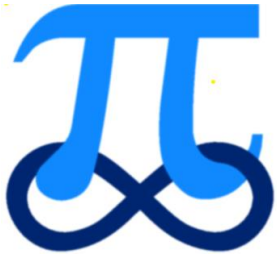
Edexcel
New GCE A Level Maths workbook
Trigonometry 1



Edited by: K V Kumaran

Trigonometry

The sine and cosine rules, and the area of a triangle in the form $\frac{1}{2}ab\sin C$.

Pythagoras Theorem

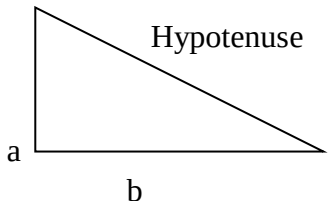
Pythagoras' Theorem is the relationship between the sides of a right angled triangle. It shows that the squares of the two shortest sides are equal to the square of the longest side.

The longest side is called the hypotenuse

The formula is:

$$a^2 + b^2 = c^2$$

where c is always the hypotenuse



Example 1. In a right angled triangle, AB = 23·5*cm* , BC = 19·3*cm* and $\angle ABC = 90°$. Find the length of AC.

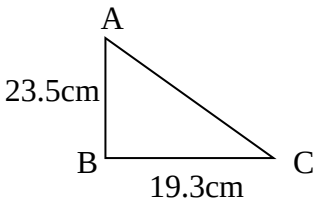
$a^2 + b^2 = c^2$ where $a = 23{\cdot}5$, $b = 19{\cdot}3$

$23{\cdot}5^2 + 19{\cdot}3^2 = x^2$

$552{\cdot}25 + 372{\cdot}49 = x^2$

$924{\cdot}75 = x$

$x = 30{\cdot}41cm$ (2*dp*)



You need to remember that if you don't know the longest side you add

$$a^2 + b^2 = c^2$$

If you know the longest side you subtract

$$a^2 = c^2 - b^2$$

Trigonometry

The ratio of any two sides of a right angled triangle will always remain the same if the angles stay the same. From this we can find three ratios:-

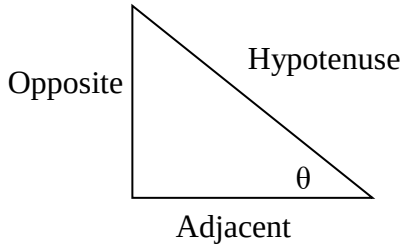
$$\sin\theta = \frac{opp}{hyp} \qquad os\,\theta = \frac{ady}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

It is important the sides are always named relative to the angle given (this does not include the right angle)

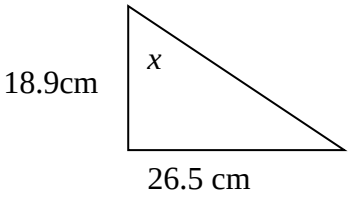
Hypotenuse is always across from the right angle, it never touches it

Opposite faces the given angle but never touches it.

Adjacent is next to the angle and it touches both the right angle and the given angle.



Example 1. Find the size of angle x



$$\tan\theta = \frac{opp}{adj}$$

$$\tan x = \frac{26{\cdot}5}{18{\cdot}9}$$

$$\tan x = 1{\cdot}402$$

$$x = 54{\cdot}5cm\ (1dp)$$

If x is the angle then work out the fraction and use shift tan

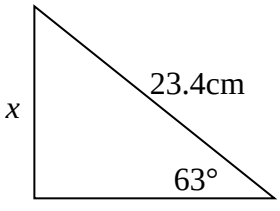
Example 2. Find the size of length x

$$\sin\theta = \frac{opp}{hyp}$$

$$\sin 63° = \frac{x}{23{\cdot}4}$$

$$\sin 63° \times 23{\cdot}4 = x$$

$$x = 20{\cdot}85cm$$



If x is at the top of the fractions then you need to multiply

"top means times"

