OCR Core Maths 2

Past paper questions Binomial Theorem

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Binomial Theorem

- Binomial expansion allows us to expand $(a+b)^n$ for any integer n. Best explained by means of an example; expand $(2x-y)^5$.

  1. Begin by considering 'prototype' expansion of $(a+b)^5$.
  2. So $(a+b)^5 = \binom{5}{0}a^5 + \binom{5}{1}a^4b + \binom{5}{2}a^3b^2 + \binom{5}{3}a^2b^3 + \binom{5}{4}ab^4 + \binom{5}{5}b^5$.
  3. Calculate binomial coefficients either on calculator or by drawing a mini Pascal's Triangle to give $(a+b)^5 = 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$.
  4. Next notice that in our case $a = 2x$ and $b = -y$ and substitute in to get $(2x-y)^5 = 1(2x)^5 + 5(2x)^4(-y) + 10(2x)^3(-y)^2 + 10(2x)^2(-y)^3 + 5(2x)(-y)^4 + 1(-y)^5$.
  5. Tidying up we get $(2x-y)^5 = 32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$.

- It is worth noting that when the expansion is of the form $(\text{something} - \text{another thing})^n$, then the signs will alternate.

- Also of note is the way each *individual* component is constructed. For example; find the x^5 coefficient in the expansion of $(2-3x)^7$. The component with x^5 is given by $\binom{7}{5}(2)^2(-3x)^5 = -20412x^5$, so the coefficient is -20412.

- $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$. For example $\binom{5}{2} = \frac{5!}{2!\,3!} = \frac{5\times4\times3\times2\times1}{(2\times1)\times(3\times2\times1)} = 10$.

- Know that $(1+x)^n$ expands thus:

$$\begin{aligned}(1+x)^n &= 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \cdots + x^n \\ &= 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + x^n.\end{aligned}$$

1.

(i) Find the first three terms of the expansion, in ascending powers of x, of $(1-2x)^{12}$. [3]

(ii) Hence find the coefficient of x^2 in the expansion of

$$(1+3x)(1-2x)^{12}.$$ [3]

Q3 Jan 2006

2.

Find the binomial expansion of $(3x-2)^4$. [4]

Q1 June 2006

3.

(i) Find and simplify the first four terms in the expansion of $(1+4x)^7$ in ascending powers of x. [4]

(ii) In the expansion of

$$(3+ax)(1+4x)^7,$$

the coefficient of x^2 is 1001. Find the value of a. [3]

Q6 Jan 2007

4.

Expand $\left(x+\frac{2}{x}\right)^4$ completely, simplifying the terms. [5]

Q2 June 2007

5.

(i) Find the binomial expansion of $(2x+5)^4$, simplifying the terms. [4]

(ii) Hence show that $(2x+5)^4-(2x-5)^4$ can be written as

$$320x^3+kx,$$

where the value of the constant k is to be stated. [2]

(iii) Verify that $x=2$ is a root of the equation

$$(2x+5)^4-(2x-5)^4=3680x-800,$$

and find the other possible values of x. [6]

Q10 Jan 2008

6.

Find and simplify the first three terms in the expansion of $(2-3x)^6$ in ascending powers of x. [4]

Q1 June 2008

7.

In the binomial expansion of $(k + ax)^4$ the coefficient of x^2 is 24.

(i) Given that a and k are both positive, show that $ak = 2$. **[3]**

(ii) Given also that the coefficient of x in the expansion is 128, find the values of a and k. **[4]**

(iii) Hence find the coefficient of x^3 in the expansion. **[2]**

Q7 Jan 2009

8.

(i) Find and simplify the first four terms in the expansion of $(2 - x)^7$ in ascending powers of x. **[4]**

(ii) Hence find the coefficient of w^6 in the expansion of $\left(2 - \frac{1}{4}w^2\right)^7$. **[2]**

Q3 Jan 2010

9.

(i) Find and simplify the first four terms in the binomial expansion of $(1 + \frac{1}{2}x)^{10}$ in ascending powers of x. **[4]**

(ii) Hence find the coefficient of x^3 in the expansion of $(3 + 4x + 2x^2)(1 + \frac{1}{2}x)^{10}$. **[3]**

Q3 June 2010

10.

(i) Find and simplify the first three terms, in ascending powers of x, in the binomial expansion of $(1 + 2x)^7$. **[3]**

(ii) Hence find the coefficient of x^2 in the expansion of $(2 - 5x)(1 + 2x)^7$. **[3]**

Q1 Jan 2011

11.

The first four terms in the binomial expansion of $(3 + kx)^5$, in ascending powers of x, can be written as $a + bx + cx^2 + dx^3$.

(i) State the value of a. **[1]**

(ii) Given that $b = c$, find the value of k. **[5]**

(iii) Hence find the value of d. **[2]**

Q5 June 2011

12.

One of the terms in the binomial expansion of $(4 + ax)^6$ is $160x^3$.

(i) Find the value of a. **[4]**

(ii) Using this value of a, find the first two terms in the expansion of $(4 + ax)^6$ in ascending powers of x. **[2]**

Q3 Jan 2012

13.

(i) Find the binomial expansion of $(3 + 2x)^5$, simplifying the terms. **[4]**

(ii) Hence find the binomial expansion of $(3 + 2x)^5 + (3 - 2x)^5$. **[2]**

Q1 June 2012

14.

(i) Find the binomial expansion of $(2 + x)^5$, simplifying the terms. **[4]**

(ii) Hence find the coefficient of y^3 in the expansion of $(2 + 3y + y^2)^5$. **[3]**

Q4 Jan 2013

15.

(i) Find and simplify the first three terms in the expansion of $(2 + 5x)^6$ in ascending powers of x. **[4]**

(ii) In the expansion of $(3 + cx)^2(2 + 5x)^6$, the coefficient of x is 4416. Find the value of c. **[3]**

Q3 June 2013

16.

(i) Find the binomial expansion of $\left(x^3 + \frac{2}{x^2}\right)^4$, simplifying the terms. **[5]**

(ii) Hence find $\int \left(x^3 + \frac{2}{x^2}\right)^4 \mathrm{d}x$. **[4]**

Q6 June 2014

17.

(i) Find and simplify the first three terms in the binomial expansion of $(2 + ax)^6$ in ascending powers of x. **[4]**

(ii) In the expansion of $(3 - 5x)(2 + ax)^6$, the coefficient of x is 64. Find the value of a. **[3]**

Q4 June 2015