Edexcel

Pure Mathematics

Year 1

Differentiation 1

Past paper questions from Core Maths 1 and IAL C12

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Past paper questions from

Edexcel Core Maths 1 and IAL C12.

From Jan 2005 to Oct 2019.

Differentiation 01

This Section 1 has 37 Questions on differentiations, finding the equations of the Tangent and Normal.

Please check the Edexcel website for the solutions.

1. (*a*) Show that $\frac{(3-\sqrt{x})^2}{\sqrt{x}}$ can be written as $9x^{-\frac{1}{2}} - 6 + x^{\frac{1}{2}}$.

(2)

Given that $\frac{dy}{dx} = \frac{(3-\sqrt{x})^2}{\sqrt{x}}$, $x > 0$, and that $y = \frac{2}{3}$ at $x = 1$,

(*b*) find y in terms of x.

(6)

(C1 May 2005, Q7)

2. The curve C has equation $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$.

The point P has coordinates (3, 0).

(*a*) Show that P lies on C.

(1)

(*b*) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(5)

Another point Q also lies on C. The tangent to C at Q is parallel to the tangent to C at P.

(*c*) Find the coordinates of Q. **(5)**

(C1 May 2005, Q10)

3. Differentiate with respect to x

(*a*) $x^4 + 6\sqrt{x}$,

(3)

(*b*) $\frac{(x+4)^2}{x}$.

(4)

(C1 May 2006, Q5)

4. Given that

$$y = 4x^3 - 1 + 2x^{\frac{1}{2}}, \quad x > 0,$$

find $\frac{dy}{dx}$.

(4)

(C1 Jan 2007, Q1)

5.

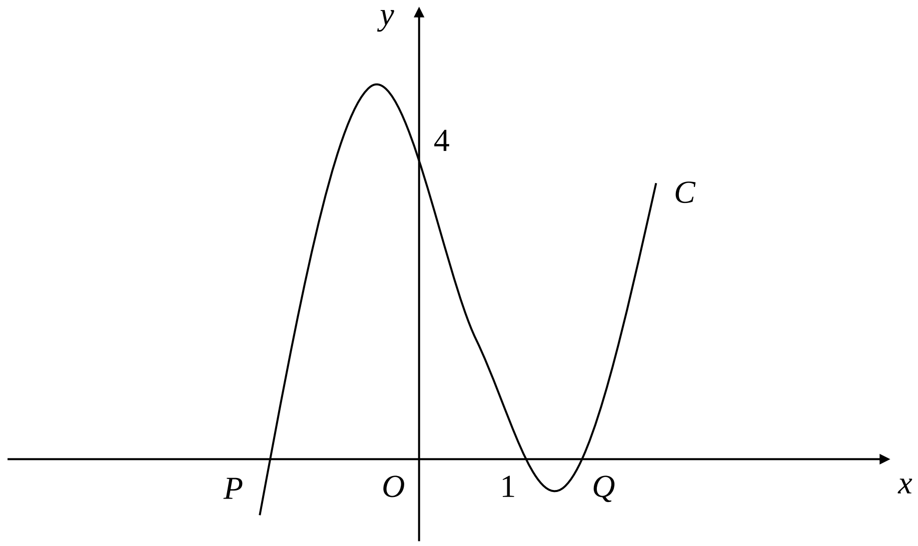


Figure 2 shows part of the curve C with equation

$$y = (x - 1)(x^2 - 4).$$

The curve cuts the x-axis at the points P, (1, 0) and Q, as shown in Figure 2.

(*a*) Write down the x-coordinate of P and the x-coordinate of Q.

(2)

(*b*) Show that $\frac{dy}{dx} = 3x^2 - 2x - 4$.

(3)

(*c*) Show that $y = x + 7$ is an equation of the tangent to C at the point (–1, 6).

(2)

The tangent to C at the point R is parallel to the tangent at the point (–1, 6).

(*d*) Find the exact coordinates of R.

(5)

(C1 Jan 2006, Q9)

6. The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2, \quad x > 0$.

(*a*) Find an expression for $\frac{dy}{dx}$.

(3)

(*b*) Show that the point $P(4, 8)$ lies on C.

(1)

(*c*) Show that an equation of the normal to C at the point P is

$$3y = x + 20.$$

(4)

The normal to C at P cuts the x-axis at the point Q.

