Edexcel
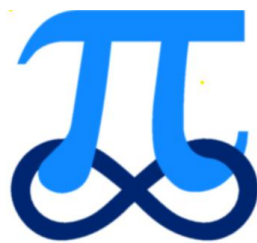
New GCE A Level Maths workbook

Solving Linear and Quadratic Inequalities

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Linear inequalities

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

Examples

Example 1   Solve $-8 \leq 4x < 16$

$-8 \leq 4x < 16$ $-2 \leq \ x \ < 4$	Divide all three terms by 4.

Example 2   Solve $4 \leq 5x < 10$

$4 \leq 5x < 10$ $\frac{4}{5} \leq x < 2$	Divide all three terms by 5.

Example 3   Solve $2x - 5 < 7$

$2x - 5 < 7$ $\quad 2x < 12$ $\quad\ x < 6$	1 Add 5 to both sides. 2 Divide both sides by 2.

Example 4   Solve $2 - 5x \geq -8$

$2 - 5x \geq -8$ $\quad -5x \geq -10$ $\qquad x \leq 2$	1 Subtract 2 from both sides. 2 Divide both sides by -5. Remember to reverse the inequality when dividing by a negative number.

Example 5   Solve $4(x - 2) > 3(9 - x)$

$4(x - 2) > 3(9 - x)$ $\ 4x - 8 > 27 - 3x$ $\ 7x - 8 > 27$ $\qquad 7x > 35$ $\qquad\ x > 5$	1 Expand the brackets. 2 Add $3x$ to both sides. 3 Add 8 to both sides. 4 Divide both sides by 7.

Practice

1 Solve these inequalities.

    a $4x > 16$ **b** $5x - 7 \leq 3$ **c** $1 \geq 3x + 4$

    d $5 - 2x < 12$ **e** $\dfrac{x}{2} \geq 5$ **f** $8 < 3 - \dfrac{x}{3}$

2 Solve these inequalities.

    a $\dfrac{x}{5} < -4$ **b** $10 \geq 2x + 3$ **c** $7 - 3x > -5$

3 Solve

    a $2 - 4x \geq 18$ **b** $3 \leq 7x + 10 < 45$ **c** $6 - 2x \geq 4$

    d $4x + 17 < 2 - x$ **e** $4 - 5x < -3x$ **f** $-4x \geq 24$

4 Solve these inequalities.

    a $3t + 1 < t + 6$ **b** $2(3n - 1) \geq n + 5$

5 Solve.

    a $3(2 - x) > 2(4 - x) + 4$ **b** $5(4 - x) > 3(5 - x) + 2$

Extend

6 Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

Answers

1 **a** $x > 4$ **b** $x \leq 2$ **c** $x \leq -1$

  d $x > -\dfrac{7}{2}$ **e** $x \geq 10$ **f** $x < -15$

2 **a** $x < -20$ **b** $x \leq 3.5$ **c** $x < 4$

3 **a** $x \leq -4$ **b** $-1 \leq x < 5$ **c** $x \leq 1$

  d $x < -3$ **e** $x > 2$ **f** $x \leq -6$

4 **a** $t < \dfrac{5}{2}$ **b** $n \geq \dfrac{7}{5}$

5 **a** $x < -6$ **b** $x < \dfrac{3}{2}$

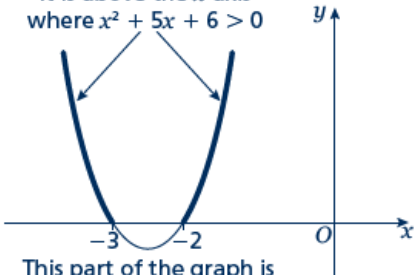**6** $x > 5$ (which also satisfies $x > 3$)
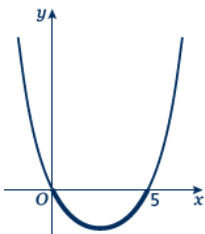
Quadratic inequalities

Key points

- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

Examples

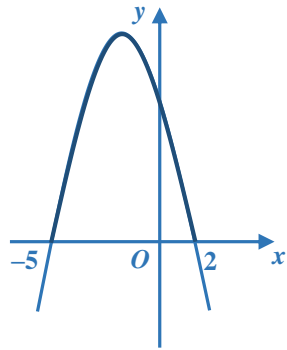**Example 1** Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$

