Edexcel

Pure Mathematics

Year 2

Differentiations.

Past paper questions from Core Maths 3 and IAL C34

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Past paper questions from

Edexcel Core Maths 3 and IAL C34.

From June 2005 to Nov 2019.

Please check the Edexcel website for the solutions.

1. (*a*) Differentiate with respect to x

(i) $3\sin^2 x + \sec 2x$, **(3)**

(ii) $\{x + \ln(2x)\}^3$.

(3)

Given that $y = \dfrac{5x^2 - 10x + 9}{(x-1)^2}$, $x \neq 1$,

(*b*) show that $\dfrac{dy}{dx} = -\dfrac{8}{(x-1)^3}$. **(6)**

[2005 June Q2]

2. The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$. The x-coordinate of P is 3.

Find an equation of the normal to the curve at the point P in the form $y = ax + b$, where a and b are constants.

(5)

[2006 Jan Q3]

3. (*a*) Differentiate with respect to x

(i) x^2e^{3x+2},

(4)

(ii) $\dfrac{\cos(2x^3)}{3x}$.

(4)

(*b*) Given that $x = 4\sin(2y + 6)$, find $\dfrac{dy}{dx}$ in terms of x.

(5)

[2006 Jan Q4]

4. Differentiate, with respect to x,

(a) $e^{3x} + \ln 2x$,

(3)

(*b*) $(5 + x^2)^{\frac{3}{2}}$.

(3)

[2006June Q2]

5. The curve C has equation $x = 2 \sin y$.

(a) Show that the point $P\left(\sqrt{2}, \frac{\pi}{4}\right)$ lies on C.

(1)

(b) Show that $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ at P.

(4)

(c) Find an equation of the normal to C at P. Give your answer in the form $y = mx + c$, where m and c are exact constants. **(4)**

[2007 Jan Q3]

6. (i) The curve C has equation $y = \frac{x}{9 + x^2}$.

Use calculus to find the coordinates of the turning points of C. **(6)**

(ii) Given that $y = (1 + e^{2x})^{\frac{3}{2}}$, find the value of $\frac{dy}{dx}$ at $x = \frac{1}{2} \ln 3$.

(5)

[2007 Jan Q4]

7. A curve C has equation $y = x^2 e^x$

(a) Find $\frac{dy}{dx}$, using the product rule for differentiation.

(3)

(b) Hence find the coordinates of the turning points of C. **(3)**

(c) Find $\frac{d^2y}{dx^2}$.

(2)

(d) Determine the nature of each turning point of the curve C.

(2)

[2007June Q3]

8. A curve C has equation

$$y = e^{2x} \tan x, \quad x \neq (2n + 1)\frac{\pi}{2}.$$

(a) Show that the turning points on C occur where $\tan x = -1$.

(6)

(b) Find an equation of the tangent to C at the point where $x = 0$. **(2)**

[2008 Jan Q2]

9. The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y-coordinate of P is 8.

(a) Find, in terms of ln 2, the x-coordinate of P. **(2)**

(b) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found. **(4)**

[2008June Q1]

10. (a) Differentiate with respect to x,

(i) $e^{3x}(\sin x + 2\cos x)$, **(3)**

(ii) $x^3 \ln(5x + 2)$. **(3)**

Given that $y = \dfrac{3x^2 + 6x - 7}{(x+1)^2}$, $x \neq -1$,

(b) show that $\dfrac{dy}{dx} = \dfrac{20}{(x+1)^3}$. **(5)**

(c) Hence find $\dfrac{d^2y}{dx^2}$ and the real values of x for which $\dfrac{d^2y}{dx^2} = -\dfrac{15}{4}$. **(3)**

[2008June Q6]

11. (a) Find the value of $\dfrac{dy}{dx}$ at the point where $x = 2$ on the curve with equation

$$y = x^2\sqrt{(5x - 1)}.$$

(6)

(b) Differentiate $\dfrac{\sin 2x}{x^2}$ with respect to x. **(4)**

[2009 Jan Q1]

12. Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \dfrac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found. **(6)**

[2009 Jan Q4]