(*d*) Find the length PQ, giving your answer in a simplified surd form. **(3)**

(C1 Jan 2007, Q8)

7. The curve C has equation $y = x^2(x - 6) + \frac{4}{x}$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 2 respectively.

(a) Show that the length of PQ is $\sqrt{170}$.

(4)

(b) Show that the tangents to C at P and Q are parallel.

(5)

(c) Find an equation for the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

(C1 May 2007, Q10)

8. (a) Write $\frac{2\sqrt{x}+3}{x}$ in the form $2x^p + 3x^q$, where p and q are constants.

(2)

Given that $y = 5x - 7 + \frac{2\sqrt{x}+3}{x}$, $x > 0$,

(b) find $\frac{dy}{dx}$, simplifying the coefficient of each term. **(4)**

(C1 Jan 2008, Q5)

9. The curve C has equation

$$y = (x + 3)(x - 1)^2.$$

(a) Sketch C, showing clearly the coordinates of the points where the curve meets the coordinate axes.

(4)

(b) Show that the equation of C can be written in the form

$$y = x^3 + x^2 - 5x + k,$$

where k is a positive integer, and state the value of k.

(2)

There are two points on C where the gradient of the tangent to C is equal to 3.

(c) Find the x-coordinates of these two points.

(6)

(C1 Jan 2008, Q10)

10. $$f(x) = 3x + x^3, \qquad x > 0.$$

(*a*) Differentiate to find $f'(x)$.

(2)

Given that $f'(x) = 15$,

(*b*) find the value of x.

(3)

(C1 June 2008, Q4)

11. The curve C has equation $y = kx^3 - x^2 + x - 5$, where k is a constant.

(*a*) Find $\frac{dy}{dx}$.

(2)

The point A with x-coordinate $-\frac{1}{2}$ lies on C. The tangent to C at A is parallel to the line with equation $2y - 7x + 1 = 0$.

Find

(*b*) the value of k,

(4)

(*c*) the value of the y-coordinate of A.

(2)
(C1 June 2008, Q9)

12. Given that $\frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(*a*) write down the value of p and the value of q.

(2)

Given that $y = 5x^4 - 3 + \frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$,

(*b*) find $\frac{dy}{dx}$, simplifying the coefficient of each term.

(4)
(C1 Jan 2009, Q6)

13. Given that $y = x^4 + x^{\frac{1}{3}} + 3$, find $\frac{dy}{dx}$.

(3)
(C1 Jan 2010, Q1)

14. The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \quad x > 0.$$

The point P on C has x-coordinate equal to 2.

(a) Show that the equation of the tangent to C at the point P is $y = 1 - 2x$.

(6)

(b) Find an equation of the normal to C at the point P.

(3)

The tangent at P meets the x-axis at A and the normal at P meets the x-axis at B.

(c) Find the area of the triangle APB.

(4)

(C1 Jan 2009, Q11)

15. $$\mathrm{f}(x) = \frac{(3 - 4\sqrt{x})^2}{\sqrt{x}}, \quad x > 0.$$

(a) Show that $\mathrm{f}(x) = 9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, where A and B are constants to be found.

(3)

(b) Find $\mathrm{f}'(x)$.

(3)

(c) Evaluate $\mathrm{f}'(9)$.

(2)

(C1 June 2009, Q9)

16. The curve C has equation

$$y = x^3 - 2x^2 - x + 9, \quad x > 0.$$

The point P has coordinates (2, 7).

(a) Show that P lies on C.

(1)

(b) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(5)

The point Q also lies on C.

Given that the tangent to C at Q is perpendicular to the tangent to C at P,

(c) show that the x-coordinate of Q is $\frac{1}{3}(2 + \sqrt{6})$.

(5)

(C1 June 2009, Q11)

17. The curve C has equation

$$y = \frac{(x+3)(x-8)}{x}, \quad x > 0.$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in its simplest form.

(4)

(*b*) Find an equation of the tangent to C at the point where $x = 2$.

(4)

(C1 Jan 2010, Q6)

18. Given that

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2 + 2}{x}, \qquad x > 0,$$

find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(6)

(C1 May 2010, Q7)

19. The curve C has equation

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \qquad x > 0.$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(4)

(*b*) Show that the point $P(4, -8)$ lies on C.

