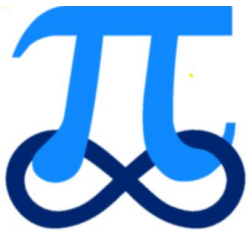
Edexcel

Pure Mathematics

Year 2

Binomial Expansions.



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1. Use the binomial theorem to expand

$$\sqrt{(4 - 9x)}, \qquad |x| < \frac{4}{9},$$

in ascending powers of x, up to and including the term in x^3, simplifying each term. **(5)**

(C4 June 2005 Q1)

2. $$f(x) = \frac{3x^2 + 16}{(1-3x)(2+x)^2} = \frac{A}{(1-3x)} + \frac{B}{(2+x)} + \frac{C}{(2+x)^2}, \quad |x| < \tfrac{1}{3}.$$

(*a*) Find the values of A and C and show that $B = 0$. **(4)**

(*b*) Hence, or otherwise, find the series expansion of f(x), in ascending powers of x, up to and including the term in x^3. Simplify each term. **(7)**

(C4 Jan 2006 Q5)

3. $$f(x) = \frac{3x - 1}{(1-2x)^2}, \qquad |x| < \tfrac{1}{2}.$$

Given that, for $x \neq \frac{1}{2}$, $\dfrac{3x-1}{(1-2x)^2} = \dfrac{A}{(1-2x)} + \dfrac{B}{(1-2x)^2}$, where A and B are constants,

(*a*) find the values of A and B. **(3)**

(*b*) Hence, or otherwise, find the series expansion of f(x), in ascending powers of x, up to and including the term in x^3, simplifying each term. **(6)**

(C4 June 2006 Q2)

4. $$f(x) = (2 - 5x)^{-2}, \quad |x| < \tfrac{2}{5}.$$

Find the binomial expansion of f(x), in ascending powers of x, as far as the term in x^3, giving each coefficient as a simplified fraction. **(5)**

(C4 Jan 2007 Q1)

5. $$f(x) = (3 + 2x)^{-3}, \quad |x| < \tfrac{3}{2}.$$

Find the binomial expansion of f(x), in ascending powers of x, as far as the term in x^3.

Give each coefficient as a simplified fraction. **(5)**

(C4 June 2007 Q1)

6. (*a*) Use the binomial theorem to expand

$$(8-3x)^{\frac{1}{3}}, \qquad |x| < \tfrac{8}{3},$$

in ascending powers of *x*, up to and including the term in x^3, giving each term as a simplified fraction. **(5)**

(*b*) Use your expansion, with a suitable value of *x*, to obtain an approximation to $\sqrt[3]{(7.7)}$.

Give your answer to 7 decimal places. **(2)**

(C4 Jan 2008 Q2)

7. (*a*) Expand $\dfrac{1}{\sqrt{(4-3x)}}$, where $|x| < \frac{4}{3}$, in ascending powers of *x* up to and including the term in x^2. Simplify each term. **(5)**

(*b*) Hence, or otherwise, find the first 3 terms in the expansion of $\dfrac{x+8}{\sqrt{(4-3x)}}$ as a series in ascending powers of *x*. **(4)**

(C4 June 2008 Q5)

8. $$f(x) = \frac{27x^2+32x+16}{(3x+2)^2(1-x)}, \quad |x| < \frac{2}{3}.$$

Given that f(*x*) can be expressed in the form

$$f(x) = \frac{A}{(3x+2)} + \frac{B}{(3x+2)^2} + \frac{C}{(1-x)},$$

(*a*) find the values of *B* and *C* and show that *A* = 0. **(4)**

(*b*) Hence, or otherwise, find the series expansion of f(*x*), in ascending powers of *x*, up to and including the term in x^2. Simplify each term. **(6)**

(*c*) Find the percentage error made in using the series expansion in part (*b*) to estimate the value of f(0.2). Give your answer to 2 significant figures. **(4)**

(C4 Jan 2009 Q3)

9. $$f(x) = \frac{1}{\sqrt{(4+x)}}, \quad |x| < 4.$$

Find the binomial expansion of f (*x*) in ascending powers of *x*, up to and including the term in x^3. Give each coefficient as a simplified fraction. **(6)**