13. $$f(x) = \frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3}.$$

(*a*) Express f(*x*) as a single fraction in its simplest form. **(4)**

(*b*) Hence show that $f'(x) = \frac{2}{(x-3)^2}$. **(3)**

[2009 Jan Q2]

14. (i) Differentiate with respect to *x*

(*a*) $x^2 \cos 3x$, **(3)**

(*b*) $\frac{\ln(x^2+1)}{x^2+1}$. **(4)**

(ii) A curve *C* has the equation

$$y = \sqrt{(4x+1)}, \quad x > -\tfrac{1}{4}, \quad y > 0.$$

The point *P* on the curve has *x*-coordinate 2. Find an equation of the tangent to *C* at *P* in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers. **(6)**

[2009June Q4]

15. (i) Given that $y = \frac{\ln(x^2+1)}{x}$, find $\frac{dy}{dx}$. **(4)**

(ii) Given that $x = \tan y$, show that $\frac{dy}{dx} = \frac{1}{1+x^2}$. **(5)**

[2010 Jan Q4]

16. (*a*) By writing sec *x* as $\frac{1}{\cos x}$, show that $\frac{d(\sec x)}{dx} = \sec x \tan x$. **(3)**

Given that $y = e^{2x} \sec 3x$,

(*b*) find $\frac{dy}{dx}$. **(4)**

The curve with equation $y = e^{2x} \sec 3x$, $-\frac{\pi}{6} < x < \frac{\pi}{6}$, has a minimum turning point at (*a*, *b*).

(*c*) Find the values of the constants *a* and *b*, giving your answers to 3 significant figures. **(4)**

[2010 Jan Q7]

17. A curve C has equation

$$y = \frac{3}{(5-3x)^2}, \qquad x \neq \frac{5}{3}.$$

The point P on C has x-coordinate 2.
Find an equation of the normal to C at P in the form $ax + by + c = 0$, where a, b and c are integers. **(7)**

[2010 June Q2]

18.

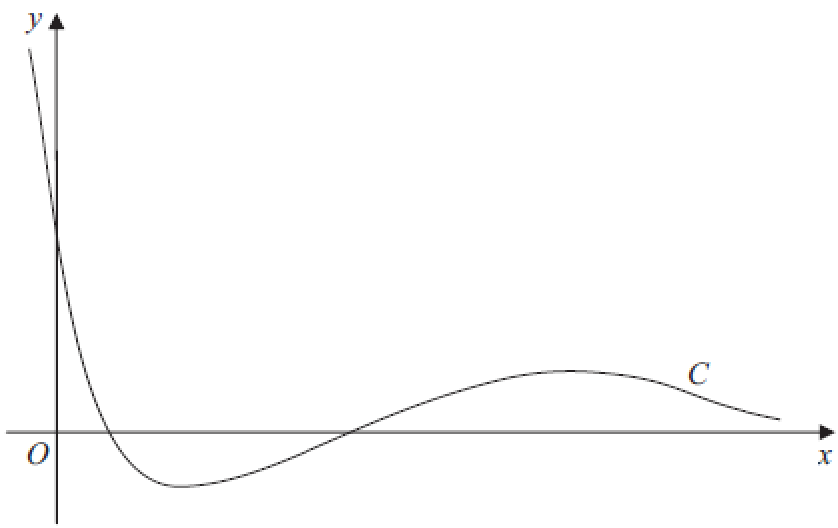

Figure 1

Figure 1 shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

(a) Find the coordinates of the point where C crosses the y-axis.

(1)

(b) Show that C crosses the x-axis at $x = 2$ and find the x-coordinate of the other point where C crosses the x-axis.

(3)

(c) Find $\frac{dy}{dx}$.

(3)

(d) Hence find the exact coordinates of the turning points of C.

(5)

[2010 June Q5]

19. The curve C has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}.$$

(a) Show that

$$\frac{dy}{dx} = \frac{6\sin 2x + 4\cos 2x + 2}{(2 + \cos 2x)^2}$$

(4)

(b) Find an equation of the tangent to C at the point on C where $x = \frac{\pi}{2}$.

Write your answer in the form $y = ax + b$, where a and b are exact constants.

