Edexcel

Pure Mathematics

Year 1

Differentiation 2

Past paper questions from Core Maths 2 and IAL C12

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Past paper questions from

Edexcel Core Maths 2 and IAL C12.

From Jan 2005 to Oct 2019.

This Section 2 has 38 Questions on application on differentiations, Min, Max problems.

Please check the Edexcel website for the solutions.

1. **Figure 3**

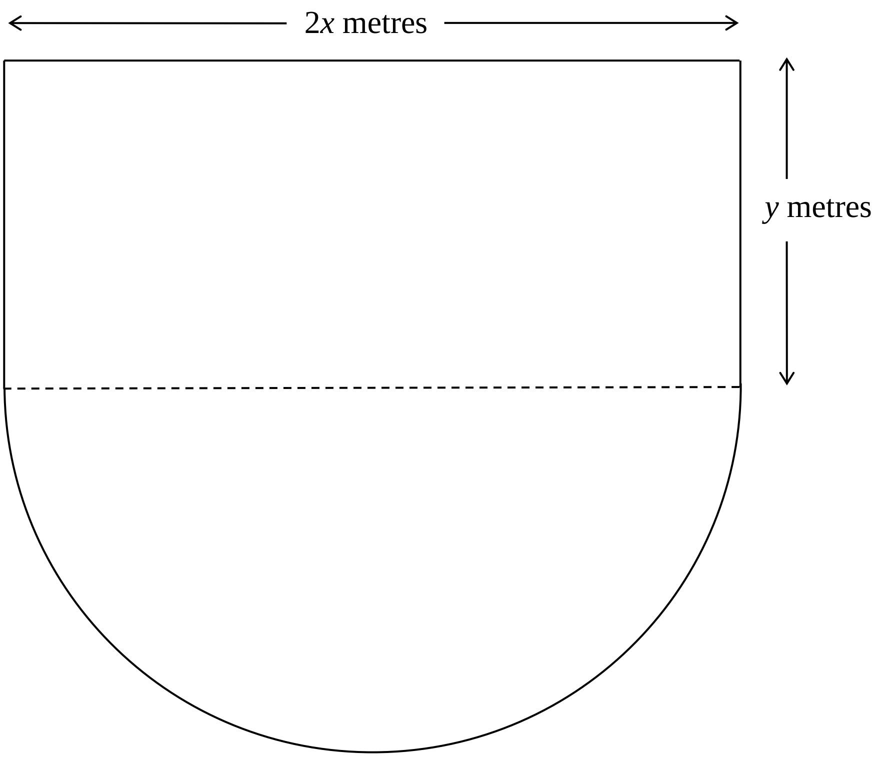

Figure 3 shows the plan of a stage in the shape of a rectangle joined to a semicircle. The length of the rectangular part is $2x$ metres and the width is y metres. The diameter of the semicircular part is $2x$ metres. The perimeter of the stage is 80 m.

(*a*) Show that the area, A m^2, of the stage is given by

$$A = 80x - \left(2 + \frac{\pi}{2}\right)x^2.$$

(4)

(*b*) Use calculus to find the value of x at which A has a stationary value.

(4)

(*c*) Prove that the value of x you found in part (*b*) gives the maximum value of A.

(2)

(*d*) Calculate, to the nearest m^2, the maximum area of the stage.

(2)

(C2, Jan 2005 Q9)

2. The curve C has equation

$$y = 2x^3 - 5x^2 - 4x + 2.$$

(*a*) Find $\frac{dy}{dx}$.

(2)

(*b*) Using the result from part (*a*), find the coordinates of the turning points of C.

(4)

(*c*) Find $\frac{d^2y}{dx^2}$.

(2)

(*d*) Hence, or otherwise, determine the nature of the turning points of C.

(2)

(C2, Jan 2006 Q7)

3. A diesel lorry is driven from Birmingham to Bury at a steady speed of v kilometres per hour. The total cost of the journey, £C, is given by

$$C = \frac{1400}{v} + \frac{2v}{7}.$$

(*a*) Find the value of v for which C is a minimum.

(5)

(*b*) Find $\frac{d^2C}{dv^2}$ and hence verify that C is a minimum for this value of v.

(2)

(*c*) Calculate the minimum total cost of the journey.

(2)

(C2, Jan 2007 Q8)

4.

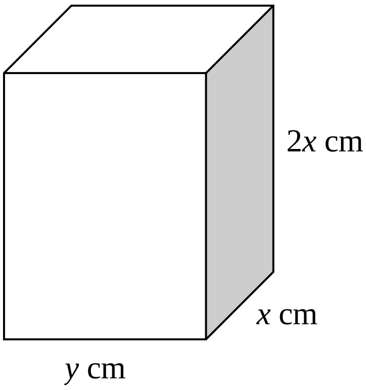

Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

(*a*) Show that the volume, V cm^3, of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that x can vary,

(*b*) use calculus to find the maximum value of V, giving your answer to the nearest cm^3.

(5)

(*c*) Justify that the value of V you have found is a maximum.

(2)

(C2, May 2007 Q10)

5.

