OCR Core Maths 2

Past paper questions
Circles

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Circles

- Circles with centre $(0, 0)$ and radius r are expressed by $x^2 + y^2 = r^2$.
- Circles with centre (a, b) and radius r are expressed by $(x - a)^2 + (y - b)^2 = r^2$.
- By 'completing the square' you can convert circles of the form $x^2 + y^2 + \alpha x + \beta y + \gamma = 0$ into the form $(x - a)^2 + (y - b)^2 = r^2$. For example

$$\begin{aligned} x^2 + y^2 + 6x - 4y + 9 &= 0 \\ x^2 + 6x + y^2 - 4y + 9 &= 0 \\ (x + 3)^2 - 9 + (y - 2)^2 - 4 + 9 &= 0 \\ (x + 3)^2 + (y - 2)^2 &= 4. \end{aligned}$$

- When finding the intersection of a line and a circle it is easiest to substitute in the value of y from the line into the circle and solve the resulting quadratic. For example; find where the line $y = 2x - 1$ intersects to circle $(x - 3)^2 + (y - 2)^2 = 25$.

$$\begin{aligned} (x - 3)^2 + (y - 2)^2 &= 25 \\ (x - 3)^2 + (2x - 3)^2 &= 25 \\ x^2 - 6x + 9 + 4x^2 - 12x + 9 - 25 &= 0 \\ 5x^2 - 18x - 7 &= 0. \end{aligned}$$

  Solve the quadratic (in this case by the formula) and then find the y values by substituting both x values into $y = 2x - 1$ (the original line). There will usually be 2 points of intersection (where the discriminant of the resulting quadratic will be positive) except if the line doesn't intersect the circle at all (discriminant negative) or if the line is a tangent to the circle (discriminant equals zero).

- The gradient of the tangent to a circle is perpendicular to the radius of the circle at that point. For example: The point $B(1, 7)$ lies on the circle $(x - 3)^2 + (y - 4)^2 = 13$. Find the equation of the tangent to the circle at B. The centre of the circle is $(3, 4)$, so the gradient of the radius at B is $-\frac{3}{2}$. Therefore the gradient of the tangent is $\frac{2}{3}$ and will pass through B, so the tangent will be:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 7 &= \tfrac{2}{3}(x - 1) \\ 0 &= 2x - 3y + 19. \end{aligned}$$

- You must always remember the GCSE theorem that if a triangle is constructed within a circle with one side being a diameter of the circle, then it is a right angled triangle. To demonstrate this one is often required to show that the gradients of certain line segments are perpendicular (i.e. $m_1 \times m_2 = -1$).

  For example; $A(2,1)$, $B(4,13)$ and $C(-3,8)$. The line segment AB is the diameter of a circle and C is a point on its circumference. Find the area of triangle ABC. We know angle $A\hat{C}B$ must be a right angle, so

$$\begin{aligned}\text{Area } ABC &= \tfrac{1}{2} \times \text{base} \times \text{height}\\ &= \tfrac{1}{2} \times (\text{length } AC) \times (\text{length } CB)\\ &= \tfrac{1}{2} \times \sqrt{5^2+7^2} \times \sqrt{7^2+5^2}\\ &= 37 \text{ units}^2.\end{aligned}$$

- Also, given two points that lie on a circle's circumference, the centre of the circle lies on the perpendicular bisector of the two points.

1.

(i) Describe completely the curve $x^2 + y^2 = 25$. [2]

(ii) Find the coordinates of the points of intersection of the curve $x^2 + y^2 = 25$ and the line $2x + y - 5 = 0$. [6]

Q8 June 2005

2.

(i) Express $x^2 + 3x$ in the form $(x + a)^2 + b$. [2]

(ii) Express $y^2 - 4y - \frac{11}{4}$ in the form $(y + p)^2 + q$. [2]

A circle has equation $x^2 + y^2 + 3x - 4y - \frac{11}{4} = 0$.

(iii) Write down the coordinates of the centre of the circle. [1]

(iv) Find the radius of the circle. [2]

Q5 Jan 2006

3.

The points A and B have coordinates $(4, -2)$ and $(10, 6)$ respectively. C is the mid-point of AB. Find

(i) the coordinates of C, [2]

(ii) the length of AC, [2]

(iii) the equation of the circle that has AB as a diameter, [3]

(iv) the equation of the tangent to the circle in part **(iii)** at the point A, giving your answer in the form $ax + by = c$. [5]

4.

The points A and B have coordinates $(4, -2)$ and $(10, 6)$ respectively. C is the mid-point of AB. Find

(i) the coordinates of C, [2]

(ii) the length of AC, [2]

(iii) the equation of the circle that has AB as a diameter, [3]

(iv) the equation of the tangent to the circle in part **(iii)** at the point A, giving your answer in the form $ax + by = c$. [5]

Q9 June 2006

5.

A circle has equation $x^2 + y^2 + 2x - 4y - 8 = 0$.

(i) Find the centre and radius of the circle. [3]

(ii) The circle passes through the point $(-3, k)$, where $k < 0$. Find the value of k. [3]

(iii) Find the coordinates of the points where the circle meets the line with equation $x + y = 6$. [6]

Q10 Jan 2007

6.

The circle with equation $x^2 + y^2 - 6x - k = 0$ has radius 4.

(i) Find the centre of the circle and the value of k. [4]

The points $A\ (3, a)$ and $B\ (-1, 0)$ lie on the circumference of the circle, with $a > 0$.