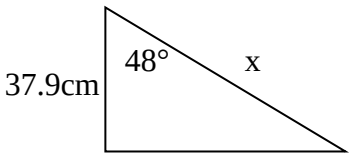
Example 3. Find the size of length x

$$\cos\theta = \frac{adj}{hyp}$$

$$\cos 48° = \frac{37{\cdot}9}{x}$$

$$x = \frac{37{\cdot}9}{\cos 48}$$

$$x = 56{\cdot}64cm\ (2dp)$$



If x is at the bottom then you need to divide the number by sin/cos or tan

"bottom means divide"

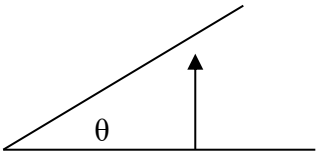
Remember:

If x is at the top then times
If x is at the bottom then divide
If x is the angle then work out the fraction and then shift sin/cos/tan

Angles of Elevation and Depression

Angles of Elevation

This is an angle always taken from the horizontal upwards. It is often used in trigonometry questions to describe the position of the angle.



The angle of elevation is the angle from the horizontal upwards

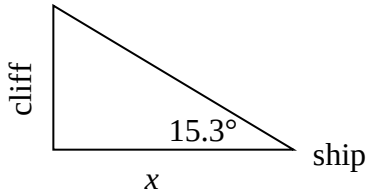
Example 1. A ship (S) is out at sea. The angle of elevation from the ship to the top of a cliff is 15.3°. If the vertical height of the cliff is 680m, how far away from the foot of the cliff is the ship.

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 15{\cdot}3° = \frac{680}{x}$$

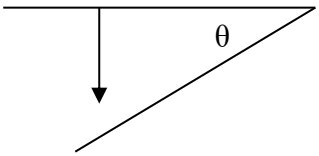
$$x = \frac{680}{\tan 15{\cdot}3}$$

$$x = 2485{\cdot}66m$$



Angles of Depression

This is an angle always taken from the horizontal downwards. It is also often used in trigonometry questions to describe the position of the angle.



The angle of elevation is the angle from the horizontal downwards

Example 2. A man (M) is standing on the top of a cliff. He is looking out to sea at a boat.

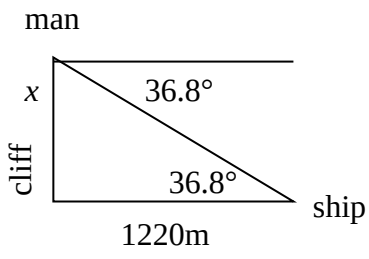
The angle of depression from the man to the boat is 36.8°. If the boat is 1220m away from the foot of the cliff, how high is the cliff.

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 36{\cdot}8° = \frac{x}{1220}$$

$$x = \tan 36{\cdot}8 \times 1220$$

$$x = 921{\cdot}68m$$



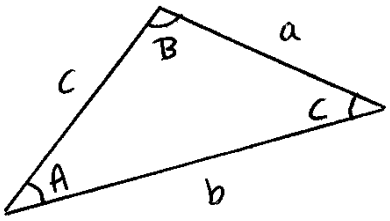
Sine Rule

The sine rule is:-

$$\frac{a}{Sin\ A} = \frac{b}{Sin\ B} = \frac{c}{Sin\ C}$$ ← Choose this formula for an unknown side

or $$\frac{SinA}{a} = \frac{Sin\ B}{b} = \frac{Sin\ C}{c}$$ ← Choose this formula for an unknown angle

To use the sine rule you must have a ***complete ratio*** – by that I mean you must know one side and its corresponding angle



How to prove the sine rule using trigonometry.