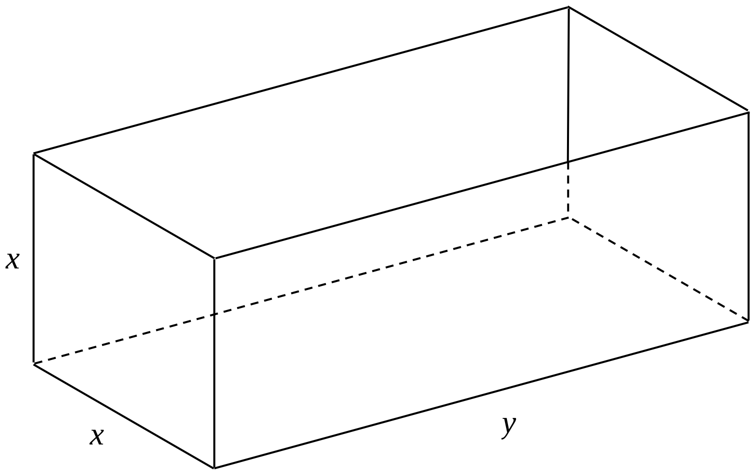

Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is 100 m^3.

(*a*) Show that the area A m^2 of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2.$$

(4)

(*b*) Use calculus to find the value of x for which A is stationary.

(4)

(*c*) Prove that this value of x gives a minimum value of A.

(2)

(*d*) Calculate the minimum area of sheet metal needed to make the tank.

(2)

(C2, Jan 2008 Q9)

6. A solid right circular cylinder has radius r cm and height h cm.

The total surface area of the cylinder is 800 cm^2 .

(*a*) Show that the volume, V cm^3 , of the cylinder is given by

$$V = 400r - \pi r^3.$$

(4)

Given that r varies,

(*b*) use calculus to find the maximum value of V, to the nearest cm^3.

(6)

(*c*) Justify that the value of V you have found is a maximum. **(2)**

(C2, Jan 2009 Q10)

8. The curve C has equation $y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10, \; x > 0$.

(*a*) Use calculus to find the coordinates of the turning point on C.

(7)

(*b*) Find $\dfrac{d^2y}{dx^2}$.

(2)

(*c*) State the nature of the turning point. **(1)**

(C2, Jan 2010 Q9)

9. $$y = x^2 - k\sqrt{x}, \quad \text{where } k \text{ is a constant.}$$

(*a*) Find $\dfrac{dy}{dx}$.

(2)

(*b*) Given that y is decreasing at $x = 4$, find the set of possible values of k.

(2)

(C2, June2010 Q3)

10. The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5 - x)^2, \quad 0 < x < 5.$$

(*a*) Find $\dfrac{dV}{dx}$.

(4)

(*b*) Hence find the maximum volume of the box.

(4)

(*c*) Use calculus to justify that the volume that you found in part (*b*) is a maximum.

(2)

(C2, Jan 2011 Q10)

11.

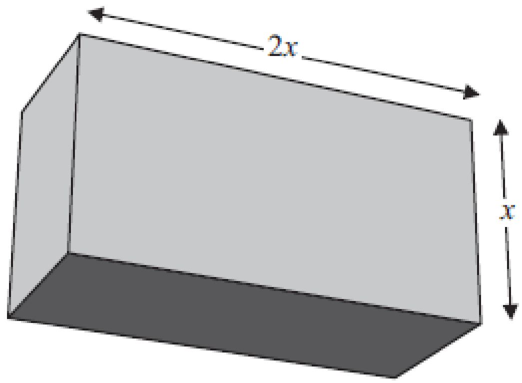

Figure 2

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 2.

The volume of the cuboid is 81 cubic centimetres.

(*a*) Show that the total length, L cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2}.$$

(3)

(*b*) Use calculus to find the minimum value of L.

(6)

(*c*) Justify, by further differentiation, that the value of L that you have found is a minimum.

(2)

(C2, May 2011 Q8)

12.

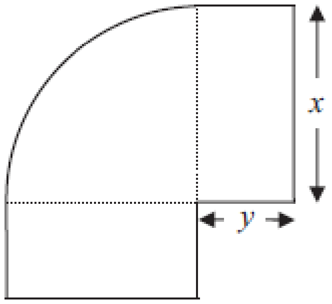

Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is 4m^2,

(*a*) show that

$$y = \frac{16 - \pi x^2}{8x}.$$

(3)

(*b*) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x.$$

(3)

(*c*) Use calculus to find the minimum value of P.

(5)

(*d*) Find the width of each rectangle when the perimeter is a minimum.
Give your answer to the nearest centimetre.

(2)

(C2, Jan 2012 Q8)

13.

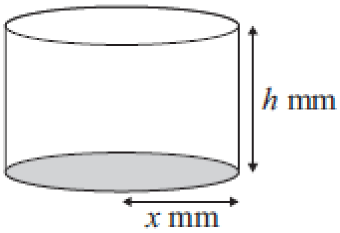

Figure 3

A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and height h mm, as shown in Figure 3.

Given that the volume of each tablet has to be 60 mm^3,

(a) express h in terms of x,

(1)

(b) show that the surface area, A mm^2, of a tablet is given by $A = 2\pi x^2 + \dfrac{120}{x}$.

(3)

The manufacturer needs to minimise the surface area A mm^2, of a tablet.

(c) Use calculus to find the value of x for which A is a minimum.

(5)

(d) Calculate the minimum value of A, giving your answer to the nearest integer.

(2)

(e) Show that this value of A is a minimum.

(2)

(C2, May 2012 Q8)

14. The curve C has equation $y = 6 - 3x - \dfrac{4}{x^3}$, $x \neq 0$.

(a) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$.

(4)

(b) Find the x-coordinate of the other turning point Q on the curve.

(1)

(c) Find $\dfrac{d^2y}{dx^2}$.

(1)

(d) Hence or otherwise, state with justification, the nature of each of these turning points P and Q.

(3)

(C2, Jan 2013 Q8)

15. The curve with equation

$$y = x^2 - 32\sqrt{x} + 20, \qquad x > 0,$$

has a stationary point P.

