OCR Statistics 01

Past paper questions on

Discrete random variables

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Probability Distributions

- A random variable is a quantity whose value depends on chance. The outcome of a random variable is usually denoted by a capital letter (e.g. X). We read $\mathbb{P}(X = 2)$ as the probability that the random variable takes the value 2. For a fair die, $\mathbb{P}(X = 5) = \frac{1}{6}$.

- For discrete random variables they are usually presented in a table. For example for a fair die:

x	1	2	3	4	5	6
$\mathbb{P}(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

- In general, for any event, the probability distribution is of the form

x	x_1	x_2	x_3	x_4	x_5	x_6	$\cdots$
$\mathbb{P}(X = x)$	p_1	p_2	p_3	p_4	p_5	p_6	$\cdots$

- As before, it is crucial that we remember the probabilities sum to one. This can be useful at the start or problems where a constant must be evaluated. For example in:

x	1	2	3	4
$\mathbb{P}(X = x)$	k	k	$2k$	$4k$

We discover $k + k + 2k + 4k = 1$, so $k = \frac{1}{8}$.

Expectation And Variance Of A Random Variable

- The expected value of the event is denoted $\mathbb{E}(X)$ or μ. It is defined

$$\mathbb{E}(X) = \mu = \boxed{\sum x\mathbb{P}(X = x)}.$$

For example for a fair die with

x	1	2	3	4	5	6
$\mathbb{P}(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

we find:

$$\begin{aligned}\mathbb{E}(X) &= \left(1 \times \tfrac{1}{6}\right) + \left(2 \times \tfrac{1}{6}\right) + \left(3 \times \tfrac{1}{6}\right) + \left(4 \times \tfrac{1}{6}\right) + \left(5 \times \tfrac{1}{6}\right) + \left(6 \times \tfrac{1}{6}\right) \\ &= 3\tfrac{1}{2}.\end{aligned}$$

- The variance of an event is denoted $\mathrm{Var}(X)$ or σ^2 and is defined

$$\mathrm{Var}(X) = \sigma^2 = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \mathbb{E}(X^2) - \mu^2 = \boxed{\sum x^2 \mathbb{P}(X = x) - \mu^2}.$$

So for the *biased* die with distribution

x	1	2	3	4	5	6
$\mathbb{P}(X = x)$	$\frac{1}{3}$	$\frac{1}{6}$	0	0	$\frac{1}{6}$	$\frac{1}{3}$

we find that

$$\mathbb{E}(X) = \left(1 \times \tfrac{1}{3}\right) + \left(2 \times \tfrac{1}{6}\right) + (3 \times 0) + (4 \times 0) + \left(5 \times \tfrac{1}{6}\right) + \left(6 \times \tfrac{1}{3}\right) = 3\tfrac{1}{2}$$