$x^2 + 5x + 6 = 0$ $(x + 3)(x + 2) = 0$ $x = -3$ or $x = -2$	**1** Solve the quadratic equation by factorising.
It is above the x-axis where $x^2 + 5x + 6 > 0$ This part of the graph is not needed as this is where $x^2 + 5x + 6 < 0$	**2** Sketch the graph of $y = (x + 3)(x + 2)$ **3** Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$
$x < -3$ or $x > -2$	**4** Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$

Example 2 Find the set of values of x which satisfy $x^2 - 5x \leq 0$

$x^2 - 5x = 0$ $x(x - 5) = 0$ $x = 0$ or $x = 5$	**1** Solve the quadratic equation by factorising. **2** Sketch the graph of $y = x(x - 5)$ **3** Identify on the graph where $x^2 - 5x \leq 0$, i.e. where $y \leq 0$
$0 \leq x \leq 5$	**4** Write down the values which satisfy the inequality $x^2 - 5x \leq 0$

Example 3   Find the set of values of x which satisfy $-x^2 - 3x + 10 \geq 0$

$-x^2 - 3x + 10 = 0$ $(-x + 2)(x + 5) = 0$ $x = 2$ or $x = -5$ $-5 \leq x \leq 2$	**1** Solve the quadratic equation by factorising. **2** Sketch the graph of $y = (-x + 2)(x + 5) = 0$ **3** Identify on the graph where $-x^2 - 3x + 10 \geq 0$, i.e. where $y \geq 0$ **3** Write down the values which satisfy the inequality $-x^2 - 3x + 10 \geq 0$

Practice

1   Find the set of values of x for which $(x + 7)(x - 4) \leq 0$

2   Find the set of values of x for which $x^2 - 4x - 12 \geq 0$

3   Find the set of values of x for which $2x^2 - 7x + 3 < 0$

4   Find the set of values of x for which $4x^2 + 4x - 3 > 0$

5   Find the set of values of x for which $12 + x - x^2 \geq 0$

Extend

Find the set of values which satisfy the following inequalities.

6 $x^2 + x \leq 6$

7 $x(2x - 9) < -10$

8 $6x^2 \geq 15 + x$

Answers

1 $-7 \leq x \leq 4$

2 $x \leq -2$ or $x \geq 6$

3 $\dfrac{1}{2} < x < 3$

4 $x < -\dfrac{3}{2}$ or $x > \dfrac{1}{2}$

5 $-3 \leq x \leq 4$

6 $-3 \leq x \leq 2$

7 $2 < x < 2\dfrac{1}{2}$

8 $x \leq -\dfrac{3}{2}$ or $x \geq \dfrac{5}{3}$

Q1.

Find the set values of x for which

(a) $4x - 5 > 15 - x$

(2)

(b) $x(x - 4) > 12$

(4)

Q2.

Find the set of values of x for which

(a) $3(x - 2) < 8 - 2x$

(2)

(b) $(2x - 7)(1 + x) < 0$

(3)

(c) both $3(x - 2) < 8 - 2x$ **and** $(2x - 7)(1 + x) < 0$

(1)

Q3.

Find the set of values of x for which

(a) $4x - 3 > 7 - x$

(2)

(b) $2x^2 - 5x - 12 < 0$

(4)

(c) **both** $4x - 3 > 7 - x$ **and** $2x^2 - 5x - 12 < 0$

(1)

Q4.

Find the set of values of x for which

(a) $2(3x + 4) > 1 - x$

(2)

(b) $3x^2 + 8x - 3 < 0$

(4)

Q5.

A rectangular room has a width of x m.

The length of the room is 4 m longer than its width.

Given that the perimeter of the room is greater than 19.2 m,

(a) show that $x > 2.8$

(3)

Given also that the area of the room is less than 21 m²,

(b) (i)   write down an inequality, in terms of x, for the area of the room.

   (ii)   Solve this inequality.