Use calculus

(*a*) to find the coordinates of P,

(6)

(*b*) to determine the nature of the stationary point P.

(3)

(C2, May 2013 Q9)

16. Using calculus, find the coordinates of the stationary point on the curve with equation

$$y = 2x + 3 + \frac{8}{x^2}, \qquad x > 0$$

(6)

(C2, May 2013_R, Q1)

17. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of $75\pi\,\text{cm}^3$.

The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

(*a*) show that the cost of the polishing, £C, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}.$$

(4)

(*b*) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound.

(5)

(*c*) Justify that the answer that you have obtained in part (*b*) is a minimum.

(1)

(C2, May 2015 Q9)

18.

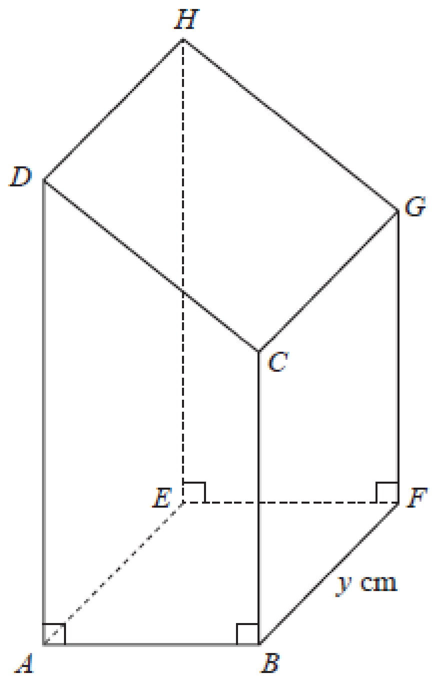

Figure 4

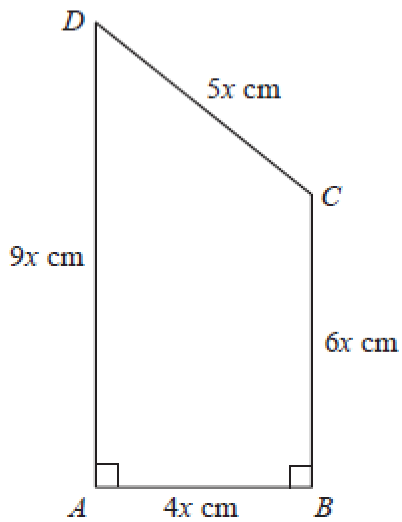

Figure 5

Figure 4 shows a closed letter box *ABFEHGCD*, which is made to be attached to a wall of a house.

The letter box is a right prism of length y cm as shown in Figure 4. The base *ABFE* of the prism is a rectangle. The total surface area of the six faces of the prism is S cm^2.

The cross section *ABCD* of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm and $CD = 5x$ cm as shown in Figure 5.

The angle $DAB = 90°$ and the angle $ABC = 90°$. The volume of the letter box is 9600 cm^3.

(*a*) Show that $y = \frac{320}{x^2}$.

(2)

(*b*) Hence show that the surface area of the letter box, S cm^2, is given by $S = 60x^2 + \frac{7680}{x}$.

(4)

(*c*) Use calculus to find the minimum value of S.

(6)

(*d*) Justify, by further differentiation, that the value of S you have found is a minimum.

(2)

(C2, May 2014 Q10)

19.

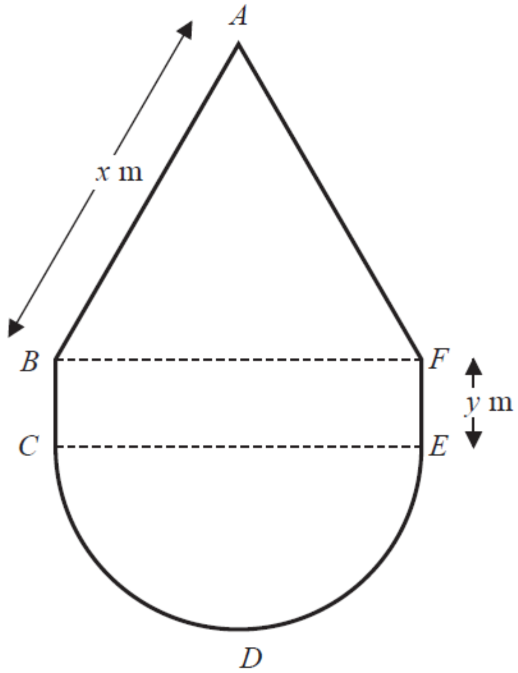

Figure 4

Figure 4 shows the plan of a pool.

The shape of the pool *ABCDEFA* consists of a rectangle *BCEF* joined to an equilateral triangle *BFA* and a semi-circle *CDE*, as shown in Figure 4.

Given that *AB* = *x* metres, *EF* = *y* metres, and the area of the pool is 50 m^2,

(*a*) show that

$$y = \frac{50}{x} - \frac{x}{8}\left(\pi + 2\sqrt{3}\right)$$

(3)

(*b*) Hence show that the perimeter, *P* metres, of the pool is given by

$$P = \frac{100}{x} + \frac{x}{4}\left(\pi + 8 - 2\sqrt{3}\right)$$

(3)

(*c*) Use calculus to find the minimum value of *P*, giving your answer to 3 significant figures.
(5)

(*d*) Justify, by further differentiation, that the value of *P* that you have found is a minimum.
(2)

(C2, May 2014_R, Q9)

20.