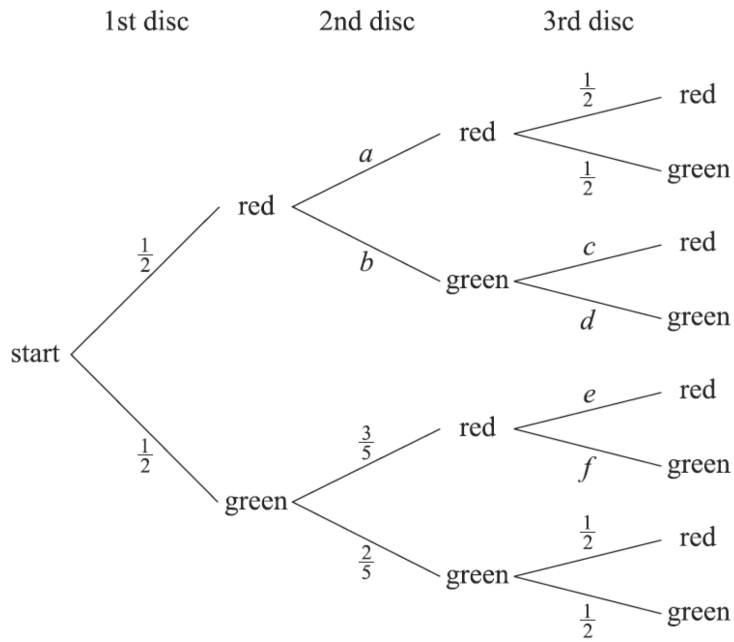
and

$$\begin{aligned}
\mathrm{Var}(X) &= \sum x^2 \mathbb{P}(X = x) - \mu^2 \\
&= \left(1^2 \times \tfrac{1}{3}\right) + \left(2^2 \times \tfrac{1}{6}\right) + \left(3^2 \times 0\right) + \left(4^2 \times 0\right) + \left(5^2 \times \tfrac{1}{6}\right) + \left(6^2 \times \tfrac{1}{3}\right) - 3\tfrac{1}{2}^2 \\
&= 17\tfrac{1}{6} - 3\tfrac{1}{2}^2 = 4\tfrac{11}{12}.
\end{aligned}$$

1.

Two bags contain coloured discs. At first, bag P contains 2 red discs and 2 green discs, and bag Q contains 3 red discs and 1 green disc. A disc is chosen at random from bag P, its colour is noted and it is placed in bag Q. A disc is then chosen at random from bag Q, its colour is noted and it is placed in bag P. A disc is then chosen at random from bag P.

The tree diagram shows the different combinations of three coloured discs chosen.



(i) Write down the values of a, b, c, d, e and f. [4]

The total number of red discs chosen, out of 3, is denoted by R. The table shows the probability distribution of R.

r	0	1	2	3
$\mathrm{P}(R = r)$	$\frac{1}{10}$	k	$\frac{9}{20}$	$\frac{1}{5}$

(ii) Show how to obtain the value $\mathrm{P}(R = 2) = \frac{9}{20}$. [3]

(iii) Find the value of k. [2]

(iv) Calculate the mean and variance of R. [5]

Q6 June 2005

2.

In Mr Kendall's cupboard there are 3 tins of baked beans and 2 tins of pineapple. Unfortunately his daughter has removed all the labels for a school project and so the tins are identical in appearance. Mr Kendall wishes to use both tins of pineapple for a fruit salad. He opens tins at random until he has opened the two tins of pineapples.

Let X be the number of tins that Mr Kendall opens.

(i) Show that $P(X = 3) = \frac{1}{5}$. [4]

(ii) The probability distribution of X is given in the table below.

x	2	3	4	5
$P(X = x)$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

Find $E(X)$ and $Var(X)$. [5]

Q3 Jan 2006

3.

The probability distribution of a discrete random variable, X, is given in the table.

x	0	1	2	3
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{4}$	p	q

It is given that the expectation, $E(X)$, is $1\frac{1}{4}$.

(i) Calculate the values of p and q. [5]

(ii) Calculate the standard deviation of X. [4]

Q5 June 2006

4.

Part of the probability distribution of a variable, X, is given in the table.

x	0	1	2	3
$P(X = x)$		$\frac{3}{10}$	$\frac{1}{5}$	$\frac{2}{5}$

(i) Find $P(X = 0)$. [2]

(ii) Find $E(X)$. [2]

Q1 Jan 2007

5.

The table shows the probability distribution for a random variable X.

x	0	1	2	3
$P(X = x)$	0.1	0.2	0.3	0.4

Calculate $E(X)$ and $Var(X)$. [5]

Q1 June 2007

6.

The probability distribution for a random variable Y is shown in the table.

y	1	2	3
$P(Y = y)$	0.2	0.3	0.5

(i) Calculate $E(Y)$ and $Var(Y)$. [5]

Another random variable, Z, is independent of Y. The probability distribution for Z is shown in the table.

z	1	2	3
$P(Z = z)$	0.1	0.25	0.65

One value of Y and one value of Z are chosen at random. Find the probability that

(ii) $Y + Z = 3$, [3]

(iii) $Y \times Z$ is even. [3]

Q6 Jan 2008

7.