(C4 June 2009 Q1)

10. (*a*) Find the binomial expansion of

$$\sqrt{(1 - 8x)}, \quad |x| < \frac{1}{8},$$

in ascending powers of x up to and including the term in x^3, simplifying each term. **(6)**

(*b*) Show that, when $x = \frac{1}{100}$, the exact value of $\sqrt{(1 - 8x)}$ is $\frac{\sqrt{23}}{5}$. **(2)**

(*c*) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (*a*) and hence obtain an approximation to $\sqrt{23}$. Give your answer to 5 decimal places. **(3)**

(C4 Jan 2010 Q1)

11.
$$\frac{2x^2 + 5x - 10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}.$$

(*a*) Find the values of the constants A, B and C. **(4)**

(*b*) Hence, or otherwise, expand $\frac{2x^2 + 5x - 10}{(x-1)(x+2)}$ in ascending powers of x, as far as the term in x^2. Give each coefficient as a simplified fraction. **(7)**

(C4 June 2010 Q5)

12. (*a*) Use the binomial theorem to expand

$$(2 - 3x)^{-2}, \quad |x| < \frac{2}{3},$$

in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction. **(5)**

$$f(x) = \frac{a + bx}{(2 - 3x)^2}, \quad |x| < \frac{2}{3}, \quad \text{where } a \text{ and } b \text{ are constants.}$$

In the binomial expansion of f(x), in ascending powers of x, the coefficient of x is 0 and the coefficient of x^2 is $\frac{9}{16}$.

(*b*) the value of a and the value of b, **(5)**

(*c*) the coefficient of x^3, giving your answer as a simplified fraction. **(3)**

(C4 Jan 2011 Q5)

13. $f(x) = \dfrac{1}{\sqrt{(9+4x^2)}}, \quad |x| < \dfrac{3}{2}.$

Find the first three non-zero terms of the binomial expansion of f(x) in ascending powers of x. Give each coefficient as a simplified fraction. **(6)**

(C4 June 2011 Q2)

14. (a) Expand

$$\frac{1}{(2-5x)^2}, \quad |x| < \frac{2}{5},$$

in ascending powers of x, up to and including the term in x^2, giving each term as a simplified fraction. **(5)**

Given that the binomial expansion of $\dfrac{2+kx}{(2-5x)^2}$, $|x| < \dfrac{2}{5}$, is

$$\frac{1}{2} + \frac{7}{4}x + Ax^2 + \ldots,$$

(b) find the value of the constant k, **(2)**

(c) find the value of the constant A. **(2)**

(C4 Jan 2012 Q3)

15. $f(x) = \dfrac{6}{\sqrt{(9-4x)}}, \quad |x| < \dfrac{9}{4}.$

(a) Find the binomial expansion of f(x) in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form. **(6)**

Use your answer to part (a) to find the binomial expansion in ascending powers of x, up to and including the term in x^3, of

(b) $g(x) = \dfrac{6}{\sqrt{(9+4x)}}, \quad |x| < \dfrac{9}{4},$ **(1)**

(c) $h(x) = \dfrac{6}{\sqrt{(9-8x)}}, \quad |x| < \dfrac{9}{8}.$ **(2)**

(C4 June 2012 Q3)

16. Given

$$f(x) = (2 + 3x)^{-3}, \quad |x| < \frac{2}{3},$$

find the binomial expansion of f(x), in ascending powers of x, up to and including the term in x^3.

Give each coefficient as a simplified fraction.

(5)

(C4 Jan 2013 Q1)

17. (*a*) Use the binomial expansion to show that

$$\sqrt{\left(\frac{1+x}{1-x}\right)} \approx 1 + x + \frac{1}{2}x^2, \qquad |x| < 1$$

(6)

(*b*) Substitute $x = \frac{1}{26}$ into

$$\sqrt{\left(\frac{1+x}{1-x}\right)} = 1 + x + \frac{1}{2}x^2$$

to obtain an approximation to $\sqrt{3}$.

Give your answer in the form $\frac{a}{b}$ where a and b are integers.