(4)

[2011 Jan Q7]

20. Given that

$$\frac{d}{dx}(\cos x) = -\sin x,$$

(a) show that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

(3)

Given that $x = \sec 2y$,

(b) find $\frac{dx}{dy}$ in terms of y.

(2)

(c) Hence find $\frac{dy}{dx}$ in terms of x.

(4)

[2010 Jan Q8]

21. Differentiate with respect to x

(a) $\ln(x^2 + 3x + 5)$,

(2)

(b) $\frac{\cos x}{x^2}$.

(3)

[2011 June Q1]

22. $$\mathrm{f}(x) = \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{x^2-9}, \qquad x \neq \pm 3,\ x \neq -\frac{1}{2}.$$

(*a*) Show that

$$\mathrm{f}(x) = \frac{5}{(2x+1)(x-3)}.$$

(5)

The curve C has equation $y = \mathrm{f}(x)$. The point $P\left(-1, -\frac{5}{2}\right)$ lies on C.

(*b*) Find an equation of the normal to C at P.

(8)

[2011 June Q7]

23. Differentiate with respect to x, giving your answer in its simplest form,

(*a*) $x^2 \ln(3x)$,

(4)

(*b*) $\dfrac{\sin 4x}{x^3}$.

(5)

[2012 Jan Q1]

24. The point P is the point on the curve $x = 2\tan\left(y + \frac{\pi}{12}\right)$ with y-coordinate $\frac{\pi}{4}$.

Find an equation of the normal to the curve at P.

(7)

[2012 Jan Q4]

25.

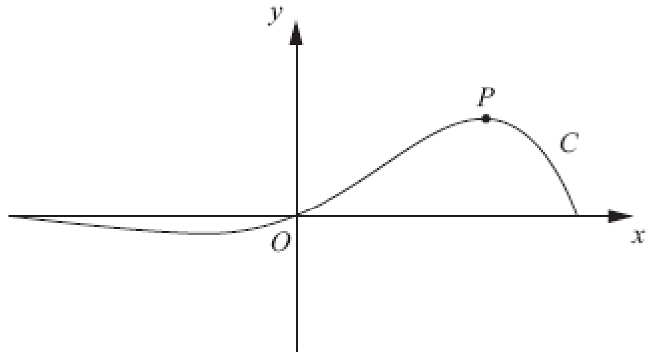

Figure 1

Figure 1 shows a sketch of the curve C which has equation

$$y = e^{x\sqrt{3}} \sin 3x, \quad -\frac{\pi}{3} \leq x \leq \frac{\pi}{3}.$$

(a) Find the x-coordinate of the turning point P on C, for which $x > 0$. Give your answer as a multiple of π.

(6)

(b) Find an equation of the normal to C at the point where $x = 0$.

(3)

[2012 June Q3]

26. (a) Differentiate with respect to x,

(i) $x^{\frac{1}{2}} \ln (3x)$,

(ii) $\dfrac{1-10x}{(2x-1)^5}$, giving your answer in its simplest form.

(6)

(b) Given that $x = 3 \tan 2y$ find $\dfrac{dy}{dx}$ in terms of x.

(5)

[2012 June Q7]

27. The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

(*a*) the value of w,

(2)

(*b*) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants.

(5)
[2013 Jan Q1]

28. (i) Differentiate with respect to x

(*a*) $y = x^3 \ln 2x$,

(*b*) $y = (x + \sin 2x)^3$.

(6)

Given that $x = \cot y$,

(ii) show that $\dfrac{dy}{dx} = \dfrac{-1}{1+x^2}$.

(5)
[2013 Jan Q5]

29. $$\mathrm{h}(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \quad x \geq 0.$$

(*a*) Show that $\mathrm{h}(x) = \dfrac{2x}{x^2+5}$.

(4)

(*b*) Hence, or otherwise, find $\mathrm{h}'(x)$ in its simplest form.