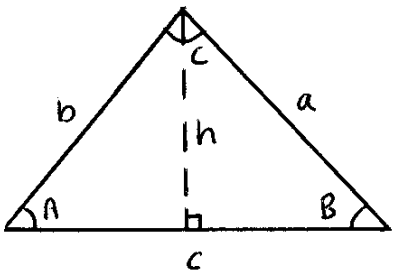
$$Sin\ B = \frac{h}{a} \qquad Sin\ A = \frac{h}{b}$$

$$h = a \sin B \qquad h = b \sin A$$

$$\therefore a\ Sin\ B = b \sin A$$

$$a = \frac{b \sin A}{Sin\ B}$$

$$\frac{a}{Sin\ A} = \frac{b}{Sin\ B}$$



Finding an Unknown Length

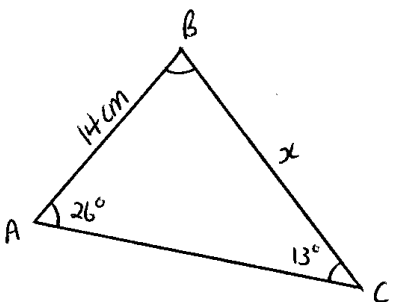
Example 1 *Find the size of BC, given that AB = 14cm , ∠BAC = 26°, ∠ACB = 13°.*

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{x}{\sin 26^\circ} = \frac{14}{\sin 13^\circ}$$

$$x = \frac{14}{\sin 13^\circ} \times \sin 26^\circ$$

$$x = 27{\cdot}3cm\ (1dp)$$



Sine Rule

Finding an unknown Angle

Example 1. Find the size of $\angle BAC$, given that AC = 13*cm* , $\angle ABC = 16°$, and BC = 18*cm*

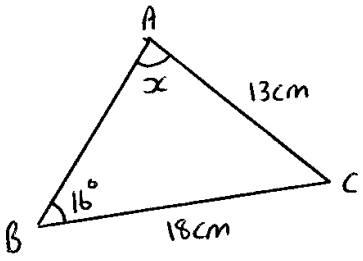
$$\frac{Sin\ A}{a} = \frac{Sin\ B}{b}$$

$$\frac{Sin\ x}{18} = \frac{\sin 16°}{13}$$

$$\sin x = \frac{\sin 16°}{13} \times 18$$

$$\sin x = 0{\cdot}3817$$

$$x = 22{\cdot}4°$$



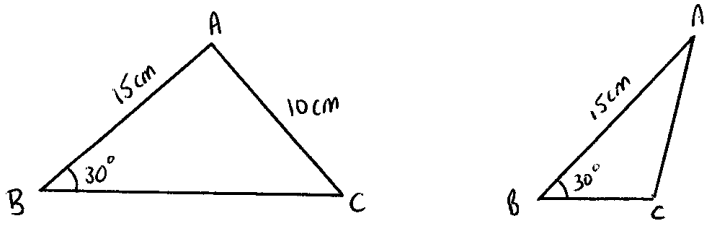
Leave it in the calculator to keep your accuracy

Finding Two Solutions for a Missing Angle

In general if $x = y°$ then x is also equal to $180 - y°$

This is because sometimes you can draw a triangle in 2 different ways.

Example 1. Given triangle ABC, where $\angle ABC = 30°$, AB = 15cm and BC = 10cm



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin x}{15} = \frac{\sin 30°}{10}$$

$$\sin x = \frac{\sin 30°}{10} \times 15$$

$$\sin x = 0{\cdot}75$$

$$x = 48{\cdot}6°$$

Also as $x = 180 - y°$ $\quad x = 180 - 48{\cdot}6$

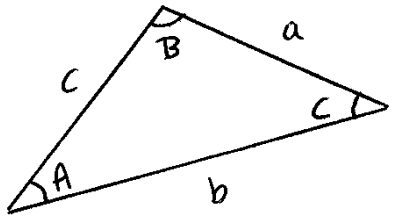
$$x = 131{\cdot}4°$$

so two solutions are $48{\cdot}6°$ and $131{\cdot}4°$

Note: This only occurs if the angle you are finding is larger than the angle given.

The Cosine Rule

This is the last of the triangle formulas. You should only use this if you know all 3 sides or you do not have a complete ratio.



