OCR Core Maths 2

Past paper questions

Equations & Inequalities

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Inequalities

- Treat linear inequalities like equations except when multiplying or dividing by a negative number when you reverse the sign. For example

$$\begin{aligned} 2x + 4 &< 3x + 2 \\ -x &< -2 \\ x &> 2. \end{aligned}$$

- To solve quadratic inequalities:

  1. Get all terms over one side so that quadratic > 0 or quadratic < 0 in such a way that the x^2 term is always positive. This will ensure a 'happy' curve.
  2. Solve quadratic $= 0$ to find where it crosses x-axis.
  3. Sketch the graph and read off solution. If it is quadratic > 0 then it is the region(s) above the x-axis, and if quadratic < 0 then it is region below the x-axis.
  4. If one region then express as one triple inequality (e.g. $-2 < x < 5$) and if two regions then two *separate* inequalities (e.g. $x > 5$ or $x < -2$).

- For example solve the inequality $-7x \geqslant 4 - 2x^2$. Firstly get the $2x^2$ on the other side to make it positive to get $2x^2 - 7x - 4 \geqslant 0$. Then solve the equality $2x^2 - 7x - 4 = 0 = (2x+1)(x-4)$, so $x = -\frac{1}{2}$ or $x = 4$. So we have a happy quadratic that crosses the x-axis at $-\frac{1}{2}$ and 4. The inequality is asking for where the curve is bigger than (or equal to) zero, and this is to the right of $x = 4$ and the left of $x = -\frac{1}{2}$. Therefore the solution is $x \leqslant -\frac{1}{2}$ or $x \geqslant 4$.

- Don't fall into the trap of seeing $x^2 < 16$ and saying $x < \pm 4$! Be disciplined and get zero on one side; $x^2 - 16 < 0$ so $(x-4)(x+4) < 0$ so we have happy curve that crosses at 4 and -4. Where is the curve less than zero? Between -4 and 4 so solution is $-4 < x < 4$.

1.

Solve the inequality $x^2 - 6x - 40 \geqslant 0$. [4]

Q1 June 2005

2.

(i) Given that $y = x^2 - 5x + 15$ and $5x - y = 10$, show that $x^2 - 10x + 25 = 0$. [2]

(ii) Find the discriminant of $x^2 - 10x + 25$. [1]

(iii) What can you deduce from the answer to part **(ii)** about the line $5x - y = 10$ and the curve $y = x^2 - 5x + 15$? [1]

(iv) Solve the simultaneous equations

$$y = x^2 - 5x + 15 \quad \text{and} \quad 5x - y = 10.$$ [3]

(v) Hence, or otherwise, find the equation of the normal to the curve $y = x^2 - 5x + 15$ at the point (5, 15), giving your answer in the form $ax + by = c$, where a, b and c are integers. [4]

Q8 Jan 2006

3.

Solve the inequalities

(i) $1 < 4x - 9 < 5$, [3]

(ii) $y^2 \geqslant 4y + 5$. [5]

Q5 June 2006

4.

(i) Solve the simultaneous equations

$$y = x^2 - 5x + 4, \qquad y = x - 1.$$ [4]

(ii) State the number of points of intersection of the curve $y = x^2 - 5x + 4$ and the line $y = x - 1$. [1]

(iii) Find the value of c for which the line $y = x + c$ is a tangent to the curve $y = x^2 - 5x + 4$. [4]

Q7 June 2006

5.

Solve the inequalities

(i) $3(x - 5) \leqslant 24$, [2]

(ii) $5x^2 - 2 > 78$. [3]

Q3 Jan 2006

6.

(i) Find the gradient of the line l which has equation $x + 2y = 4$. [1]

(ii) Find the equation of the line parallel to l which passes through the point (6, 5), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [3]

(iii) Solve the simultaneous equations

$$y = x^2 + x + 1 \qquad \text{and} \qquad x + 2y = 4. \qquad [4]$$

Q7 Jan 2008

7.

Solve the inequalities

(i) $8 < 3x - 2 < 11$, [3]

(ii) $y^2 + 2y \geqslant 0$. [4]

Q7 June 2008

8.