(3)

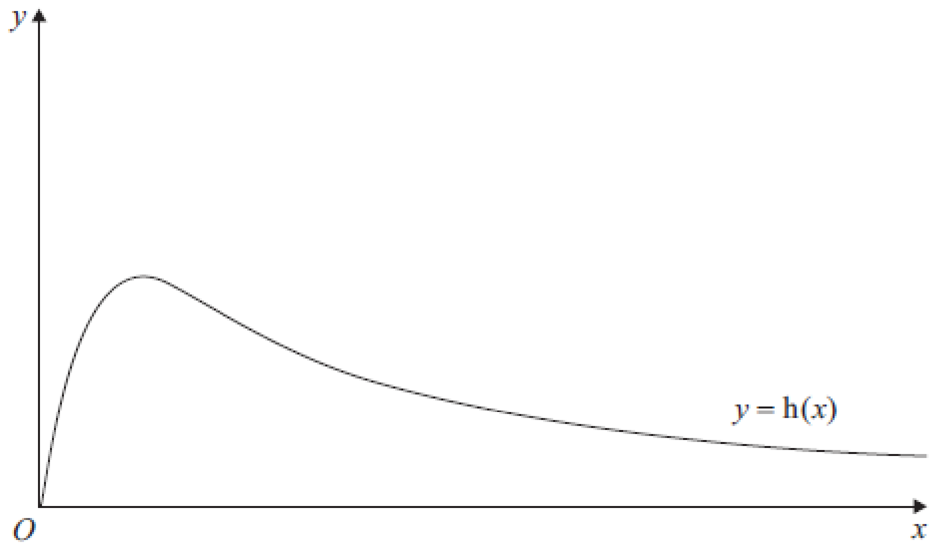

Figure 2

Figure 2 shows a graph of the curve with equation $y = \mathrm{h}(x)$.

(*c*) Calculate the range of $\mathrm{h}(x)$.

(5)

[2013 Jan Q7]

30. Given that

$$x = \sec^2 3y, \qquad 0 < y < \frac{\pi}{6}$$

(*a*) find $\dfrac{dx}{dy}$ in terms of y. **(2)**

(*b*) Hence show that

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$ **(4)**

(*c*) Find an expression for $\dfrac{d^2y}{dx^2}$ in terms of x. Give your answer in its simplest form. **(4)**

[2013 June Q5]

31.

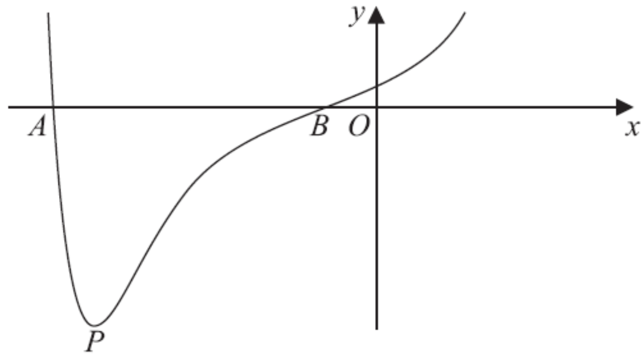

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (x^2 + 3x + 1)e^{x^2}$$

The curve cuts the x-axis at points A and B as shown in Figure 2.

(a) Calculate the x-coordinate of A and the x-coordinate of B, giving your answers to 3 decimal places.

(2)

(b) Find $\mathrm{f}'(x)$.

(3)

The curve has a minimum turning point P as shown in Figure 2.

(c) Show that the x-coordinate of P is the solution of

$$x = -\frac{3(2x^2+1)}{2(x^2+2)}$$

(3)

[2013_R June Q5]

32. The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{4x+1}{x-2}, \qquad x > 2$$

(a) Show that

$$\mathrm{f}'(x) = \frac{-9}{(x-2)^2}$$

(3)

Given that P is a point on C such that $\mathrm{f}'(x) = -1$,

(b) find the coordinates of P.

(3)

[2014 June Q1]

33. The curve C has equation $x = 8y \tan 2y$.

The point P has coordinates $\left(\pi, \frac{\pi}{8}\right)$.

(a) Verify that P lies on C.

(1)

(b) Find the equation of the tangent to C at P in the form $ay = x + b$, where the constants a and b are to be found in terms of π.

(7)

[2014 June Q3]

34. (i) Given that

$$x = \sec^2 2y, \quad 0 < y < \frac{\pi}{4}$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4x\sqrt{(x-1)}}$$

(4)

(ii) Given that

$$y = \left(x^2 + x^3\right)\ln 2x$$

find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = \frac{\mathrm{e}}{2}$, giving your answer in its simplest form.