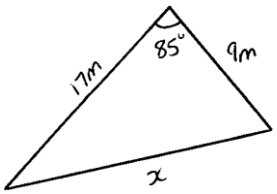
Formulas:

To find a missing angle $\quad Cos\,A = \dfrac{b^2 + c^2 - a^2}{2bc}$

To find a missing side $\quad a^2 = b^2 + c^2 - (2bc\,Cos\,A)$

Finding an Unknown Side

Example 1. Find the value of x



$a^2 = b^2 + c^2 - (abc\,Cos\,A)$

$a^2 = 9^2 + 17^2 - (2 \times 9 \times 17 \times \cos 85)$

$a^2 = 343{\cdot}33$

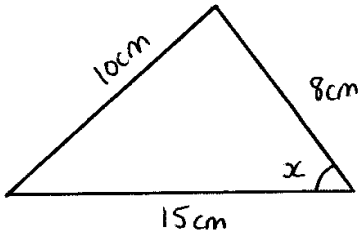
$a = 18{\cdot}53m\ (2dp)$

Remember to leave this value in your calculator to keep your accuracy

The Cosine Rule

Finding an Unknown Angle

Example 1. Find the value of x



$$Cos\ A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$Cos\ x = \frac{15^2 + 8 - 10}{2 \times 15 \times 8}$$

$$Cos\ x = 0{\cdot}7875$$

$$x = 38{\cdot}05° \ (2dp)$$

Remember to leave this value in your calculator to keep your accuracy

Using all Formulas to Solve Problems

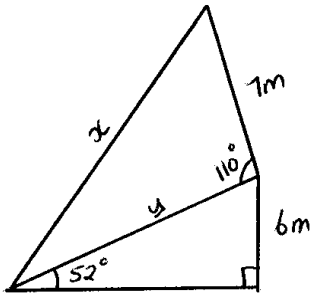
The order in which you should attempt triangle problems is:-

1. Pythagoras
2. Trigonometry — use these two for right angled triangles

3. Sine Rule
4. Cosine Rule — use these two for **non** right angled triangles

Remember also your angle facts such as angles in a triangle equal 180°

Example 1. Find the size of length x

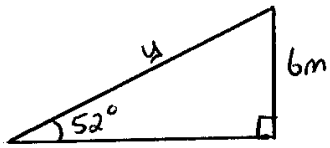


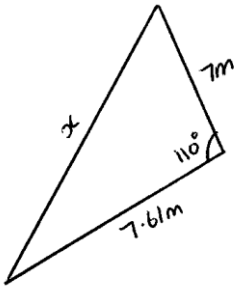
using $\sin\theta = \frac{opp}{hyp}$

$$\sin = \frac{6}{y}$$

$$y = \frac{6}{\sin 52}$$

$$y = 7{\cdot}61cm$$





$$a^2 = b^2 + c^2 - (2bc\cos A)$$

$$x^2 = 7^2 + 7{\cdot}61^2 - (2 \times 7 \times 7{\cdot}61 \times \cos 110)$$

$$x^2 = 143{\cdot}35$$

$$x = 11{\cdot}97m \ \ (2dp)$$

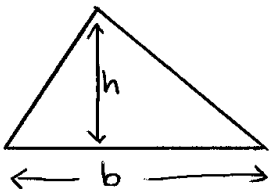
Areas of Triangles

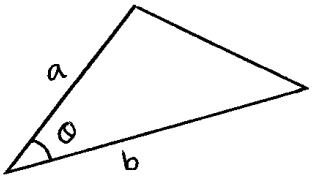
There are 2 formulas for finding the area of a triangle.

Formulas:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{perpendicular height} \qquad \text{or} \qquad \text{Area} = \frac{1}{2}ab\sin\theta$$

$$A = \frac{b \times h}{2}$$



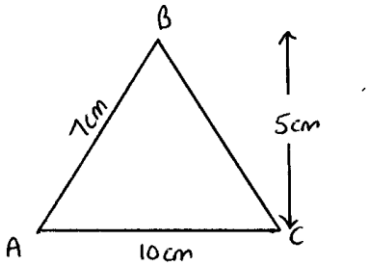


Example 1. Find the area of triangle ABC (diagram not to scale)

$$A = \frac{b \times h}{2}$$

$$A = \frac{10 \times 5}{2}$$

$$A = 25cm^2$$



Example 2. Find the area of triangle ABC (diagram not to scale)

$$Cos\,A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$Cos\,\theta = \frac{9^2 + 11^2 - 14^2}{2 \times 9 \times 11}$$