(2)

(*c*) Find an equation of the normal to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

(C1 Jan 2011, Q11)

20. The curve C has equation

$$y = (x + 1)(x + 3)^2.$$

(*a*) Sketch C, showing the coordinates of the points at which C meets the axes.

(4)

(*b*) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 14x + 15$.

(3)

The point A, with x-coordinate -5, lies on C.

(*c*) Find the equation of the tangent to C at A, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Another point B also lies on C. The tangents to C at A and B are parallel.

(*d*) Find the x-coordinate of B. **(3)**

(C1 May 2011, Q10)

21. The curve C_1 has equation

$$y = x^2(x + 2).$$

(*a*) Find $\dfrac{dy}{dx}$.

(2)

(*b*) Sketch C_1, showing the coordinates of the points where C_1 meets the x-axis.

(3)

(*c*) Find the gradient of C_1 at each point where C_1 meets the x-axis.

(2)

The curve C_2 has equation

$$y = (x - k)^2(x - k + 2),$$

where k is a constant and $k > 2$.

(*d*) Sketch C_2, showing the coordinates of the points where C_2 meets the x and y axes.

(3)

(C1 Jan 2012, Q8)

22. The curve C has equation

$$y = 2x - 8\sqrt{x} + 5, \quad x \geq 0.$$

(*a*) Find $\dfrac{dy}{dx}$, giving each term in its simplest form.

(3)

The point P on C has x-coordinate equal to $\dfrac{1}{4}$.

(*b*) Find the equation of the tangent to C at the point P, giving your answer in the form $y = ax + b$, where a and b are constants.

(4)

The tangent to C at the point Q is parallel to the line with equation $2x - 3y + 18 = 0$.

(*c*) Find the coordinates of Q.

(5)

(C1 Jan 2013, Q11)

23.

$$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3.$$

(*a*) Find $\dfrac{dy}{dx}$, giving each term in its simplest form.

(4)

(*b*) Find $\dfrac{d^2y}{dx^2}$.

(2)

(C1 May 2012, Q4)

24.

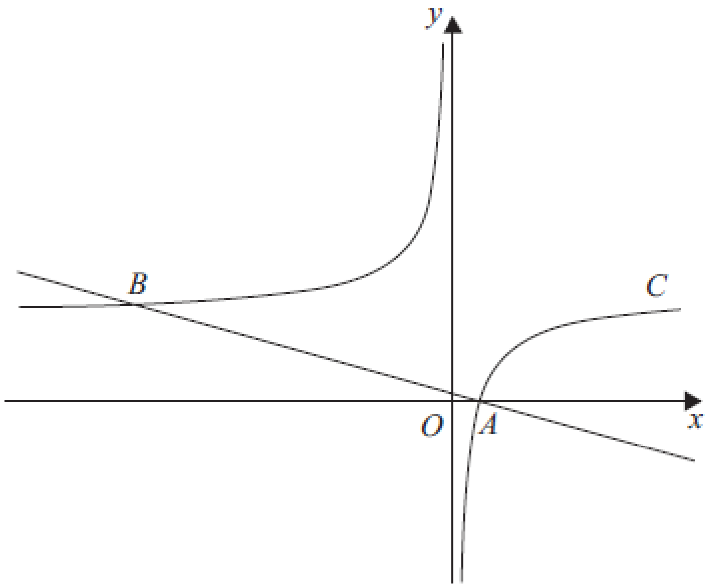


Figure 2

Figure 2 shows a sketch of the curve C with equation

$$y = 2 - \frac{1}{x}, \qquad x \neq 0.$$

The curve crosses the x-axis at the point A.

(a) Find the coordinates of A.

(1)

(b) Show that the equation of the normal to C at A can be written as

$$2x + 8y - 1 = 0.$$

(6)

The normal to C at A meets C again at the point B, as shown in Figure 2.

(c) Find the coordinates of B.

(4)

(C1 Jan 2012, Q10)

25. Given $y = x^3 + 4x + 1$, find the value of $\frac{dy}{dx}$ when $x = 3$.

(4)

(C1 May 2013R, Q1)

26.