(5)

(iii) Given that

$$\mathrm{f}(x) = \frac{3\cos x}{(x+1)^{\frac{1}{3}}}, \qquad x \neq -1$$

show that

$$\mathrm{f}'(x) = \frac{\mathrm{g}(x)}{(x+1)^{\frac{4}{3}}}, \qquad x \neq -1$$

where $\mathrm{g}(x)$ is an expression to be found.

(3)

[2014_R June Q4]

35. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2.$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2}\right)$, where p is a constant,

(a) find the exact value of p.

(1)

The tangent to the curve at P cuts the y-axis at the point A.

(b) Use calculus to find the coordinates of A.

(6)

[2015 June Q5]

36. Given that k is a **negative** constant and that the function $f(x)$ is defined by

$$f(x) = 2 - \frac{(x-5k)(x-k)}{x^2 - 3kx + 2k^2}, \qquad x \geq 0,$$

(a) show that $f(x) = \dfrac{x+k}{x-2k}$.

(3)

(b) Hence find $f'(x)$, giving your answer in its simplest form.

(3)

(c) State, with a reason, whether $f(x)$ is an increasing or a decreasing function. Justify your answer.

(2)

[2015 June Q9]

37.

$$y = \frac{4x}{x^2 + 5}.$$

(a) Find $\dfrac{dy}{dx}$, writing your answer as a single fraction in its simplest form.

(4)

(b) Hence find the set of values of x for which $\dfrac{dy}{dx} < 0$.

(3)

[2016 June Q2]

38. (i) Find, using calculus, the x coordinate of the turning point of the curve with equation

$$y = e^{3x} \cos 4x, \quad \frac{\pi}{4} \le x < \frac{\pi}{2}.$$

Give your answer to 4 decimal places.

(5)

(ii) Given $x = \sin^2 2y$, $0 < y < \frac{\pi}{4}$, find $\frac{dy}{dx}$ as a function of y.

Write your answer in the form

$$\frac{dy}{dx} = p \operatorname{cosec}(qy), \quad 0 < y < \frac{\pi}{4},$$

where p and q are constants to be determined.

(5)

[2016 June Q5]

39. $$f(x) = \frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6}, \quad x > 2, \quad x \in \mathbb{R}.$$

(*a*) Given that

$$\frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6} \equiv x^2 + A + \frac{B}{x - 2},$$

find the values of the constants A and B.

(4)

(*b*) Hence or otherwise, using calculus, find an equation of the normal to the curve with equation $y = f(x)$ at the point where $x = 3$.

(5)

[2016 June Q6]

40. (i) Given $y = 2x(x^2 - 1)^5$, show that

(*a*) $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{g}(x)(x^2 - 1)^4$ where $\mathrm{g}(x)$ is a function to be determined.

(4)

(*b*) Hence find the set of values of x for which $\frac{\mathrm{d}y}{\mathrm{d}x} \geqslant 0$

(2)

(ii) Given

$$x = \ln(\sec 2y), \qquad 0 < y < \frac{\pi}{4}$$

find $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a function of x in its simplest form.

(4)

[2017 June Q7]

41.

Given $y = 2x(3x - 1)^5$,

(a) find $\frac{\mathrm{d}y}{\mathrm{d}x}$, giving your answer as a single fully factorised expression.

(4)

(b) Hence find the set of values of x for which $\frac{\mathrm{d}y}{\mathrm{d}x} \leqslant 0$

(2)

[2018 June Q1]

42.

The curve C has equation $y = \frac{\ln(x^2 + 1)}{x^2 + 1}$, $x \in \mathbb{R}$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a single fraction, simplifying your answer.

(3)

(b) Hence find the exact coordinates of the stationary points of C.

(6)

[2018 June Q7]

43.

(a) By writing $\sec\theta = \dfrac{1}{\cos\theta}$, show that $\dfrac{\mathrm{d}}{\mathrm{d}\theta}(\sec\theta) = \sec\theta\tan\theta$

(2)

(b) Given that

$$x = \mathrm{e}^{\sec y} \qquad x > \mathrm{e}, \quad 0 < y < \frac{\pi}{2}$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x\sqrt{\mathrm{g}(x)}}, \qquad x > \mathrm{e}$$

where $\mathrm{g}(x)$ is a function of $\ln x$.

