On each diagram, show the coordinates of each point at which the curve meets or cuts the axes.

On each diagram state the equation of the asymptote.

(6)

(b) Deduce the set of values of x for which $\mathrm{f}(x) = |\mathrm{f}(x)|$.

(1)

(c) Find the exact solutions of the equation $|\mathrm{f}(x)| = 2$.

(3)

(Q2, June 2015)

53. Given that a and b are constants and that $a > b > 0$

(a) on separate diagrams, sketch the graph with equation

(i) $y = |x - a|$

(ii) $y = |x - a| - b$

Show on each sketch the coordinates of each point at which the graph crosses or meets

the x-axis and the y-axis.

(5)

(b) Hence or otherwise find the complete set of values of x for which

$$|x - a| - b < \frac{1}{2}x$$

giving your answer in terms of a and b.

(4)

(Q6, IAL. June 2016

54.

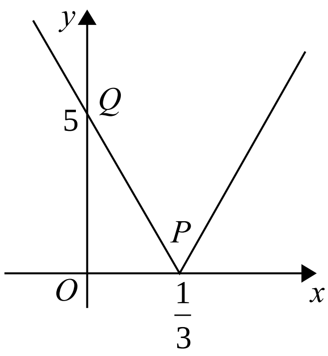


Figure 2

Figure 2 shows a sketch of the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The point $P\left(\frac{1}{3}, 0\right)$ is the vertex of the graph.

The point Q (0, 5) is the intercept with the y-axis.

Given that $\mathrm{f}(x) = |ax + b|$, where a and b are constants,

(a) (i) find all possible values for a and b,

(ii) hence find an equation for the graph.

(4)

(b) Sketch the graph with equation

$$y = \mathrm{f}\left(\frac{1}{2}x\right) + 3$$

showing the coordinates of its vertex and its intercept with the y-axis.

(3)

(Q7, IAL. Jan 2017)

55.

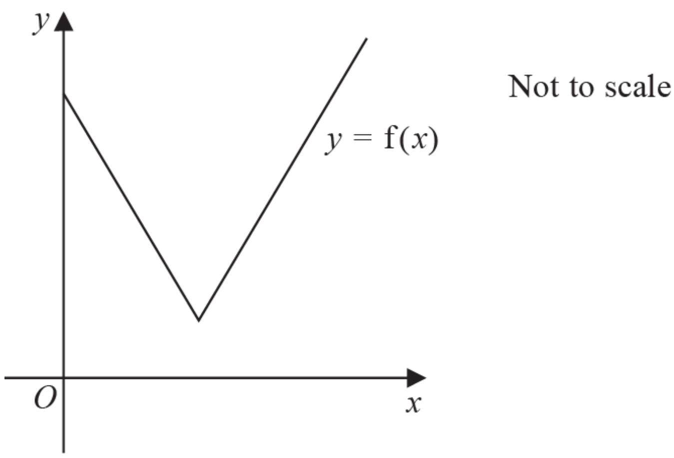


Figure 2

Figure 2 shows part of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|5 - x| + 3, \qquad x \geqslant 0$$

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has exactly one root,

(*a*) state the set of possible values of k.

(2)

(*b*) Solve the equation $\mathrm{f}(x) = \frac{1}{2}x + 10$

(4)

The graph with equation $y = \mathrm{f}(x)$ is transformed onto the graph with equation $y = 4\mathrm{f}(x - 1)$.
The vertex on the graph with equation $y = 4\mathrm{f}(x - 1)$ has coordinates (p, q).

(*c*) State the value of p and the value of q.

(2)

(Q5, C3 June 2018)

56. $$\mathrm{f}(x) = 2\ln(x) - 4, \qquad x > 0, \qquad x \in \mathbb{R}$$

(*a*) Sketch, on separate diagrams, the curve with equation

(i) $y = \mathrm{f}(x)$

(ii) $y = |\mathrm{f}(x)|$

On each diagram, show the coordinates of each point at which the curve meets or cuts the axes.

On each diagram state the equation of the asymptote.

(5)

(*b*) Find the exact solutions of the equation $|\mathrm{f}(x)| = 4$

(4)

$$\mathrm{g}(x) = \mathrm{e}^{x+5} - 2, \qquad x \in \mathbb{R}$$

(*c*) Find $\mathrm{gf}(x)$, giving your answer in its simplest form.

(3)

(*d*) Hence, or otherwise, state the range of gf.

(1)

(Q9, IAL C34 Oct 2017)

57.

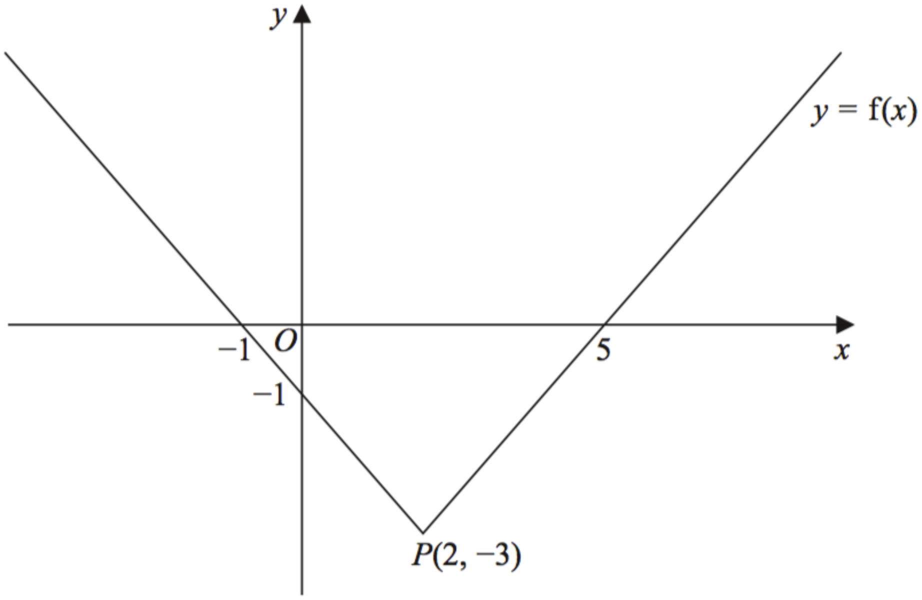


Figure 1

Figure 1 shows a sketch of part of the graph with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$

The graph consists of two half lines that meet at the point P (2, −3), the vertex of the graph.