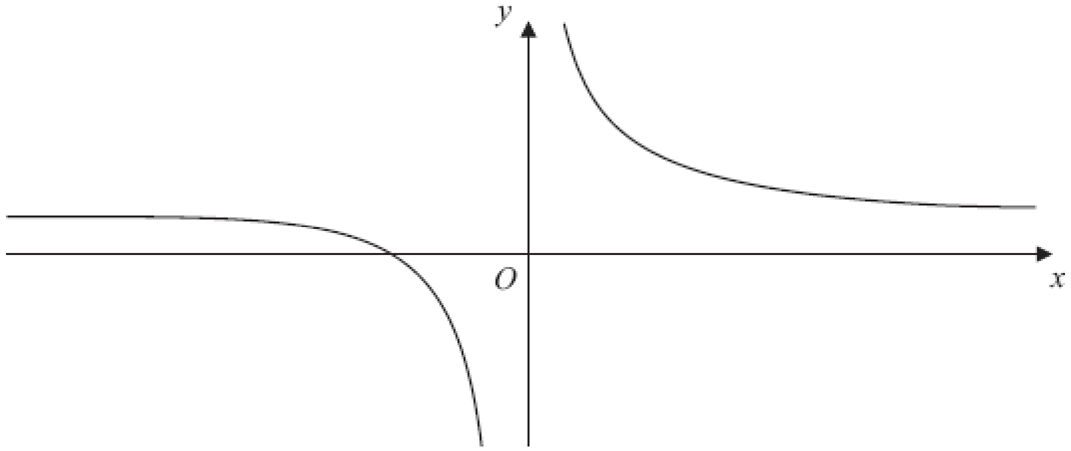


Figure 2

Figure 2 shows a sketch of the curve H with equation $y = \frac{3}{x} + 4$, $x \neq 0$.

(a) Give the coordinates of the point where H crosses the x-axis.

(1)

(b) Give the equations of the asymptotes to H.

(2)

(c) Find an equation for the normal to H at the point $P(-3, 3)$.

(5)

This normal crosses the x-axis at A and the y-axis at B.

(d) Find the length of the line segment AB. Give your answer as a surd.

(3)

(C1 May 2013, Q11)

27. The curve C has equation

$$y = \frac{(x^2 + 4)(x - 3)}{2x}, \quad x \neq 0.$$

(a) Find $\frac{dy}{dx}$ in its simplest form.

(5)

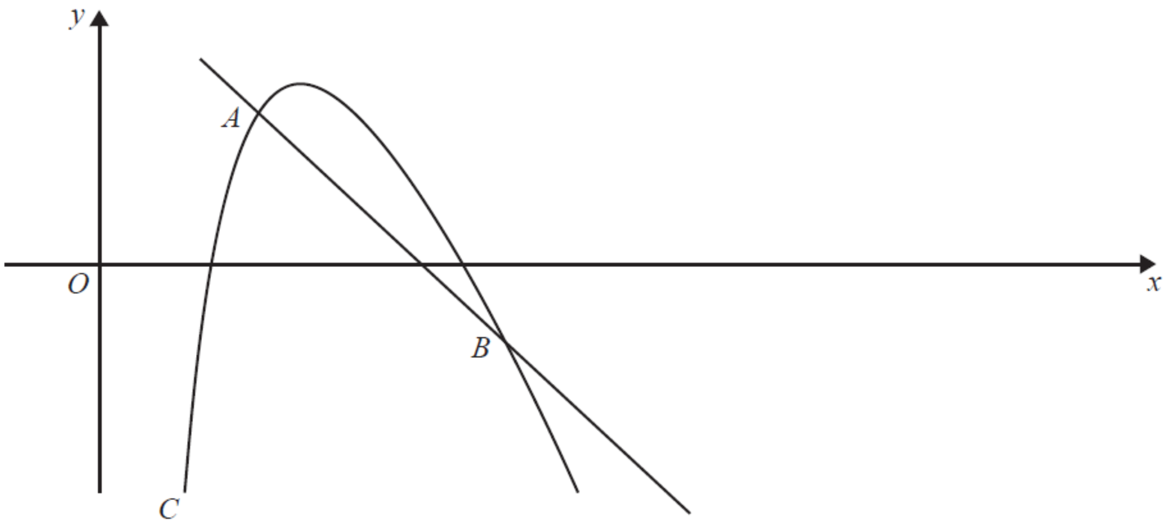
(b) Find an equation of the tangent to C at the point where $x = -1$.