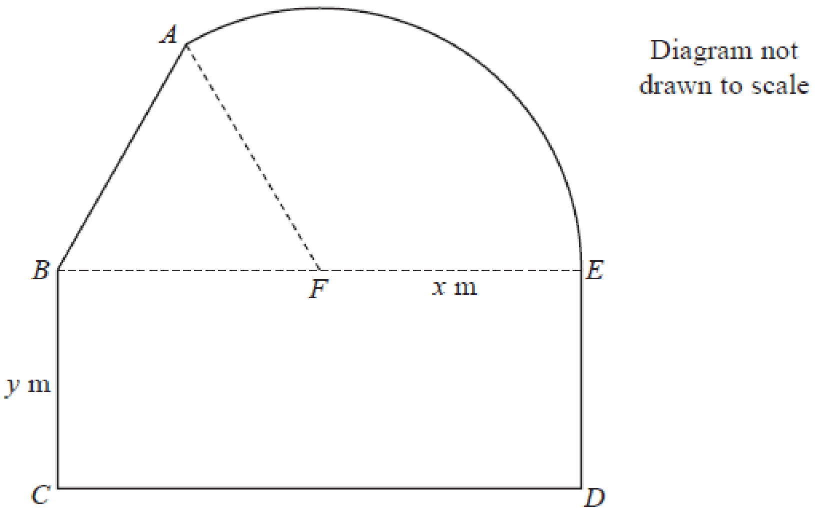

Figure 4

Figure 4 shows a plan view of a sheep enclosure.

The enclosure *ABCDEA*, as shown in Figure 4, consists of a rectangle *BCDE* joined to an equilateral triangle *BFA* and a sector *FEA* of a circle with radius x metres and centre *F*.

The points *B*, *F* and *E* lie on a straight line with $FE = x$ metres and $10 \le x \le 25$.

(*a*) Find, in m^2, the exact area of the sector *FEA*, giving your answer in terms of x, in its simplest form.

(2)

Given that $BC = y$ metres, where $y > 0$, and the area of the enclosure is 1000 m^2,

(*b*) show that

$$y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right).$$

(3)

(*c*) Hence show that the perimeter P metres of the enclosure is given by

$$P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right).$$

(3)

(*d*) Use calculus to find the minimum value of P, giving your answer to the nearest metre.

(5)

(*e*) Justify, by further differentiation, that the value of P you have found is a minimum.

(2)

(C2, May 2016 Q9)

21.

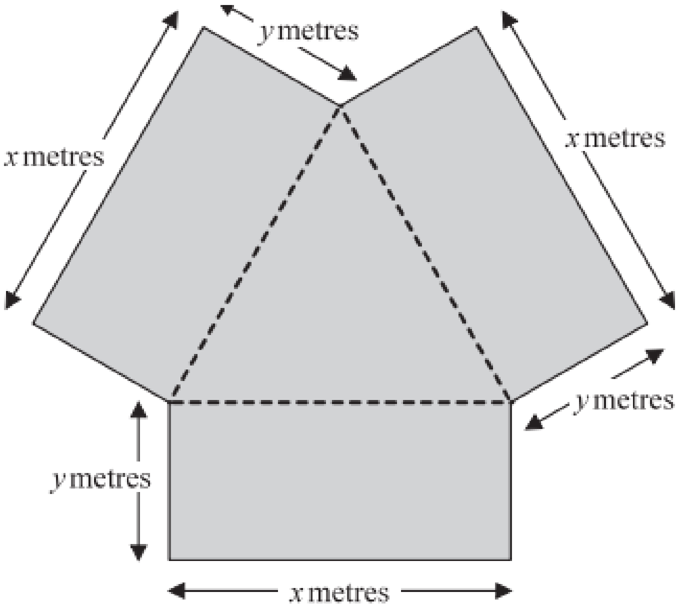

Figure 4

Figure 4 shows a plan view for a flower bed. Its shape is an equilateral triangle of side x metres with three congruent rectangles attached to the triangle along its sides. Each rectangle has length x metres and width y metres, as shown in Figure 4.

Given that the total area of the flower bed is $3\,\text{m}^2$ and that $0 < x < 2.632$ (3d.p.),

(a) show that the perimeter P metres, around the outside of the flower bed, is given by the equation

$$P = 3x + \frac{6}{x} - \frac{\sqrt{3}}{2}x$$

(6)

(b) Use calculus to find the minimum value of P, giving your answer to 3 significant figures.

(5)

(c) Justify, using calculus, that the value you have found in part (b) is a minimum value.

(2)

(C2, May 2019 Q10)

22. The curve C has equation

$$y = \frac{(x-3)(3x-25)}{x}, \quad x > 0$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in a fully simplified form.

(3)

(*b*) Hence find the coordinates of the turning point on the curve C.

(4)

(*c*) Determine whether this turning point is a minimum or maximum, justifying your answer.

(2)

The point P, with x coordinate $2\frac{1}{2}$, lies on the curve C.

(*d*) Find the equation of the normal at P, in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

(IAL C12, Jan 2014 Q13)

23.

