OCR Core Maths 4

Past paper questions

Binomial Expansion

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- Review binomial expansion from C2 for $(x+y)^n$ for positive integer n. Notice that it is valid for *any* x and y and that the expansion has $n+1$ terms.
- The general binomial expansion is given by

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

and is valid for any n (fractional or negative) but $-1 < x < 1$ (i.e. $|x| < 1$). Notice also it must start with a 1 in the brackets. For example expand $(4-x)^{-1/2}$.

$$\begin{aligned}
(4-x)^{-1/2} &= \left(4\left(1-\frac{x}{4}\right)\right)^{-1/2} \\
&= \frac{1}{2}\left(1-\frac{x}{4}\right)^{-1/2} \\
&= \frac{1}{2}\left[1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}\left(-\frac{x}{4}\right)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}\left(-\frac{x}{4}\right)^3 + \cdots\right] \\
&= \frac{1}{2}\left[1 + \frac{x}{8} + \frac{3x^2}{128} + \frac{15x^3}{3072} + \cdots\right] = \frac{1}{2} + \frac{x}{16} + \frac{3x^2}{256} + \frac{15x^3}{6144} + \cdots .
\end{aligned}$$

It is only valid for $|x/4| < 1 \quad \Rightarrow \quad |x| < 4$.

- Another example: Find first 3 terms in the expansion for $\dfrac{(3+x)^2}{1+\frac{x}{2}}$.

$$(3+x)^2\left(1+\frac{x}{2}\right)^{-1} = (9+6x+x^2)\left(1-\frac{x}{2}+\frac{x^2}{4}+\ldots\right) = 9 + \frac{3}{2}x + \frac{1}{4}x^2 + \ldots .$$

1.

(i) Given that $\frac{3x+4}{(1+x)(2+x)^2} \equiv \frac{A}{1+x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$, find A, B and C. [5]

(ii) Hence or otherwise expand $\frac{3x+4}{(1+x)(2+x)^2}$ in ascending powers of x, up to and including the term in x^2. [5]

(iii) State the set of values of x for which the expansion in part **(ii)** is valid. [1]

Q8 June 2005

2.

The expression $\frac{11+8x}{(2-x)(1+x)^2}$ is denoted by $\mathrm{f}(x)$.

(i) Express $\mathrm{f}(x)$ in the form $\frac{A}{2-x} + \frac{B}{1+x} + \frac{C}{(1+x)^2}$, where A, B and C are constants. [5]

(ii) Given that $|x| < 1$, find the first 3 terms in the expansion of $\mathrm{f}(x)$ in ascending powers of x. [5]

Q7 Jan 2006

3.

(i) Expand $(1-3x)^{-2}$ in ascending powers of x, up to and including the term in x^2. [3]

(ii) Find the coefficient of x^2 in the expansion of $\frac{(1+2x)^2}{(1-3x)^2}$ in ascending powers of x. [4]

Q2 June 2006

4.

(i) Expand $(1-3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3. [4]

(ii) Hence find the coefficient of x^3 in the expansion of $\left(1-3(x+x^3)\right)^{-\frac{1}{3}}$. [3]

Q5 Jan 2007

5.

(i) Expand $(2+x)^{-2}$ in ascending powers of x up to and including the term in x^3, and state the set of values of x for which the expansion is valid. [5]

(ii) Hence find the coefficient of x^3 in the expansion of $\frac{1+x^2}{(2+x)^2}$. [2]

Q4 June 2007

6.

(i) Expand $(1 + ax)^{-4}$ in ascending powers of x, up to and including the term in x^2. [3]

(ii) The coefficients of x and x^2 in the expansion of $(1 + bx)(1 + ax)^{-4}$ are 1 and -2 respectively. Given that $a > 0$, find the values of a and b. [5]

Q6 Jan 2008

7.

(i) Show that $\sqrt{\dfrac{1-x}{1+x}} \approx 1 - x + \frac{1}{2}x^2$, for $|x| < 1$. [5]

(ii) By taking $x = \frac{2}{7}$, show that $\sqrt{5} \approx \frac{111}{49}$. [3]

Q5 June 2008

8.