The graph cuts the y-axis at the point (0, −1) and the x-axis at the points (−1, 0) and (5, 0).

Sketch, on separate diagrams, the graph of

(a) $y = \mathrm{f}(|x|)$,

(3)

(b) $y = 2\mathrm{f}(x + 5)$.

(3)

In each case, give the coordinates of the points where the graph crosses or meets the coordinate axes.

Also give the coordinates of any vertices corresponding to the point P.

(Total 6 marks)

(Q4, IAL C34 Jan 2018)

58. (i) The functions f and g are defined by

$$\mathrm{f} : x \to \mathrm{e}^{2x} - 5, \qquad x \in \mathbb{R}$$

$$\mathrm{g} : x \to \ln(3x - 1), \qquad x \in \mathbb{R}, \quad x > \frac{1}{3}$$

(a) Find f^{-1} and state its domain.

(3)

(b) Find fg(3), giving your answer in its simplest form.

(2)

(ii) (*a*) Sketch the graph with equation

$$y = | 4x - a |$$

where a is a positive constant. State the coordinates of each point where the graph cuts or meets the coordinate axes.

(2)

Given that

$$| 4x - a | = 9a$$

where a is a positive constant,

(*b*) find the possible values of

$$| x - 6a | = 3 |x|$$

giving your answers, in terms of a, in their simplest form.

(5)

(Q5, IAL C34 June 2018)

59.

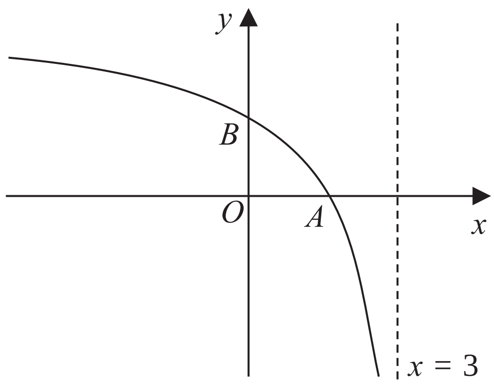


Figure 2

Figure 2 shows a sketch of part of the graph with equation $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \frac{3x - 4}{x - 3}, \; x \in \mathbb{R}, \; x < 3$$

The graph cuts the x-axis at the point A and the y-axis at the point B, as shown in Figure 2.

(*a*) State the range of g.

(1)

(*b*) State the coordinates of

(i) point A

(ii)point B

(2)

(*c*) Find gg(x) in its simplest form.

(3)

(*d*) Sketch the graph with equation $y = | \mathrm{g}(x) |$

On your sketch, show the coordinates of each point at which the graph meets or cuts

the axes and state the equation of each asymptote. **(3)**

(*e*) Find the exact solution of the equation $| \mathrm{g}(x) | = 8$ **(3)**

(Q10, IAL C34 Oct 2018)

60.

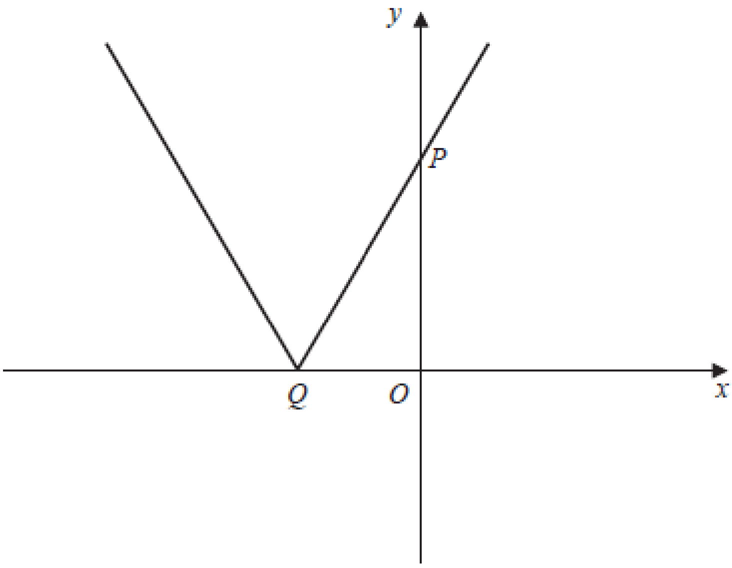


Figure 1

Figure 1 shows a sketch of the graph with equation $y = | 4x + 10a |$, where a is a positive constant.

The graph cuts the y-axis at the point P and meets the x-axis at the point Q as shown.

(a) (i) State the coordinates of P.

(ii) State the coordinates of Q.

(2)

(b) A copy of Figure 1 is shown on page 15. On this copy, sketch the graph with equation

$$y = | x | - a$$

Show on the sketch the coordinates of each point where your graph cuts or meets the coordinate axes.

(2)

(c) Hence, or otherwise, solve the equation

$$| 4x + 10a | = | x | - a$$

giving your answers in terms of a.

(3)

(Q6, IAL C34 June 2019)