(3)

(C4 June 2013 Q2)

18. (*a*) Find the binomial expansion of

$$\sqrt[3]{(8-9x)}, \qquad |x| < \frac{8}{9}$$

in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(6)

(*b*) Use your expansion to estimate an approximate value for $\sqrt[3]{7100}$, giving your answer to 4 decimal places. State the value of x, which you use in your expansion, and show all your working.

(C4 June 2013_R Q4)

19. Given that the binomial expansion of $(1 + kx)^{-4}$, $|kx| < 1$, is

$$1 - 6x + Ax^2 + \ldots$$

(*a*) find the value of the constant *k*, **(2)**

(*b*) find the value of the constant *A*, giving your answer in its simplest form. **(3)**

(C4 June 2014 Q2)

20. (*a*) Find the binomial expansion of

$$\frac{1}{\sqrt{(9-10x)}}, \qquad |x| < \frac{9}{10}$$

in ascending powers of *x* up to and including the term in x^2.

Give each coefficient as a simplified fraction. **(5)**

(*b*) Hence, or otherwise, find the expansion of

$$\frac{3+x}{\sqrt{(9-10x)}}, \qquad |x| < \frac{9}{10}$$

in ascending powers of *x*, up to and including the term in x^2.

Give each coefficient as a simplified fraction. **(3)**

(C4 June 2014_R Q1)

21. (*a*) Find the binomial expansion of

$$(4+5x)^{\frac{1}{2}}, \quad |x| < \frac{4}{5},$$

in ascending powers of *x*, up to and including the term in x^2.

Give each coefficient in its simplest form. **(5)**

(*b*) Find the exact value of $(4+5x)^{\frac{1}{2}}$ when $x = \frac{1}{10}$.

Give your answer in the form $k\sqrt{2}$, where *k* is a constant to be determined. **(1)**

(*c*) Substitute $x = \frac{1}{10}$ into your binomial expansion from part (*a*) and hence find an approximate value for $\sqrt{2}$. Give your answer in the form $\frac{p}{q}$, where *p* and *q* are integers. **(2)**

(C4 June 2015 Q1)

22. Use the binomial series to find the expansion of

$$\frac{1}{(2+5x)^3}, \quad |x| < \frac{2}{5},$$

in ascending powers of x, up to and including the term in x^3.
Give each coefficient as a fraction in its simplest form.

(6)

(C4 June 2016 Q1)

23. $f(x) = (2 + kx)^{-3}$, $|kx| < 2$, where k is a positive constant

The binomial expansion of $f(x)$, in ascending powers of x, up to and including the term in x^2 is

$$A + Bx + \frac{243}{16}x^2$$

where A and B are constants.

(a) Write down the value of A.

(1)

(b) Find the value of k.

(3)

(c) Find the value of B.

(2)

(C4 June 2017 Q2)

24. Given that the binomial expansion, in ascending powers of x, of

$$\frac{6}{\sqrt{\left(9+Ax^2\right)}}, \qquad |x| < \frac{3}{\sqrt{|A|}}$$

is

$$B - \frac{2}{3}x^2 + Cx^4 + \ldots$$

(a) find the values of the constants A, B and C. **(7)**

(b) Hence find the coefficient of x^6. **(2)**

(IAL, C34 Jan 2014 Q6)

25. $f(x) = \left(8+27x^3\right)^{\frac{1}{3}}, \qquad |x| < \frac{2}{3}$

Find the first three non-zero terms of the binomial expansion of $f(x)$ in ascending powers of x. Give each coefficient as a simplified fraction. **(5)**

(IAL, C34 June 2014 Q5)

26. (*a*) Use the binomial expansion, in ascending powers of *x*, of $\frac{1}{\sqrt{(1-2x)}}$ to show that

$$\frac{2+3x}{\sqrt{(1-2x)}} \approx 2+5x+6x^2, \qquad |x| < 0.5$$ **(4)**

(*b*) Substitute $x = \frac{1}{20}$ into

$$\frac{2+3x}{\sqrt{(1-2x)}} = 2+5x+6x^2$$

to obtain an approximation to $\sqrt{10}$.