(5)

[2018 June Q8]

44. (i)

$$y = \frac{(2x-1)^3}{(3x-2)} \qquad x \neq \frac{2}{3}$$

(*a*) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ writing your answer as a single fraction in simplest form.

(4)

(*b*) Hence find the set of values of x for which $\dfrac{\mathrm{d}y}{\mathrm{d}x} \geq 0$

(2)

(ii) Given

$$y = \ln(1 + \cos 2x) \qquad x \neq (2n+1)\frac{\pi}{2} \quad n \in \square$$

show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = C\tan x$, where C is a constant to be determined.

(You may assume the double angle formulae.)

(4)

[2019 June Q2]

45. The curve C has equation

$$x = \frac{1}{1+\cot y} \qquad 0 < y < \frac{3\pi}{4}$$

(a) Show that

$$\frac{dx}{dy} = 2x^2 - 2x + 1$$

(5)

The point A with y coordinate $\arctan\left(\frac{1}{3}\right)$ lies on C.

(b) Find the x coordinate of A.

(1)

(c) Find the value of $\frac{dy}{dx}$ at A.

(2)

[2019 June Q9]

46. Given that

$$y = \frac{\cos 2\theta}{1+\sin 2\theta}, \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{dy}{d\theta} = \frac{\alpha}{1+\sin 2\theta}, \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where α is a constant to be determined.

(4)

[2014 June, IAL Q3]

47. (a) Use the identity for $\sin(A + B)$ to prove that

$$\sin 2A \equiv 2\sin A\cos A$$

(2)

(b) Show that

$$\frac{d}{dx}\left[\ln\left(\tan\left(\tfrac{1}{2}x\right)\right)\right] = \operatorname{cosec} x$$

(4)

A curve C has the equation

$$y = \ln\left(\tan\left(\tfrac{1}{2}x\right)\right) - 3\sin x, \qquad 0 < x < \pi$$

(c) Find the x coordinates of the points on C where $\frac{dy}{dx} = 0$.

Give your answers to 3 decimal places.

(6)

[2014 June, IAL Q10]

48.

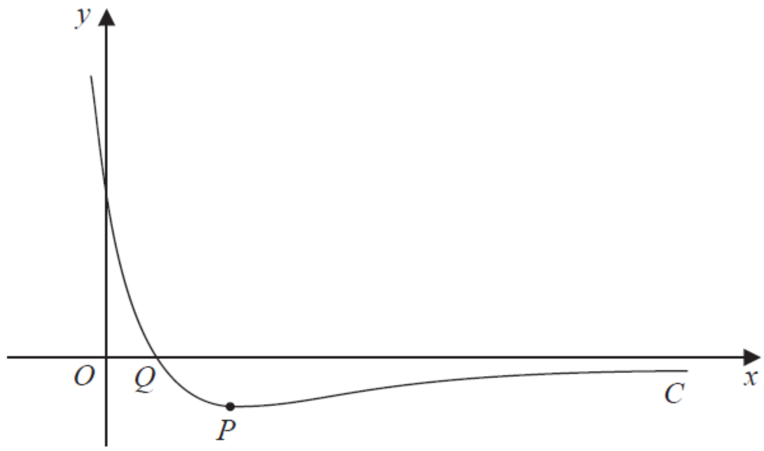

Figure 2 shows a sketch of part of the curve C with equation

$$y = e^{a-3x} - 3e^{-x}, \qquad x \in \mathbb{R}$$

where a is a constant and $a > \ln 4$.

The curve C has a turning point P and crosses the x-axis at the point Q as shown in Figure 2.

(a) Find, in terms of a, the coordinates of the point P.

(6)

(b) Find, in terms of a, the x coordinate of the point Q.

(3)

(c) Sketch the curve with equation

$$y = \left| e^{a-3x} - 3e^{-x} \right|, \qquad x \in \mathbb{R}, a > \ln 4$$

Show on your sketch the exact coordinates, in terms of a, of the points at which the curve meets or cuts the coordinate axes.