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

(C1 May 2015, Q6)

28.



A sketch of part of the curve C with equation

$$y = 20 - 4x - \frac{18}{x}, \qquad x > 0$$

is shown in Figure 3.

Point A lies on C and has an x coordinate equal to 2.

(a) Show that the equation of the normal to C at A is $y = -2x + 7$.

(6)

The normal to C at A meets C again at the point B, as shown in Figure 3.

(b) Use algebra to find the coordinates of B.

(5)

(C1 May 2014R, Q11)

29. Differentiate with respect to x, giving each answer in its simplest form,

(a) $(1 - 2x)^2$,

(3)

(b) $\dfrac{x^5 + 6\sqrt{x}}{2x^2}$.

(4)

(C1 May 2014, Q7)

30. Given that

$$y = 3x^2 + 6x^{\frac{1}{3}} + \frac{2x^3 - 7}{3\sqrt{x}}, \quad x > 0,$$

find $\dfrac{dy}{dx}$. Give each term in your answer in its simplified form.

(6)

(C1 May 2016, Q7)

31. The curve C has equation $y = 2x^3 + kx^2 + 5x + 6$, where k is a constant.

(a) Find $\frac{dy}{dx}$.

(2)

The point P, where $x = -2$, lies on C.

The tangent to C at the point P is parallel to the line with equation $2y - 17x - 1 = 0$.

Find

(b) the value of k,

(4)

(c) the value of the y coordinate of P,

(2)

(d) the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)

(C1 May 2016, Q11)

32. Given

$$y = \sqrt{x} + \frac{4}{\sqrt{x}} + 4, \qquad x > 0$$

find the value of $\frac{dy}{dx}$ when $x = 8$, writing your answer in the form $a\sqrt{2}$, where a is a rational number.

(5)

(C1 May 2017, Q2)

33. Given

$$y = 3\sqrt{x} - 6x + 4, \qquad x > 0$$

(a) find $\int y\,dx$, simplifying each term.

(3)

(b) (i) Find $\frac{dy}{dx}$

(ii) Hence find the value of x such that $\frac{dy}{dx} = 0$

(4)

(C1 May 2018, Q2)

34.

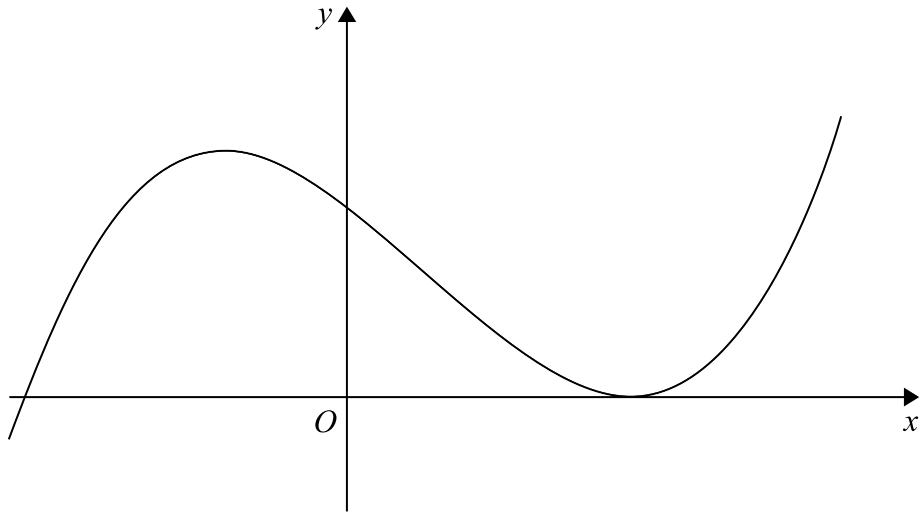


Figure 2

Figure 2 shows a sketch of part of the curve $y = f(x)$, $x \in \mathbb{R}$, where

$$f(x) = (2x - 5)^2 (x + 3)$$

(*a*) Given that

(i) the curve with equation $y = f(x) - k$, $x \in \mathbb{R}$, passes through the origin, find the value of the constant k,

(ii) the curve with equation $y = f(x + c)$, $x \in \mathbb{R}$, has a minimum point at the origin, find the value of the constant c.