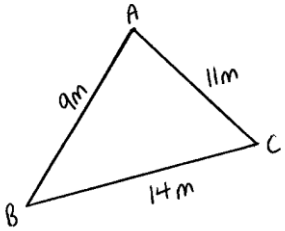
$$Cos\,\theta = 0{\cdot}030303$$

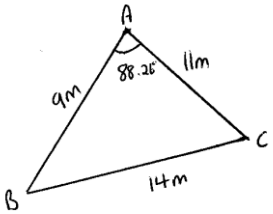
$$\theta = 88{\cdot}26°$$

$$\therefore \quad A = \frac{1}{2}ab\,Sin\,\theta$$

$$A = \frac{1}{2} \times 9 \times 11 \times \sin 88{\cdot}26°$$

$$A = 49{\cdot}48m^2$$

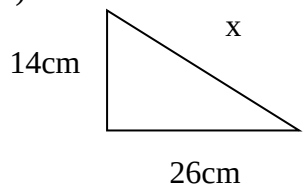




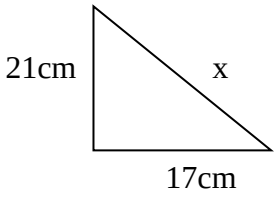
Homework Questions 1 – Pythagoras

Solve the following problems using Pythagoras. You must show all your working out
Give answers to 1 dp (all triangles shown are right angled triangles)

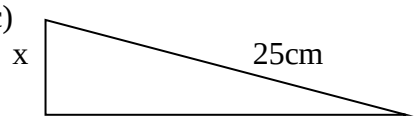
a)



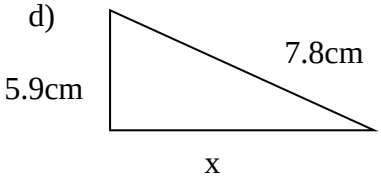
b)



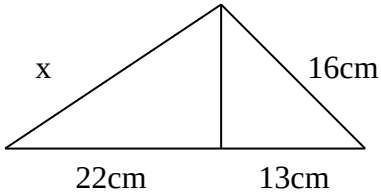
c)



d)



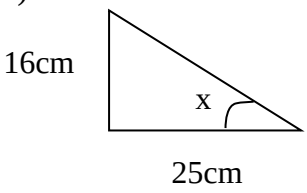
e)



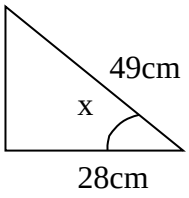
Homework Questions 2 – Trigonometry

Solve the following problems using Trigonometry. You must show all your working out
Give answers to 1dp (all triangles shown are right angled)

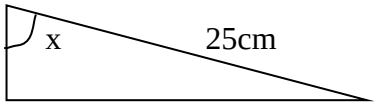
a)



b)

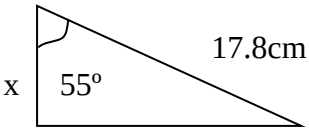


c)

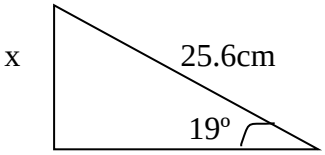


16.5cm

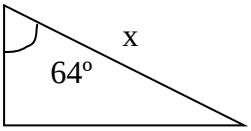
d)



e)



f)



Homework Questions 3 – Angles of Elevation and Depression

(Give all answers correct to 1dp)

1. Simon is 26m from the base of a tree. The angle of elevation of the top of the tree from Simon is 42°. Calculate the height of the tree.

2. Graham is standing 52m from a church tower. The tower is 32m high. Work out the angle of elevation of the top of the church tower from Graham.

3. Fiona is at the edge at the top of the cliff. She sees a boat on the water below. The cliff is 105m high. The angle of depression of the boat from Fiona is 7°. How far is the boat from the foot of the cliff?