(3)

[2014 June, IAL Q11]

49. The curve C has equation

$$y = \frac{3x-2}{(x-2)^2}, \quad x \neq 2$$

The point P on C has x coordinate 3.

Find an equation of the normal to C at the point P in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

[2015 Jan, IAL Q1]

50.

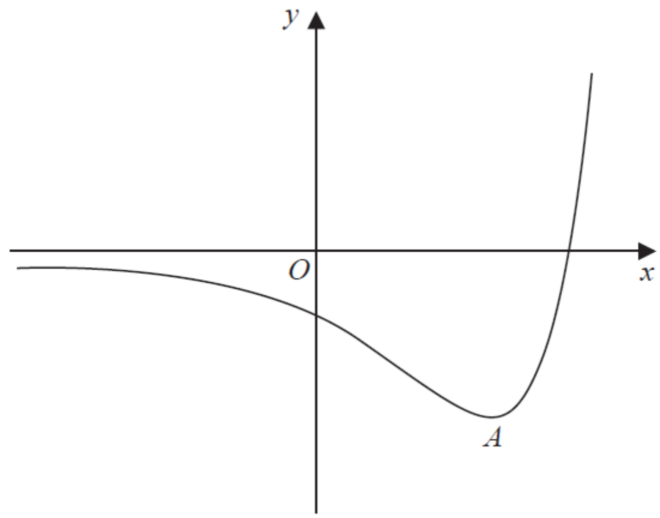

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (2x - 5)\mathrm{e}^{x}, \qquad x \in \square$$

The curve has a minimum turning point at A.

(a) Use calculus to find the exact coordinates of A.

(5)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has exactly two roots,

(b) state the range of possible values of k.

(2)

(c) Sketch the curve with equation $y = |\mathrm{f}(x)|$.

Indicate clearly on your sketch the coordinates of the points at which the curve crosses
or meets the axes.

(3)

[2015 June, IAL Q3]

51. $$g(x)=\frac{x^4+x^3-7x^2+8x-48}{x^2+x-12}, \qquad x>3, \quad x\in\mathbb{R}$$

(a) Given that

$$\frac{x^4+x^3-7x^2+8x-48}{x^2+x-12}\equiv x^2+A+\frac{B}{x-3}$$

find the values of the constants A and B.

(4)

(b) Hence, or otherwise, find the equation of the tangent to the curve with equation $y = \mathrm{g}(x)$ at the point where $x = 4$. Give your answer in the form $y = mx + c$, where m and c are constants to be determined.

(5)

[2016 June, IAL Q4]

52. (i) Differentiate $y = 5x^2 \ln 3x$, $\qquad x>0$

(2)

(ii) Given that

$$y=\frac{x}{\sin x+\cos x},$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(1+x)\sin x+(1-x)\cos x}{1+\sin 2x},$$

(4)

[2017 Jan, IAL Q6]

53. $$\frac{6-5x-4x^2}{(2-x)(1+2x)} \circ A+\frac{B}{(2-x)}+\frac{C}{(1+2x)}$$

(a) Find the values of the constants A, B and C.

(4)

$$\mathrm{f}(x)=\frac{6-5x-4x^2}{(2-x)(1+2x)} \qquad x>2$$

(b) Using part (a), find $\mathrm{f}'(x)$.

(3)

(c) Prove that $\mathrm{f}(x)$ is a decreasing function.

(1)

[2017 Jun, IAL Q5]

54. Given that

$$y = 8\tan(2x), \qquad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

show that

$$\frac{dx}{dy} = \frac{A}{B + y^2}$$

where A and B are integers to be found.

(4)

[2018 Jan, IAL Q8]

55. A curve has equation

$$y = \ln(1 - \cos 2x), \qquad x \in \mathbb{R},\ 0 < x < \pi$$

Show that

(*a*) $\frac{dy}{dx} = k\cot x$, where k is a constant to be found.

(4)

Hence find the exact coordinates of the point on the curve where

(*b*) $\frac{dy}{dx} = 2\sqrt{3}$

(4)

[2018 Oct, IAL Q7]