At a fairground stall, on each turn a player receives prize money with the following probabilities.

Prize money	£0.00	£0.50	£5.00
Probability	$\frac{17}{20}$	$\frac{1}{10}$	$\frac{1}{20}$

(i) Find the probability that a player who has two turns will receive a total of £5.50 in prize money. [3]

(ii) The stall-holder wishes to make a profit of 20p per turn on average. Calculate the amount the stall-holder should charge for each turn. [4]

Q4 June 2008

8.

Each time a certain triangular spinner is spun, it lands on one of the numbers 0, 1 and 2 with probabilities as shown in the table.

Number	Probability
0	0.7
1	0.2
2	0.1

The spinner is spun twice. The total of the two numbers on which it lands is denoted by X.

(i) Show that $P(X = 2) = 0.18$. **[3]**

The probability distribution of X is given in the table.

x	0	1	2	3	4
$P(X = x)$	0.49	0.28	0.18	0.04	0.01

(ii) Calculate $E(X)$ and $Var(X)$. **[5]**

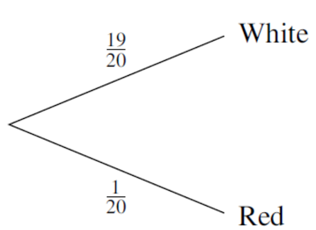
Q1 Jan 2009

9.

A game at a charity event uses a bag containing 19 white counters and 1 red counter. To play the game once a player takes counters at random from the bag, one at a time, without replacement. If the red counter is taken, the player wins a prize and the game ends. If not, the game ends when 3 white counters have been taken. Niko plays the game once.

(i) (a) Copy and complete the tree diagram showing the probabilities for Niko. **[4]**

First counter



(b) Find the probability that Niko will win a prize. **[3]**

(ii) The number of counters that Niko takes is denoted by X.

(a) Find $P(X = 3)$. **[2]**

(b) Find $E(X)$. **[4]**

Q8 June 2009

10.

A certain four-sided die is biased. The score, X, on each throw is a random variable with probability distribution as shown in the table. Throws of the die are independent.

x	0	1	2	3
$P(X = x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

(i) Calculate $E(X)$ and $Var(X)$. **[5]**

The die is thrown 10 times.

(ii) Find the probability that there are not more than 4 throws on which the score is 1. **[2]**

(iii) Find the probability that there are exactly 4 throws on which the score is 2. **[3]**

Q4 Jan 2010

11.

Each of four cards has a number printed on it as shown.

1	**2**	**3**	**3**

Two of the cards are chosen at random, without replacement. The random variable X denotes the sum of the numbers on these two cards.

(i) Show that $P(X = 6) = \frac{1}{6}$ and $P(X = 4) = \frac{1}{3}$. **[3]**

(ii) Write down all the possible values of X and find the probability distribution of X. **[4]**

(iii) Find $E(X)$ and $Var(X)$. **[5]**

Q5 June 2010

12.

The probability distribution of a discrete random variable, X, is shown below.

