OCR Core Maths 2

Past paper questions Factors & Remainders

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Factors and Remainders

- Need to know how to divide any polynomial by a linear factor of the form $ax - b$. For example divide $x^3 + 2x^2 + 3x - 6$ by $x - 2$. (*Always* devote a column to each power of x.)

$$
\begin{array}{r|rrrr}
 & & x^2 & +4x & +11 \\ \hline
x-2 & +x^3 & +2x^2 & +3x & -6 \\
 & +x^3 & -2x^2 & & \\ \cline{2-3}
 & & +4x^2 & +3x & \\
 & & +4x^2 & -8x & \\ \cline{3-4}
 & & & +11x & -6 \\
 & & & +11x & -22 \\ \cline{4-5}
 & & & & +16
\end{array}
$$

  So the remainder is $+16$. Therefore $x^3 + 2x^2 + 3x - 6 = (x - 2)(x^2 + 4x + 11) + 16$.

- If the remainder is zero, then the divisor is said to be a *factor* of the original polynomial.

- The Factor Theorem states:

$$(x - a) \text{ is a factor of } f(x) \qquad \Leftrightarrow \qquad f(a) = 0.$$

  [More generally (but used less often in exams) is:

$$(ax - b) \text{ is a factor of } f(x) \qquad \Leftrightarrow \qquad f(\tfrac{b}{a}) = 0.]$$

- For example if $x^3 + ax^2 + 8x - 4$ has $(x - 2)$ as a factor, find a. From factor theorem we know $f(2) = 0$, so we discover $2^3 + a \times 2^2 + 8 \times 2 - 4 = 0$, and therefore $a = -5$.

- The Remainder Theorem states:

$$\text{When } f(x) \text{ is divided by } (x - a) \text{ the remainder is } f(a).$$

  [More generally (but used less often in exams) is:

$$\text{When } f(x) \text{ is divided by } (ax - b) \text{ the remainder is } f(\tfrac{b}{a}).]$$

  Notice that the factor theorem is a subset of the remainder theorem. In the factor theorem all remainders are zero, by definition.

- For example if told that when $f(x) = x^3 + 2x^2 - 3x - 7$ is divided by $x - 2$ the remainder is 3, we know $f(2) = 3$.

- Worked example: $f(x) = 2x^3 + 3x^2 + kx - 2$. The remainder when $f(x)$ is divided by $(x - 2)$ is four times the remainder when $f(x)$ is divided by $(x + 1)$. Find k. We know

$$
\begin{aligned}
f(2) &= 4 \times f(-1) \\
2 \times 2^3 + 3 \times 2^2 + 2k - 2 &= 4[2 \times (-1)^3 + 3 \times (-1)^2 - k - 2] \\
k &= -5.
\end{aligned}
$$

1.

The cubic polynomial $f(x)$ is given by

$$f(x) = x^3 + ax + b,$$

where a and b are constants. It is given that $(x + 1)$ is a factor of $f(x)$ and that the remainder when $f(x)$ is divided by $(x - 3)$ is 16.

(i) Find the values of a and b. [5]

(ii) Hence verify that $f(2) = 0$, and factorise $f(x)$ completely. [3]

Q5 June 2005

2.

The cubic polynomial $2x^3 + ax^2 + bx - 10$ is denoted by $f(x)$. It is given that, when $f(x)$ is divided by $(x - 2)$, the remainder is 12. It is also given that $(x + 1)$ is a factor of $f(x)$.

(i) Find the values of a and b. [6]

(ii) Divide $f(x)$ by $(x + 2)$ to find the quotient and the remainder. [5]

Q8 June 2006

3.

The polynomial $f(x)$ is defined by $f(x) = x^3 - 9x^2 + 7x + 33$.

(i) Find the remainder when $f(x)$ is divided by $(x + 2)$. [2]

(ii) Show that $(x - 3)$ is a factor of $f(x)$. [1]

(iii) Solve the equation $f(x) = 0$, giving each root in an exact form as simply as possible. [6]

Q8 Jan 2007

4.

The cubic polynomial $ax^3 - 4x^2 - 7ax + 12$ is denoted by $f(x)$.

(i) Given that $(x - 3)$ is a factor of $f(x)$, find the value of the constant a. [3]

(ii) Using this value of a, find the remainder when $f(x)$ is divided by $(x + 2)$. [2]

Q4 Jan 2008

5.

The polynomial $f(x)$ is given by $f(x) = 2x^3 + 9x^2 + 11x - 8$.

(i) Find the remainder when $f(x)$ is divided by $(x + 2)$. **[2]**

(ii) Use the factor theorem to show that $(2x - 1)$ is a factor of $f(x)$. **[2]**

(iii) Express $f(x)$ as a product of a linear factor and a quadratic factor. **[3]**

(iv) State the number of real roots of the equation $f(x) = 0$, giving a reason for your answer. **[2]**

Q7 June 2009

6.

The cubic polynomial $f(x)$ is given by

$$f(x) = 2x^3 + ax^2 + bx + 15,$$

where a and b are constants. It is given that $(x + 3)$ is a factor of $f(x)$ and that, when $f(x)$ is divided by $(x - 2)$, the remainder is 35.

(i) Find the values of a and b. **[6]**

(ii) Using these values of a and b, divide $f(x)$ by $(x + 3)$. **[3]**

Q6 Jan 2010

7.

The cubic polynomial $f(x)$ is defined by $f(x) = x^3 + ax^2 - ax - 14$, where a is a constant.

(i) Given that $(x - 2)$ is a factor of $f(x)$, find the value of a. **[3]**

(ii) Using this value of a, find the remainder when $f(x)$ is divided by $(x + 1)$. **[2]**

Q1 June 2010

8.

The cubic polynomial $f(x)$ is defined by $f(x) = x^3 + x^2 - 11x + 10$.

(i) Use the factor theorem to find a factor of $f(x)$. **[2]**

(ii) Hence solve the equation $f(x) = 0$, giving each root in an exact form. **[6]**

Q6 June 2011

9.

The cubic polynomial f(x) is defined by $f(x) = 2x^3 + 3x^2 - 17x + 6$.

(i) Find the remainder when f(x) is divided by $(x - 3)$. **[2]**

(ii) Given that $f(2) = 0$, express f(x) as the product of a linear factor and a quadratic factor. **[4]**

(iii) Determine the number of real roots of the equation $f(x) = 0$, giving a reason for your answer. **[2]**

Q5 Jan 2012

10.

Two cubic polynomials are defined by

$$f(x) = x^3 + (a - 3)x + 2b, \quad g(x) = 3x^3 + x^2 + 5ax + 4b,$$

where a and b are constants.

(i) Given that f(x) and g(x) have a common factor of $(x - 2)$, show that $a = -4$ and find the value of b. **[6]**

(ii) Using these values of a and b, factorise f(x) fully. Hence show that f(x) and g(x) have two common factors. **[5]**

Q8 June 2012

11.

The cubic polynomial f(x) is defined by $f(x) = 12 - 22x + 9x^2 - x^3$.

(i) Find the remainder when f(x) is divided by $(x + 2)$. **[2]**

(ii) Show that $(3 - x)$ is a factor of f(x). **[1]**

(iii) Express f(x) as the product of a linear factor and a quadratic factor. **[3]**

(iv) Hence solve the equation $f(x) = 0$, giving each root in simplified surd form where appropriate. **[3]**

Q7 June 2014