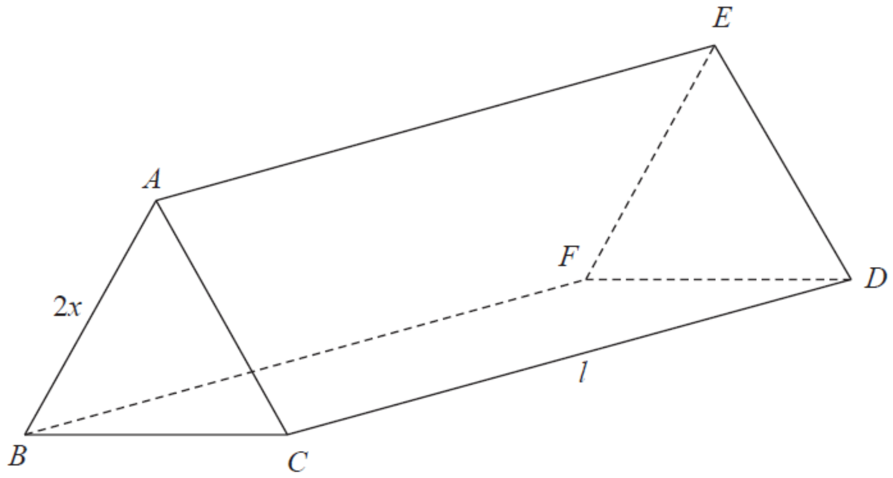

Figure 6

Figure 6 shows a solid triangular prism *ABCDEF* in which $AB = 2x$ cm and $CD = l$ cm.

The cross section *ABC* is an equilateral triangle.

The rectangle *BCDF* is horizontal and the triangles *ABC* and *DEF* are vertical.

The total surface area of the prism is S cm^2 and the volume of the prism is V cm^3.

(*a*) Show that $S = 2x^2\sqrt{3} + 6xl$

(3)

Given that $S = 960$,

(*b*) show that $V = 160x\sqrt{3} - x^3$

(5)

(*c*) Use calculus to find the maximum value of V, giving your answer to the nearest integer.

(5)

(*d*) Justify that the value of V found in part (*c*) is a maximum.

(2)

(IAL C12, May 2014 Q14)

24. [*In this question you may assume the formula for the area of a circle and the following formulae:*

a ***sphere*** *of radius r has volume* $V = \frac{4}{3}\pi r^3$ *and surface area* $S = 4\pi r^2$

a ***cylinder*** *of radius r and height h has volume* $V = \pi r^2 h$ *and curved surface area* $S = 2\pi rh$]

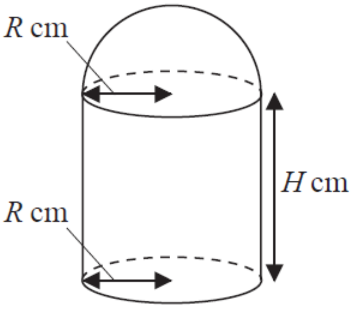

Figure 5

Figure 5 shows the model for a building. The model is made up of three parts. The roof is modelled by the curved surface of a hemisphere of radius R cm. The walls are modelled by the curved surface of a circular cylinder of radius R cm and height H cm. The floor is modelled by a circular disc of radius R cm. The model is made of material of negligible thickness, and the walls are perpendicular to the base.

It is given that the volume of the model is 800π cm^3 and that $0 < R < 10.6$

(*a*) Show that

$$H = \frac{800}{R^2} - \frac{2}{3}R$$

(2)

(*b*) Show that the surface area, A cm^2, of the model is given by

$$A = \frac{5\pi R^2}{3} + \frac{1600\pi}{R}$$

(3)

(*c*) Use calculus to find the value of R, to 3 significant figures, for which A is a minimum.

(5)

(*d*) Prove that this value of R gives a minimum value for A.

(2)

(*e*) Find, to 3 significant figures, the value of H which corresponds to this value for R.

(1)

(IAL C12, May 2015 Q16)

25. The curve C has equation

$$y = 12x^{\frac{5}{4}} - \frac{5}{18}x^2 - 1000, \qquad x > 0$$

(*a*) Find $\frac{dy}{dx}$

(2)

(*b*) Hence find the coordinates of the stationary point on C.

(5)

(*c*) Use $\frac{d^2y}{dx^2}$ to determine the nature of this stationary point. **(3)**

(IAL C12, Jan 2016 Q10)

26.

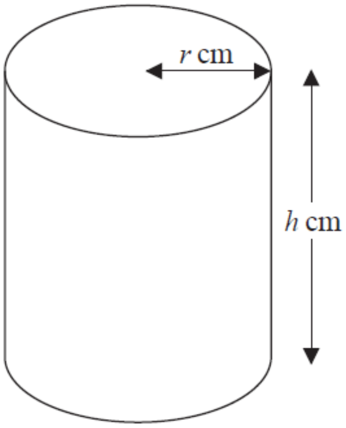

Figure 5

Figure 5 shows a design for a water barrel.

It is in the shape of a right circular cylinder with height h cm and radius r cm.

The barrel has a base but has no lid, is open at the top and is made of material of negligible thickness.

The barrel is designed to hold 60 000 cm^3 of water when full.

(*a*) Show that the total external surface area, S cm^2, of the barrel is given by the formula

$$S = \pi r^2 + \frac{120\,000}{r}$$

(3)

(*b*) Use calculus to find the minimum value of S, giving your answer to 3 significant figures.

(6)

(*c*) Justify that the value of S you found in part (*b*) is a minimum.

(2)

(IAL C12, May 2016 Q15)

27.