Solve the simultaneous equations

$$4x^2 + y^2 = 10, \qquad 2x - y = 4. \qquad \textbf{[6]}$$

Q4 June 2009

9.

Solve the inequalities

(i) $-35 < 6x + 7 < 1$, **[3]**

(ii) $3x^2 > 48$. **[3]**

Q8 June 2009

10.

Solve the simultaneous equations

$$x + 2y - 6 = 0, \qquad 2x^2 + y^2 = 57. \qquad \textbf{[6]}$$

Q7 June 2010

11.

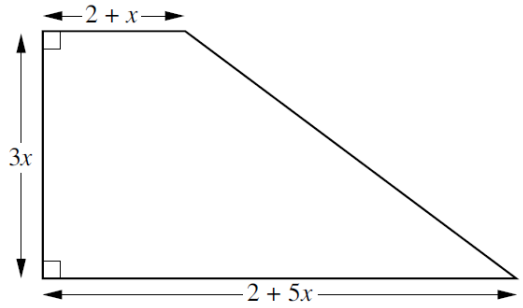
Solve the simultaneous equations

$$y = 2(x - 2)^2, \qquad 3x + y = 26. \qquad \textbf{[5]}$$

Q4 June 2011

12.

A lawn is to be made in the shape shown below. The units are metres.



(i) The perimeter of the lawn is P m. Find P in terms of x. **[2]**

(ii) Show that the area, A m^2, of the lawn is given by $A = 9x^2 + 6x$. **[2]**

The perimeter of the lawn must be at least 39 m and the area of the lawn must be less than 99 m^2.

(iii) By writing down and solving appropriate inequalities, determine the set of possible values of x. **[7]**

Q11 Jan 2010

13.

Solve the inequalities

(i) $-9 \leqslant 6x + 5 \leqslant 0$, **[3]**

(ii) $6x + 5 < x^2 + 2x - 7$. **[5]**

Q7 June 2011

14.

(i) Sketch the curve $y = 12 - x - x^2$, giving the coordinates of all intercepts with the axes. **[5]**

(ii) Solve the inequality $12 - x - x^2 > 0$. **[2]**

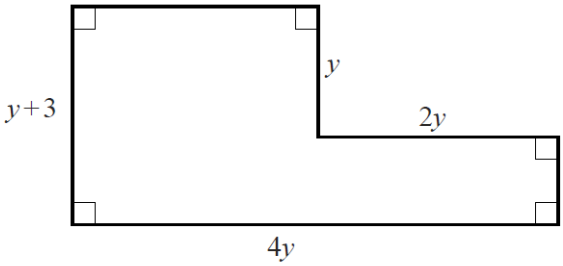
(iii) Find the coordinates of the points of intersection of the curve $y = 12 - x - x^2$ and the line $3x + y = 4$. **[5]**

Q9 Jan 2012

15.

(i) A rectangular tile has length $4x$ cm and width $(x + 3)$ cm. The area of the rectangle is less than $112\,\text{cm}^2$. By writing down and solving an inequality, determine the set of possible values of x. **[6]**

(ii) A second rectangular tile of length $4y$ cm and width $(y + 3)$ cm has a rectangle of length $2y$ cm and width y cm removed from one corner as shown in the diagram.



Given that the perimeter of this tile is between 20 cm and 54 cm, determine the set of possible values of y. **[5]**

Q9 June 2012

16.

(i) Solve the simultaneous equations

$$y = 2x^2 - 3x - 5, \qquad 10x + 2y + 11 = 0.$$ **[5]**

(ii) What can you deduce from the answer to part **(i)** about the curve $y = 2x^2 - 3x - 5$ and the line $10x + 2y + 11 = 0$? **[1]**

Q4 Jan 2013

17.

Solve the inequalities

(i) $3 - 8x > 4$, **[2]**

(ii) $(2x - 4)(x - 3) \leqslant 12$. **[5]**

Q7 June 2013

18.

Solve the following inequalities.

(i) $5 < 6x + 3 < 14$ **[3]**

(ii) $x(3x - 13) \geqslant 10$ **[5]**

Q5 June 2014

19.

Solve the simultaneous equations

$$2x+y-5=0, \qquad x^2-y^2=3. \qquad \textbf{[5]}$$

Q6 June 2015