Give your answer as a fraction in its simplest form. **(3)**

(IAL, C34 Jan 2015 Q5)

27. Given that

$$\frac{4(x^2+6)}{(1-2x)(2+x)^2} \equiv \frac{A}{(1-2x)} + \frac{B}{(2+x)} + \frac{C}{(2+x)^2}$$

(*a*) find the values of the constants *A* and *C* and show that *B* = 0. **(4)**

(*b*) Hence, or otherwise, find the series expansion of

$$\frac{4(x^2+6)}{(1-2x)(2+x)^2} \qquad |x| < \frac{1}{2}$$

in ascending powers of *x*, up to and including the term in x^2, simplifying each term. **(5)**

(IAL, C34 June 2015 Q2)

28. $f(x) = (3 - 2x)^{-4}, \qquad |x| < \frac{3}{2}$

Find the binomial expansion of f(*x*), in ascending powers of *x*, up to and including the term in x^2, giving each coefficient as a simplified fraction. **(4)**

(IAL, C34 Jan 2016 Q1)

29. (*a*) Find the binomial expansion of

$$(1 + ax)^{-3}, \qquad |ax| < 1$$

in ascending powers of x, up to and including the term in x^3, giving each coefficient as simply as possible in terms of the constant a.

(3)

$$f(x) = \frac{2+3x}{(1+ax)^3}, \qquad |ax| < 1$$

In the series expansion of f(x), the coefficient of x^2 is 3

Given that $a < 0$

(*b*) find the value of the constant a,

(4)

(*c*) find the coefficient of x^3 in the series expansion of f(x), giving your answer as a simplified fraction.

(2)

(IAL, C34 June 2016 Q3)

30. (*a*) Express $\frac{9+11x}{(1-x)(3+2x)}$ in partial fractions.

(3)

(*b*) Hence, or otherwise, find the series expansion of

$$\frac{9+11x}{(1-x)(3+2x)}, \qquad |x| < 1$$

in ascending powers of x, up to and including the term in x^3.
Give each coefficient as a simplified fraction.

(6)

(IAL, C34 Jan 2017 Q3)

31.

$$f(x) = \frac{27}{(3-5x)^2} \qquad |x| < \frac{3}{5}$$

(*a*) Find the series expansion of f(x), in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form.

(5)

Use your answer to part (*a*) to find the series expansion in ascending powers of x, up to and including the term in x^3, of

(*b*) $g(x) = \frac{27}{(3+5x)^2} \qquad |x| < \frac{3}{5}$

(1)

(*c*) $h(x) = \frac{27}{(3-x)^2} \qquad |x| < 3$

(2)

(IAL, C34 June 2017 Q4)

32. (*a*) Find the binomial series expansion of

$$\sqrt{4 - 9x}, \qquad |x| < \frac{4}{9}$$

in ascending powers of x, up to and including the term in x^2
Give each coefficient in its simplest form.

(5)

(*b*) Use the expansion from part (*a*), with a suitable value of x, to find an approximate value for $\sqrt{310}$

Show all your working and give your answer to 3 decimal places.

(3)

(C4 June 2018 Q1)

33. The binomial series expansion of

$$(1+ax)^{\frac{2}{3}} \qquad |ax|<1$$

up to and including the term in x^2 is

$$1+\frac{1}{2}x+kx^2$$

where a and k are constants.

(*a*) Find the value of a.

(2)

(*b*) Find the value of k, giving your answer in its simplest form.

(2)

(*c*) Hence find the numerical coefficient of x^2 in the series expansion of

$$(4-9x)(1+ax)^{\frac{2}{3}} \qquad |ax|<1$$

(2)

(C4 June 2019 Q1)

34.

$$f(x) = (125 - 5x)^{\frac{2}{3}} \qquad |x| < 25$$

(*a*) Find the binomial expansion of f(x), in ascending powers of x, up to and including the term in x^2, giving the coefficient of x and the coefficient of x^2 as simplified fractions.

(4)

(*b*) Use your expansion to find an approximate value for $120^{\frac{2}{3}}$, stating the value of x which you have used and showing your working. Give your answer to 5 decimal places.