(ii) Calculate the length of AB, giving your answer in simplified surd form. [5]

(iii) Find an equation for the line AB. [3]

Q9 June 2007

7.

(i) Write down the equation of the circle with centre $(0, 0)$ and radius 7. [1]

(ii) A circle with centre $(3, 5)$ has equation $x^2 + y^2 - 6x - 10y - 30 = 0$. Find the radius of the circle. [2]

Q2 Jan 2008

8.

(i) Find the equation of the circle with radius 10 and centre $(2, 1)$, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$. [3]

(ii) The circle passes through the point $(5, k)$ where $k > 0$. Find the value of k in the form $p + \sqrt{q}$. [3]

(iii) Determine, showing all working, whether the point $(-3, 9)$ lies inside or outside the circle. [3]

(iv) Find an equation of the tangent to the circle at the point $(8, 9)$. [5]

Q9 June 2008

9.

The line with equation $3x + 4y - 10 = 0$ passes through point A (2, 1) and point B (10, k).

(i) Find the value of k. **[2]**

(ii) Calculate the length of AB. **[2]**

A circle has equation $(x - 6)^2 + (y + 2)^2 = 25$.

(iii) Write down the coordinates of the centre and the radius of the circle. **[2]**

(iv) Verify that AB is a diameter of the circle. **[2]**

Q7 Jan 2009

10.

(i) Express $x^2 - 5x + \frac{1}{4}$ in the form $(x - a)^2 - b$. **[3]**

(ii) Find the centre and radius of the circle with equation $x^2 + y^2 - 5x + \frac{1}{4} = 0$. **[3]**

Q7 June 2009

11.

A circle has equation $x^2 + y^2 + 6x - 4y - 4 = 0$.

(i) Find the centre and radius of the circle. **[3]**

(ii) Find the coordinates of the points where the circle meets the line with equation $y = 3x + 4$. **[6]**

Q8 Jan 2010

12.

(i) The line joining the points A (4, 5) and B (p, q) has mid-point M (−1, 3). Find p and q. **[3]**

AB is the diameter of a circle.

(ii) Find the radius of the circle. **[2]**

(iii) Find the equation of the circle, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$. **[3]**

(iv) Find an equation of the tangent to the circle at the point (4, 5). **[5]**

Q9 June 2010

13.

A circle with centre C has equation $x^2 + y^2 - 8x - 2y - 3 = 0$.

(i) Find the coordinates of C and the radius of the circle. **[3]**

(ii) Find the values of k for which the line $y = k$ is a tangent to the circle, giving your answers in simplified surd form. **[3]**

(iii) The points S and T lie on the circumference of the circle. M is the mid-point of the chord ST. Given that the length of CM is 2, calculate the length of the chord ST. **[3]**

(iv) Find the coordinates of the point where the circle meets the line $x - 2y - 12 = 0$. **[6]**

Q9 Jan 2011

14.

The points $A\ (1, 3)$, $B\ (7, 1)$ and $C\ (-3, -9)$ are joined to form a triangle.

(i) Show that this triangle is right-angled and state whether the right angle is at A, B or C. **[5]**

(ii) The points A, B and C lie on the circumference of a circle. Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$. **[7]**

Q9 June 2011

15.

A circle has centre $C(-2, 4)$ and radius 5.

(i) Find the equation of the circle, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$. **[3]**

(ii) Show that the tangent to the circle at the point $P(-5, 8)$ has equation $3x - 4y + 47 = 0$. **[5]**

(iii) Verify that the point $T(3, 14)$ lies on this tangent. **[1]**

(iv) Find the area of the triangle CPT. **[4]**

Q10 Jan 2012

16.

A circle has equation $(x - 5)^2 + (y + 2)^2 = 25$.

(i) Find the coordinates of the centre C and the length of the diameter. **[3]**

(ii) Find the equation of the line which passes through C and the point $P\ (7, 2)$. **[4]**

(iii) Calculate the length of CP and hence determine whether P lies inside or outside the circle. **[3]**

(iv) Determine algebraically whether the line with equation $y = 2x$ meets the circle. **[5]**

Q10 June 2012

17.

A circle with centre C has equation $x^2 + y^2 - 2x + 10y - 19 = 0$.

(i) Find the coordinates of C and the radius of the circle. **[3]**

(ii) Verify that the point (7, –2) lies on the circumference of the circle. **[1]**

(iii) Find the equation of the tangent to the circle at the point (7, –2), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[5]**

Q9 Jan 2013

18.

A circle C has equation $x^2 + y^2 + 8y - 24 = 0$.

(i) Find the centre and radius of the circle. **[3]**

(ii) The point A (2, 2) lies on the circumference of C. Given that AB is a diameter of the circle, find the coordinates of B. **[2]**

Q6 June 2013

19.

A circle with centre C has equation $(x-2)^2 + (y+5)^2 = 25$.

(i) Show that no part of the circle lies above the x-axis. **[3]**

(ii) The point P has coordinates (6, k) and lies inside the circle. Find the set of possible values of k. **[5]**

(iii) Prove that the line $2y = x$ does not meet the circle. **[4]**

Q9 June 2014

20.

A circle with centre C has equation $x^2 + y^2 - 10x + 4y + 4 = 0$.

(i) Find the coordinates of C and the radius of the circle. **[3]**

(ii) Show that the tangent to the circle at the point P (8, 2) has equation $3x + 4y = 32$. **[5]**

(iii) The circle meets the y-axis at Q and the tangent meets the y-axis at R. Find the area of triangle PQR. **[4]**

Q10 June 2015