(4)

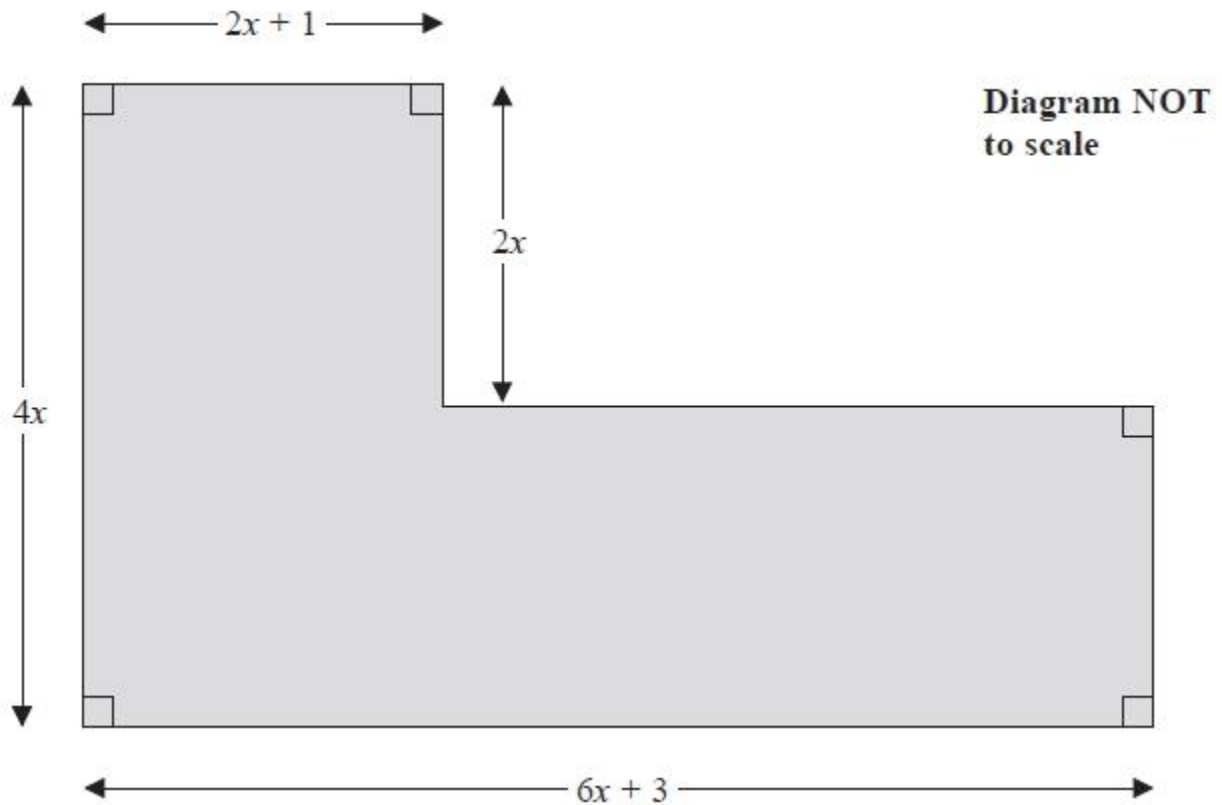(c) Hence find the range of possible values for x.

(1)

Q6.

Figure 1

Figure 1 shows the plan of a garden. The marked angles are right angles.

The six edges are straight lines.

The lengths shown in the diagram are given in metres.

Given that the perimeter of the garden is greater than 40 m,

(a) show that $x > 1.7$

(3)

Given that the area of the garden is less than 120 m^2,

(b) form and solve a quadratic inequality in x.

(5)

(c) Hence state the range of the possible values of x.

(1)

Q7. Find the set of values of x for which

    (*a*) $3(2x + 1) > 5 - 2x$,

    (2)

    (*b*) $2x^2 - 7x + 3 > 0$,

    (4)

    (*c*) **both** $3(2x + 1) > 5 - 2x$ **and** $2x^2 - 7x + 3 > 0$.

    (2)

Q8. Find the set of values of x for which

    (*a*) $4x - 3 > 7 - x$

    (2)

    (*b*) $2x^2 - 5x - 12 < 0$

    (4)

    (*c*) **both** $4x - 3 > 7 - x$ **and** $2x^2 - 5x - 12 < 0$

    (1)

Q9. Find the set of values of x for which

(a) $2(3x + 4) > 1 - x$,

(2)

(b) $3x^2 + 8x - 3 < 0$.

(4)

Q10. Find the set of values of x for which

(a) $3x - 7 > 3 - x$,

(2)

(b) $x^2 - 9x \leq 36$,

(4)

(c) **both** $3x - 7 > 3 - x$ **and** $x^2 - 9x \leq 36$.

(1)