(3)

(*b*) Show that $f'(x) = 12x^2 - 16x - 35$

(3)

Points A and B are distinct points that lie on the curve $y = f(x)$.

The gradient of the curve at A is equal to the gradient of the curve at B.

Given that point A has x coordinate 3

(*c*) find the x coordinate of point B.

(5)

(C1 May 2017, Q10)

35.

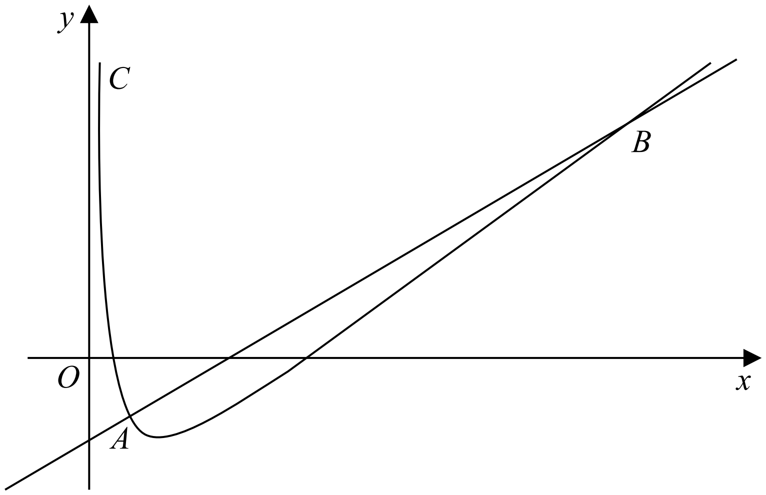


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{1}{2}x + \frac{27}{x} - 12, \quad x > 0$$

The point A lies on C and has coordinates $\left(3, -\frac{3}{2}\right)$.

(a) Show that the equation of the normal to C at A can be written as $10y = 4x - 27$

(5)

The normal to C at A meets C again at the point B, as shown in Figure 3.

(b) Use algebra to find the coordinates of B.

(5)

(C1 May 2018, Q10)

36.

It is given that

$$y = 15x + 108x^{\frac{1}{2}} + 4x^{\frac{5}{2}} \qquad x > 0$$

Find, in simplest form,

(a) $\frac{dy}{dx}$

(3)

(b) $\frac{d^2y}{dx^2}$

(2)

(c) Find the value of $\frac{d^2y}{dx^2}$ when $x = 9$

(1)

(C1 May 2019, Q2)

37.

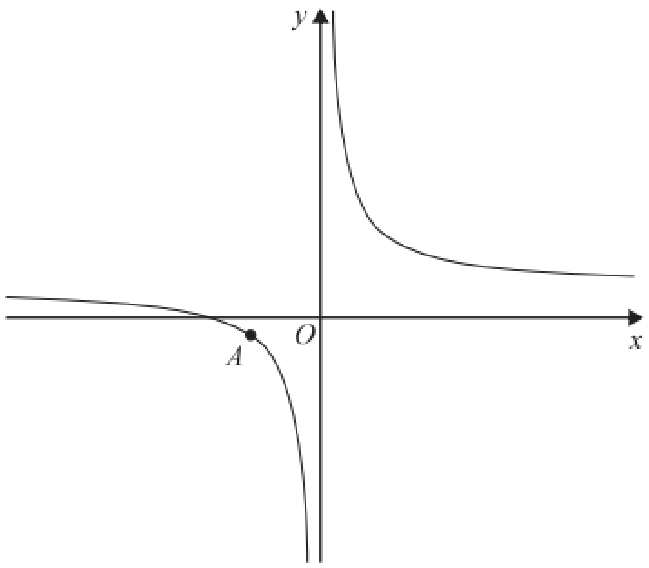


Figure 1

Figure 1 shows a sketch of part of the curve H with equation

$$y = \frac{12}{x} + 5 \quad x \neq 0$$

(a) Find an equation for the normal to H at the point A (–2, –1), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

The points B and C also lie on the curve H.
The normal to H at the point B and the normal to H at the point C are each parallel to the straight line with equation $4y = 3x + 5$

(b) Find the coordinates of the points B and C, given that the x coordinate of B is positive.

(5)

(C1 May 2019, Q9)