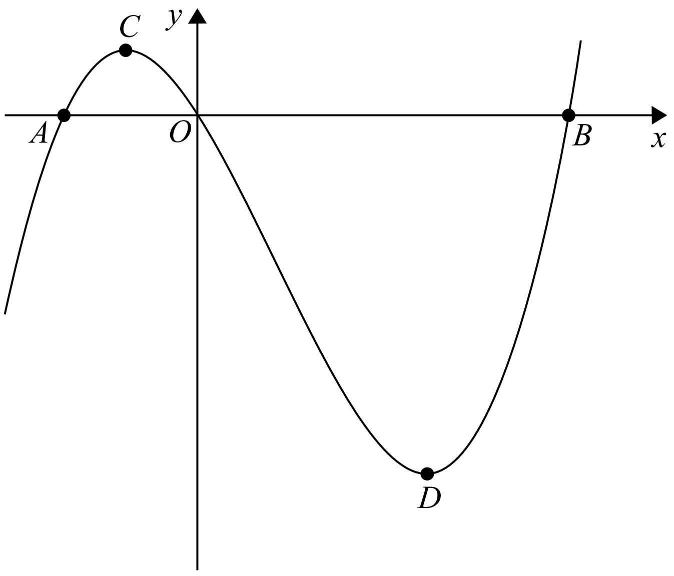

Diagram not drawn to scale

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{x^3 - 9x^2 - 81x}{27}$$

The curve crosses the x-axis at the point A, the point B and the origin O.
The curve has a maximum turning point at C and a minimum turning point at D.

(a) Use algebra to find exact values for the x coordinates of the points A and B.

(4)

(b) Use calculus to find the coordinates of the points C and D.

(6)

The graph of $y = \mathrm{f}(x + a)$, where a is a constant, has its minimum turning point on the y-axis.

(c) Write down the value of a.

(1)

(IAL C12, Oct 2016 Q12)

28. Given $y = \frac{x^3}{3} - 2x^2 + 3x + 5$

(a) find $\frac{\mathrm{d}y}{\mathrm{d}x}$, simplifying each term.

(3)

(b) Hence find the set of values of x for which $\frac{\mathrm{d}y}{\mathrm{d}x} > 0$

(4)

(IAL C12, Jan 2017 Q1)

29.

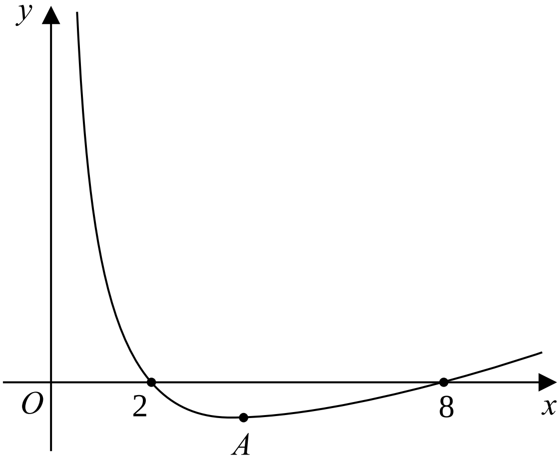

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{8}{x} + \frac{1}{2}x - 5, \qquad 0 < x \leq 12$$

The curve crosses the x-axis at (2, 0) and (8, 0) and has a minimum point at A.

(a) Use calculus to find the coordinates of point A.

(5)

(b) State
(i) the roots of the equation $2\mathrm{f}(x) = 0$

(ii) the coordinates of the turning point on the curve $y = \mathrm{f}(x) + 2$

(iii) the roots of the equation $\mathrm{f}(4x) = 0$

(3)

(IAL C12, Jan 2017 Q9)

30. The curve C has equation $y = 4x\sqrt{x} + \frac{48}{\sqrt{x}} - \sqrt{8}, \qquad x > 0$

(a) Find, simplifying each term,

(i) $\frac{\mathrm{d}y}{\mathrm{d}x}$

(ii) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

(5)

(b) Use part (a) to find the exact coordinates of the stationary point of C.

(5)

(c) Determine whether the stationary point of C is a maximum or minimum, giving a reason for your answer.

(2)

(IAL C12, May 2017 Q4)

31.

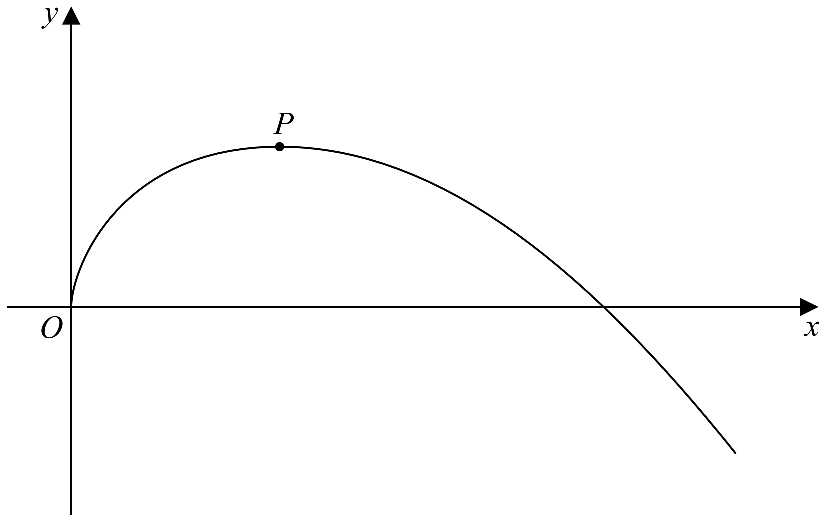

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 27\sqrt{x} - 2x^2, \qquad x \in \mathbb{R}, x > 0$$

(*a*) Find $\frac{dy}{dx}$

(3)

The curve has a maximum turning point *P*, as shown in Figure 2.

(*b*) Use the answer to part (*a*) to find the exact coordinates of *P*.

(5)

(IAL C12, Oct 2017 Q5)

32.