4. A man is standing at the top of the cliff looking out to sea at the boat. A boat has traveled 650m from the foot of the cliff, and is at an angle of depression of 35° from the man. How high is the cliff which the man is standing on?

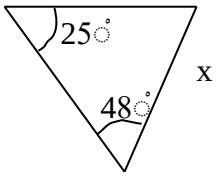
Homework Questions 4 – Finding an Unknown Side Using the Sine Rule

Solve the following problems using the sine rule, give answers to 1dp *(The diagrams are not drawn to scale*

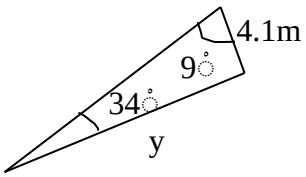
1. In the Triangle ABC, AB = 7cm, $\angle BAC = 16^\circ$ $\angle ACB = 26°$. Find the length of BC

2. In the Triangle DEF, DE = 9cm, $\angle EDF = 25^\circ$ $\angle DFE = 72°$. Find the length of DF

3. 7.6m Find the length of x



4. Find the length of y



5. In the Triangle RST, RT = 30cm, $\angle STR = 65^\circ$ $\angle RST = 18°$. Find the length of RS

6. In the Triangle ABC, BC = 19cm, $\angle BAC = 63^\circ$ $\angle ABC = 39°$.

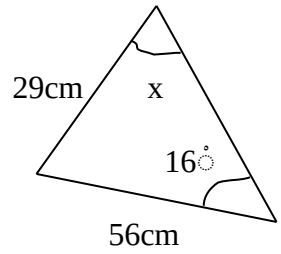
   Find the length of AC

Homework Questions 5 – Finding an Unknown Angle Using the Sine Rule

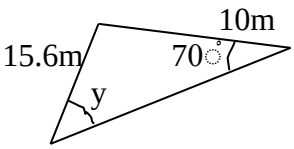
Solve the following problems using the sine rule, give answers to 1dp *(The diagrams are not drawn to scale)*

1. In the Triangle RST, RS = 10m, TS = 8m $\angle RTS$ = 55 °. Find the size of $\angle TRS$

2. Find the size of angle x



3. Find the size of angle y



4. In the Triangle ABC, AC = 9cm, BC = 18cm, $\angle ABC$ = 26 °. Find the size of $\angle BAC$

5. In the Triangle DEF, DE = 29cm, DF = 15cm $\angle DFE$ = 68 °. Find the size of $\angle DEF$

6. In the Triangle ABC, AB = 47m, AC = 32m $\angle ABC$ = 29 °. Find the size of $\angle ACB$

Homework Questions 6 – Finding Two Solutions for A Missing Angle

Solve the following problems using the sine rule, give answers to 1dp *(The diagrams are not drawn to scale)*

1. In the Triangle ABC, BC = 9m, AC = 13.5m and $\angle ABC = 55°$.

   Find the 2 possilbe values for $\angle BAC$

2. Triangle ABC is such that AB = 4cm, BC = 6cm and $\angle ACB = 38°$.

   Find the 2 possilbe values for $\angle BAC$

3. Triangle ABC is such that AB = 29cm, AC = 14cm and $\angle ACB = 45°$.

   Find the 2 possilbe values for $\angle ABC$

4. In Triangle ABC is such that AB = 14cm, AC = 7cm and $\angle ACB = 56°$.

   Find the 2 possilbe values for $\angle ABC$

5. In triangle ABC is such that AB = 49cm, BC = 26cm and $\angle BAC = 30°$.

   Find the 2 possilbe values for $\angle ACB$

Homework Questions 7 – Finding an Unknown Side Using the Cosine Rule

Solve the following, give answers to 1dp *(Diagrams are not drawn to scale)*

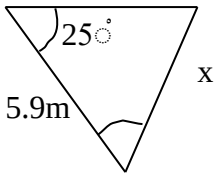
1. In triangle ABC, AB = 47cm, BC = 45cm, $\angle ABC = 62°$.