(3)

(IAL, C34 Jan 2018 Q2)

35. (*a*) Use the binomial series to expand

$$\frac{1}{(2-3x)^3} \qquad |x| < \frac{2}{3}$$

in ascending powers of x, up to and including the term in x^2, giving each term as a simplified fraction.

(5)

$$\text{f}(x) = \frac{4+kx}{(2-3x)^3} \qquad \text{where } k \text{ is a constant and } |x| < \frac{2}{3}$$

Given that the series expansion of f (x), in ascending powers of x, is

$$\frac{1}{2} + Ax + \frac{81}{16}x^2 + \ldots$$

where A is a constant,

(*b*) find the value of k,

(2)

(*c*) find the value of A.

(2)

(IAL, C34 Oct 2017 Q7)

36. (*a*) Find the binomial expansion of

$$(1 + px)^{-4}, \qquad |px| < 1$$

in ascending powers of x, up to and including the term in x^3, giving each coefficient as simply as possible in terms of the constant p.

(3)

$$\text{f}(x) = \frac{3+4x}{(1+px)^4} \qquad |px| < 1$$

where p is a positive constant.
In the series expansion of f (x), the coefficient of x^2 is twice the coefficient of x.
(*b*) Find the value of p.

(5)

(*c*) Hence find the coefficient of x^3 in the series expansion of f (x), giving your answer as a simplified fraction.

(2)

(IAL, C34 June 2018 Q4)

37. Given that

$$\frac{3x^2+4x-7}{(x+1)(x-3)} \equiv A + \frac{B}{x+1} + \frac{C}{x-3}$$

(*a*) find the values of the constants A, B and C.

(4)

(*b*) Hence, or otherwise, find the series expansion of

$$\frac{3x^2 + 4x - 7}{(x+1)(x-3)} \qquad |x| < 1$$

in ascending powers of x, up to and including the term in x^2

Give each coefficient as a simplified fraction.

(6)

(IAL, C34 Jan 2019 Q2)

38. (*a*) Use binomial expansions to show that, for $|x| < \frac{1}{2}$

$$\sqrt{\frac{1+2x}{1-x}} \approx 1 + \frac{3}{2}x + \frac{3}{8}x^2$$

(6)

(*b*) Find the exact value of $\sqrt{\frac{1+2x}{1-x}}$ when $x = \frac{1}{10}$

Give your answer in the form $k\sqrt{3}$, where k is a constant to be determined.

(1)

(*c*) Substitute $x = \frac{1}{10}$ into the expansion given in part (*a*) and hence find an approximate value for $\sqrt{3}$

Give your answer in the form $\frac{a}{b}$ where a and b are integers.

(2)

(IAL, C34 Oct 2018 Q6)

39. (*a*) Use the binomial series to find the expansion of

$$\frac{1}{(2+3x)^3} \qquad |x| < \frac{2}{3}$$

in ascending powers of x, up to and including the term in $x2$, giving each term as a simplified fraction.

(5)

(*b*) Hence or otherwise, find the coefficient of x^2 in the series expansion of

(i) $\frac{1}{(2+6x)^3} \qquad |x| < \frac{1}{3}$

(ii) $\frac{4-x}{(2+3x)^3} \qquad |x| < \frac{2}{3}$

(4)

(IAL, C34 June 2019 Q10)

40. $$\mathrm{f}(x)=\left(\frac{1}{3}-x\right)^{-2} \qquad |x|<\frac{1}{3}$$

(*a*) Find the binomial expansion of f(*x*), in ascending powers of *x*, up to and including the term in x^3, giving each coefficient in its simplest form.

(4)

$$\mathrm{g}(x)=\left(\frac{1}{3}-x\right)^{-2}(a+bx) \qquad |x|<\frac{1}{3}$$

where *a* and *b* are constants.

Given that, in the series expansion of g(*x*), the coefficient of *x* is 3 and the coefficient of x^2 is 27

(*b*) find the value of *a* and the value of *b*.

(3)

(*c*) Hence find the coefficient of x^3 in the series expansion of g(*x*).

(2)

(IAL, C34 Nov 2019 Q2)