Given that

$$y = 5x^2 + \frac{1}{2x} + \frac{2x^4 - 8}{5\sqrt{x}} \qquad x > 0$$

find $\frac{dy}{dx}$, giving each term in its simplest form.

(6)

(IAL C12, May 2019 Q4)

33.

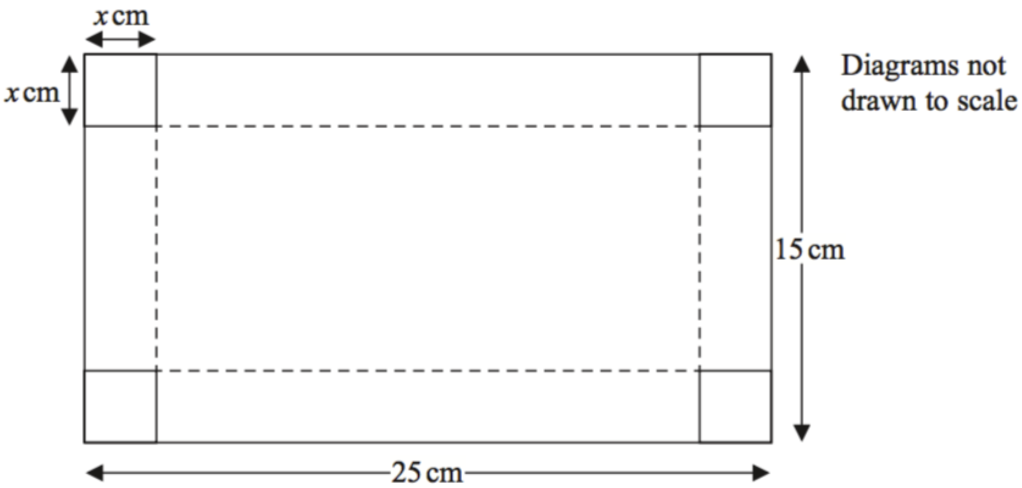

Figure 1

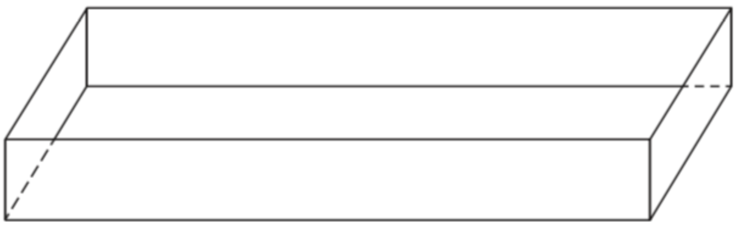

Figure 2

Figure 1 shows a rectangular sheet of metal of negligible thickness, which measures 25 cm by 15 cm. Squares of side x cm are cut from each corner of the sheet and the remainder is folded along the dotted lines to make an open cuboid box, as shown in Figure 2.

(*a*) Show that the volume, V cm^3, of the box is given by

$$V = 4x^3 - 80x^2 + 375x$$

(2)

(*b*) Use calculus to find the value of x, to 3 significant figures, for which the volume of the box is a maximum.

(4)

(*c*) Justify that this value of x gives a maximum value for V.

(2)

(*d*) Find, to 3 significant figures, the maximum volume of the box.

(2)

(IAL C12, Jan 2018 Q7)

34.

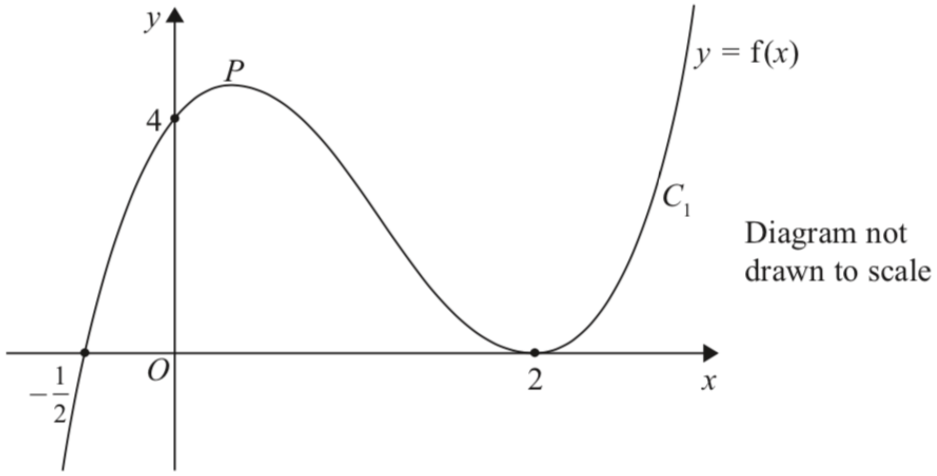

Figure 2

Figure 2 shows a sketch of the curve C_1 with equation $y = \text{f}(x)$ where

$$\text{f}(x) = (x - 2)^2(2x + 1), \qquad x \in \mathbb{R}$$

The curve crosses the x-axis at $\left(-\frac{1}{2}, 0\right)$, touches it at (2, 0) and crosses the y-axis at (0, 4). There is a maximum turning point at the point marked P.

(*a*) Use $\text{f}'(x)$ to find the exact coordinates of the turning point P.

(7)

A second curve C_2 has equation $y = \text{f}(x + 1)$.

(*b*) Write down an equation of the curve C_2
You may leave your equation in a factorised form.

(1)

(*c*) Use your answer to part (*b*) to find the coordinates of the point where the curve C_2 meets the y-axis.