   What is the size of the length AC

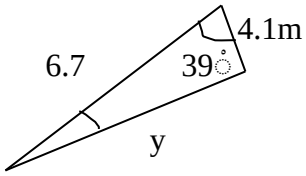
2. In triangle DEF, DE = 18cm, DF = 20cm, $\angle EDF = 46°$.

   What is the size of the length EF

3. 8.2m

   Find the size of x

4. 

   Find the size of y

5. In triangle RST, ST = 14cm, RT = 29cm, $\angle RTS = 35°$.

   What is the size of the length RS

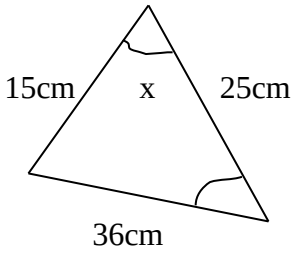
6. In triangle ABC, AC = 48cm, BC = 54cm, $\angle BCA = 68°$.

   What is the size of the length AB

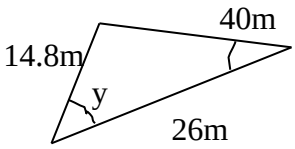
Homework Questions 8 – Finding an Unknown Angle Using the Cosine Rule

Solve the following, give answers to 1dp. (the diagrams are not drawn to scale)

1. Find the size of angle x



2. Find the size of angle y



3. In Triangle ABC, AB = 19cm, AC = 15cm and BC = 29cm.

   What is the size of $\angle ABC$

4. In Triangle DEF, DE = 21m, EF = 26m and DF = 38m.

   What is the size of $\angle DFE$

5. In Triangle RST, RS = 65cm, ST = 72cm and RT = 82cm.

   What is the size of $\angle RST$

6. In Triangle ABC, AB = 29cm, AC = 26cm and BC = 35cm.

   What is the size of $\angle ACB$

Homework Questions 9 – Mixed Problems

Solve the following, give answers to 1dp *(Diagrams are not drawn to scale)*

1. A hiker walks from A to B on a bearing of 060°. After 7km he reaches B and then walks a further 12km on a bearing of140 to point C

   a) What is the distance from A to C

   b) What is the bearing of C from A

2. In triangle ABC, AB = 10km, BC = 14km, $\angle BCA = 35°$

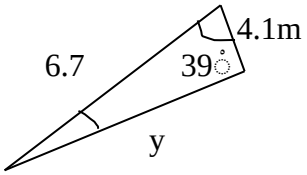
   a) What is the size of $\angle ABC$

   b) What is the size of length AC

3. In the triangle ABC, AB = 9m, BC = 9m, $\angle ACB = 62°$.

If D is the midpoint of AC, what is the length of BD

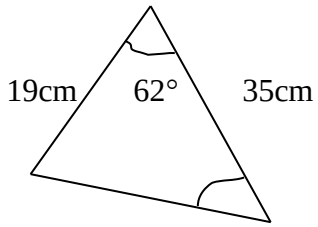
4. Find the size of y



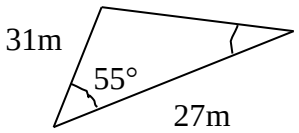
Homework Questions 10 – Area of a Triangle

Find the area of the following triangles, give answers to 1dp. *(The diagrams are not drawn to scale)*

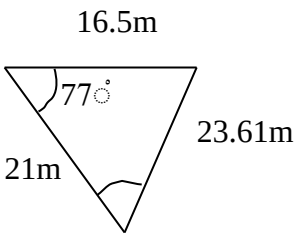
1.



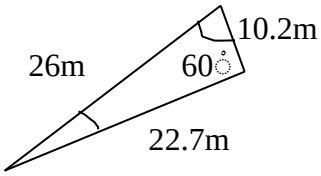
2.



3.



4.



5. In triangle ABC, AB = 54m, AC = 43m and BC = 62m.

What is the area of the triangle ABC

6. In triangle ABC, AB = 16cm, BC = 16cm and $\angle BCA = 48°$

What is the area of the triangle ABC