x	0	2
$P(X = x)$	a	$1 - a$

(i) Find $E(X)$ in terms of a. **[2]**

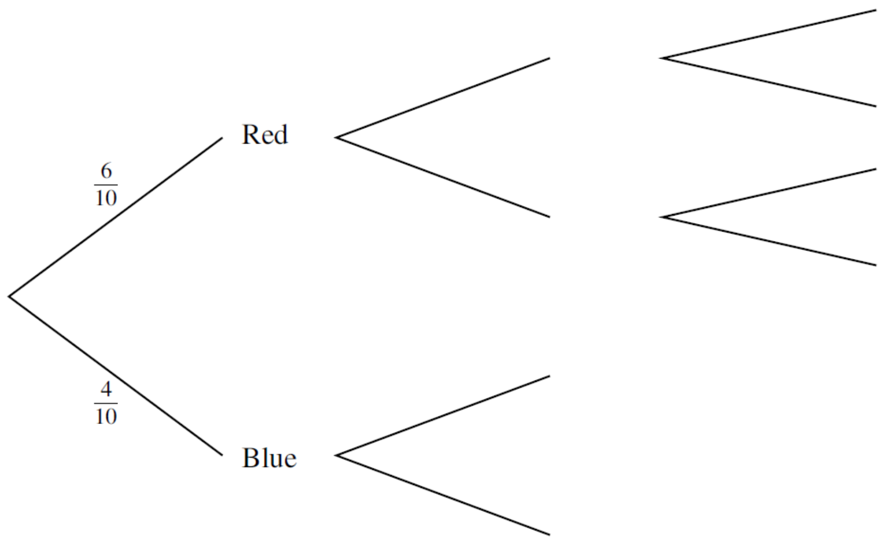
(ii) Show that $Var(X) = 4a(1 - a)$. **[3]**

Q7 Jan 2011

12.

A bag contains 4 blue discs and 6 red discs. Chloe takes a disc from the bag. If this disc is red, she takes 2 more discs. If not, she takes 1 more disc. Each disc is taken at random and no discs are replaced.

(i) Complete the probability tree diagram in your Answer Book, showing all the probabilities. **[2]**



The total number of blue discs that Chloe takes is denoted by X.

(ii) Show that $\mathrm{P}(X = 1) = \frac{3}{5}$. **[2]**

The complete probability distribution of X is given below.

x	0	1	2
$\mathrm{P}(X = x)$	$\frac{1}{6}$	$\frac{3}{5}$	$\frac{7}{30}$

(iii) Calculate $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. **[5]**

Q5 June 2011

13.

The probability distribution of a random variable X is shown in the table.

x	1	2	3	4
$\mathrm{P}(X=x)$	0.1	0.3	$2p$	p

(i) Find p. **[2]**

(ii) Find $\mathrm{E}(X)$. **[2]**

14.

A six-sided die is biased so that the probability of scoring 6 is 0.1 and the probabilities of scoring 1, 2, 3, 4, and 5 are all equal. In a game at a fête, contestants pay £3 to roll this die. If the score is 6 they receive £10 back. If the score is 5 they receive £5 back. Otherwise they receive no money back. Find the organiser's expected profit for 100 rolls of the die. **[5]**

Q6 June 2012

15.

The probability distribution of a random variable X is shown.

x	1	3	5	7
$P(X = x)$	0.4	0.3	0.2	0.1

(i) Find $E(X)$ and $Var(X)$. **[5]**

(ii) Three independent values of X, denoted by X_1, X_2 and X_3, are chosen. Given that $X_1 + X_2 + X_3 = 19$, write down all the possible sets of values for X_1, X_2 and X_3 and hence find $P(X_1 = 7)$. **[2]**

(iii) 11 independent values of X are chosen. Use an appropriate formula to find the probability that exactly 4 of these values are 5s. **[3]**

Q3 June 2013

16.

(a) The probability distribution of a random variable W is shown in the table.

w	0	2	4
$P(W = w)$	0.3	0.4	0.3

Calculate $Var(W)$. **[3]**

(b) The random variable X has probability distribution given by

$$P(X = x) = k(x + 1) \quad \text{for } x = 1, 2, 3, 4.$$

(i) Show that $k = \frac{1}{14}$. **[1]**

(ii) Calculate $E(X)$. **[3]**

Q2 June 2014

17.

The random variable X has probability distribution given by

$$P(X = x) = a + bx \qquad \text{for } x = 1, 2 \text{ and } 3,$$

where a and b are constants.

(i) Show that $3a + 6b = 1$. **[2]**

(ii) Given that $E(X) = \frac{5}{3}$, find a and b. **[4]**

Q9 June 2015