(2)

(*d*) Write down the coordinates of the two turning points on the curve C_2

(2)

(*e*) Sketch the curve C_2, with equation $y = \text{f}(x + 1)$, giving the coordinates of the points where the curve crosses or touches the x-axis.

(3)

(IAL C12, May 2018 Q14)

35.

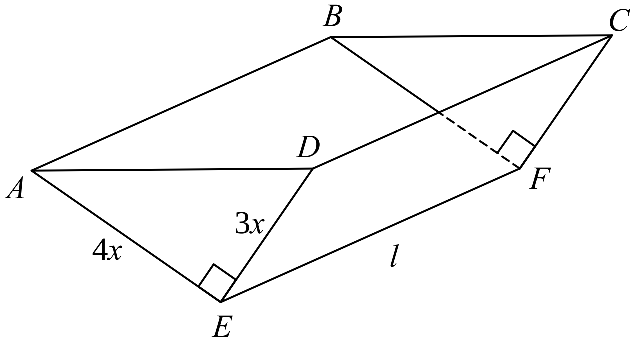

Figure 4

Figure 4 shows the design for a container in the shape of a hollow triangular prism.

The container **is open at the top**, which is labelled *ABCD*.

The sides of the container, *ABFE* and *DCFE*, are rectangles.

The ends of the container, *ADE* and *BCF*, are congruent right-angled triangles, as shown in Figure 4.

The ends of the container are vertical and the edge *EF* is horizontal.

The edges *AE*, *DE* and *EF* have lengths $4x$ metres, $3x$ metres and l metres respectively.

Given that the container has a capacity of 0.75 m^3 and is made of material of negligible thickness,

(*a*) show that the internal surface area of the container, S m^2, is given by

$$S = 12x^2 + \frac{7}{8x}$$

(5)

(*b*) Use calculus to find the value of x, for which S is a minimum.
Give your answer to 3 significant figures.

(5)

(*c*) Justify that the value of x found in part (*b*) gives a minimum value for S.

(2)

Using the value of x found in part (b), find to 2 decimal places,

(*d*) (i) the length of the edge *AD*,

(ii) the length of the edge *CD*.

(4)

(IAL C12, Jan 2019 Q16)

36.

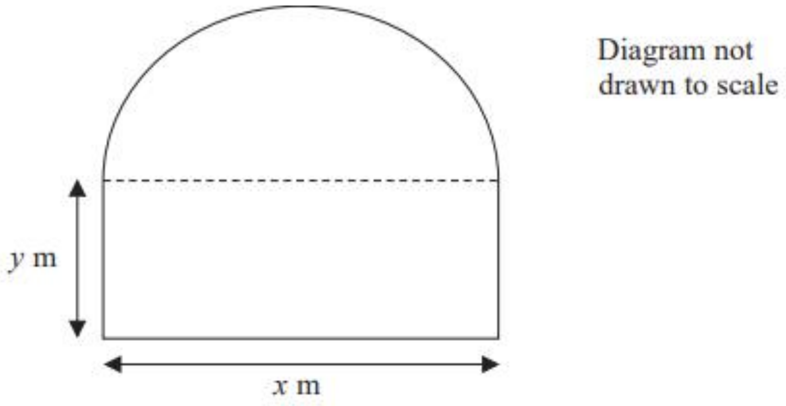

Figure 3

Figure 3 shows the plan view of a garden. The shape of this garden consists of a rectangle joined to a semicircle.

The rectangle has length x metres and width y metres.

The area of the garden is $100\,\text{m}^2$.

(a) Show that the perimeter, P metres, of the garden is given by

$$P = \frac{1}{4}x(4 + \pi) + \frac{200}{x} \qquad x > 0$$

(4)

(b) Use calculus to find the exact value of x for which the perimeter of the garden is a minimum.

(3)

(c) Justify that the value of x found in part (b) gives a minimum value for P.

(2)

(d) Find the minimum perimeter of the garden, giving your answer in metres to one decimal place.

(2)

(IAL C12, May 2019 Q15)

37. A curve has equation

$$y = 16x\sqrt{x} - 3x^2 - 78 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

(b) Hence find the equation of the normal to the curve at the point where $x = 4$, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

(IAL C12, Oct 2019 Q4)

38.

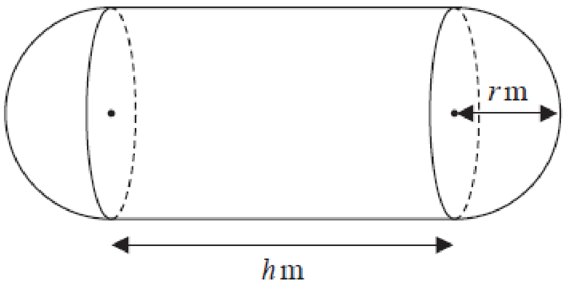

Figure 4

[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $42\pi r^2$]

A manufacturer wishes to produce a storage tank.

The tank is modelled in the shape of a hollow circular cylinder with a hemispherical shell at each end, as shown in Figure 4.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and length h metres and each hemisphere has radius r metres.

The volume of the tank will be 5 m^3.

(a) Show that, according to the model, the surface area of the tank A m^2 is given by

$$A = \frac{10}{r} + \frac{4}{3}\pi r^2$$

(4)

The manufacturer wishes to find the minimum value of A.

(b) Find the value of A when $\frac{dA}{dr} = 0$

(6)

(c) Justify, by further differentiation, that the value of A found in part (b) is a minimum.

(2)

For the minimum value of A,

(d) find the value of h.

(2)

(IAL C12, Oct 2019 Q15)