(i) Expand $(1 + 2x)^{\frac{1}{2}}$ as a series in ascending powers of x, up to and including the term in x^3. **[3]**

(ii) Hence find the expansion of $\dfrac{(1 + 2x)^{\frac{1}{2}}}{(1 + x)^3}$ as a series in ascending powers of x, up to and including the term in x^3. **[5]**

(iii) State the set of values of x for which the expansion in part **(ii)** is valid. **[1]**

Q3 Jan 2009

9.

(i) Expand $(a + x)^{-2}$ in ascending powers of x up to and including the term in x^2. **[4]**

(ii) When $(1 - x)(a + x)^{-2}$ is expanded, the coefficient of x^2 is 0. Find the value of a. **[3]**

Q3 June 2009

10.

(i) Expand $(1 + x)^{\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^2. **[2]**

(ii) (a) Hence, or otherwise, expand $(8 + 16x)^{\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^2. **[4]**

(b) State the set of values of x for which the expansion in part **(ii) (a)** is valid. **[1]**

Q5 Jan 2010

11.

Expand $(1 + 3x)^{-\frac{5}{3}}$ in ascending powers of x, up to and including the term in x^3. **[5]**

Q1 June 2010

12.

(i) Expand $(1 - x)^{\frac{1}{2}}$ in ascending powers of x as far as the term in x^2. **[3]**

(ii) Hence expand $\left(1 - 2y + 4y^2\right)^{\frac{1}{2}}$ in ascending powers of y as far as the term in y^2. **[3]**

Q1 Jan 2011

13.

Find the coefficient of x^2 in the expansion in ascending powers of x of

$$\sqrt{\frac{1 + ax}{4 - x}},$$

giving your answer in terms of a. **[8]**

Q6 June 2011

14.

(i) Expand $\left(1 - 4x\right)^{\frac{1}{4}}$ in ascending powers of x, up to and including the term in x^3. **[5]**

(ii) The term of lowest degree in the expansion of

$$\left(1 + ax\right)\left(1 + bx^2\right)^7 - \left(1 - 4x\right)^{\frac{1}{4}}$$

in ascending powers of x is the term in x^3. Find the values of the constants a and b. **[4]**

Q4 Jan 2012

15.

(i) Expand $\dfrac{1 + x^2}{\sqrt{1 + 4x}}$ in ascending powers of x, up to and including the term in x^3. **[6]**

(ii) State the set of values of x for which this expansion is valid. **[1]**

Q3 June 2012

16.

Find the first three terms in the expansion of $(9 - 16x)^{\frac{3}{2}}$ in ascending powers of x, and state the set of values for which this expansion is valid. **[5]**

Q2 Jan 2013

17.

(i) Show that $\frac{x}{(1-x)^3} \approx x + 3x^2 + 6x^3$ for small values of x. **[2]**

(ii) Use this result, together with a suitable value of x, to obtain a decimal estimate of the value of $\frac{100}{729}$. **[2]**

(iii) Show that $\frac{x}{(1-x)^3} = -\frac{1}{x^2}\left(1-\frac{1}{x}\right)^{-3}$. Hence find the first three terms of the binomial expansion of $\frac{x}{(1-x)^3}$ in powers of $\frac{1}{x}$. **[4]**

(iv) Comment on the suitability of substituting the same value of x as used in part **(ii)** in the expansion in part **(iii)** to estimate the value of $\frac{100}{729}$. **[1]**

Q10 June 2013

18.

(i) Find the first three terms in the expansion of $(1-2x)^{-\frac{1}{2}}$ in ascending powers of x, where $|x| < \frac{1}{2}$. **[3]**

(ii) Hence find the coefficient of x^2 in the expansion of $\frac{x+3}{\sqrt{1-2x}}$. **[2]**

Q3 June 2014

19.

(i) Find the first three terms in the binomial expansion of $(8-9x)^{\frac{2}{3}}$ in ascending powers of x. **[4]**

(ii) State the set of values of x for which this expansion is valid. **[1]**

Q4 June 2015