Edexcel

Pure Mathematics

Year 2

Arithmetic Sequences.

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1. The rth term of an arithmetic series is $(2r - 5)$.

(a) Write down the first three terms of this series.

(2)

(b) State the value of the common difference.

(1)

(c) Show that $\sum_{r=1}^{n}(2r-5) = n(n-4)$.

(3)

(C1, Q5 Jan 2005)

2. An arithmetic series has first term a and common difference d.

(a) Prove that the sum of the first n terms of the series is

$$\tfrac{1}{2}n[2a + (n-1)d].$$

(4)

Sean repays a loan over a period of n months. His monthly repayments form an arithmetic sequence.

He repays £149 in the first month, £147 in the second month, £145 in the third month, and so on. He makes his final repayment in the nth month, where $n > 21$.

(b) Find the amount Sean repays in the 21st month.

(2)

Over the n months, he repays a total of £5000.

(c) Form an equation in n, and show that your equation may be written as

$$n^2 - 150n + 5000 = 0.$$

(3)

(d) Solve the equation in part (c).

(3)

(e) State, with a reason, which of the solutions to the equation in part (c) is **not** a sensible solution to the repayment problem.

(1)

(C1, Q9, Jan 2005)

4. On Alice's 11th birthday she started to receive an annual allowance. The first annual allowance was £500 and on each following birthday the allowance was increased by £200.

(*a*) Show that, immediately after her 12th birthday, the total of the allowances that Alice had received was £1200. **(1)**

(*b*) Find the amount of Alice's annual allowance on her 18th birthday.

(2)

(*c*) Find the total of the allowances that Alice had received up to and including her 18th birthday.

(3)

When the total of the allowances that Alice had received reached £32 000 the allowance stopped.

(*d*) Find how old Alice was when she received her last allowance.

(7)

(C1, Q7, Jan 2006)

6. An athlete prepares for a race by completing a practice run on each of 11 consecutive days. On each day after the first day he runs further than he ran on the previous day. The lengths of his 11 practice runs form an arithmetic sequence with first term a km and common difference d km.

He runs 9 km on the 11th day, and he runs a total of 77 km over the 11 day period.

Find the value of a and the value of d.

(7)

(C1, Q7, May 2006)

7. Ann has some sticks that are all of the same length. She arranges them in squares and has made the following 3 rows of patterns:

Row 1 

Row 2 

Row 3 

She notices that 4 sticks are required to make the single square in the first row, 7 sticks to make 2 squares in the second row and in the third row she needs 10 sticks to make 3 squares.

(*a*) Find an expression, in terms of n, for the number of sticks required to make a similar arrangement of n squares in the nth row.

(3)

Ann continues to make squares following the same pattern. She makes 4 squares in the 4th row and so on until she has completed 10 rows.

(*b*) Find the total number of sticks Ann uses in making these 10 rows. **(3)**

Ann started with 1750 sticks. Given that Ann continues the pattern to complete k rows but does not have sufficient sticks to complete the $(k + 1)$th row, (*c*) show that k satisfies $(3k - 100)(k + 35) < 0$.

(4)

(*d*) Find the value of k. **(2)**

(C1, Q9 Jan 2006)

8. A girl saves money over a period of 200 weeks. She saves 5p in Week 1, 7p in Week 2, 9p in Week 3, and so on until Week 200. Her weekly savings form an arithmetic sequence.

(*a*) Find the amount she saves in Week 200. **(3)**

(*b*) Calculate her total savings over the complete 200 week period.**(3)**

(C1, Q4 May 2007)

11. The first term of an arithmetic sequence is 30 and the common difference is –1.5.

(*a*) Find the value of the 25th term.

(2)

The rth term of the sequence is 0.

(*b*) Find the value of r.

(2)

The sum of the first n terms of the sequence is S_n.

(*c*) Find the largest positive value of S_n.

(3)

(C1, Q11 Jan 2008)

13. Sue is training for a marathon. Her training includes a run every Saturday starting with a run of 5 km on the first Saturday. Each Saturday she increases the length of her run from the previous Saturday by 2 km.

(*a*) Show that on the 4th Saturday of training she runs 11 km.

(1)

(*b*) Find an expression, in terms of n, for the length of her training run on the nth Saturday. **(2)**

(*c*) Show that the total distance she runs on Saturdays in n weeks of training is $n(n + 4)$ km .**(3)**

On the nth Saturday Sue runs 43 km.

(*d*) Find the value of n. **(2)**

(*e*) Find the total distance, in km, Sue runs on Saturdays in n weeks of training.

(2)

(C1, Q7 June 2008)

14. The first term of an arithmetic series is a and the common difference is d.

The 18th term of the series is 25 and the 21st term of the series is $32\frac{1}{2}$.

(*a*) Use this information to write down two equations for a and d. **(2)**

(*b*) Show that $a = -17.5$ and find the value of d. **(2)**

The sum of the first n terms of the series is 2750.

(*c*) Show that n is given by

$$n^2 - 15n = 55 \times 40.$$ **(4)**

(*d*) Hence find the value of n. **(3)**

(C1, Q9 Jan 2009)

15. A 40-year building programme for new houses began in Oldtown in the year 1951 (Year 1) and finished in 1990 (Year 40).

The numbers of houses built each year form an arithmetic sequence with first term a and common difference d.

Given that 2400 new houses were built in 1960 and 600 new houses were built in 1990, find

(*a*) the value of d,**(3)**

(*b*) the value of a,**(2)**

(*c*) the total number of houses built in Oldtown over the 40-year period.**(3)**

(C1, Q5 June 2009)

17. Jill gave money to a charity over a 20-year period, from Year 1 to Year 20 inclusive. She gave £150 in Year 1, £160 in Year 2, £170 in Year 3, and son on, so that the amounts of money she gave each year formed an arithmetic sequence.

(*a*) Find the amount of money she gave in Year 10.

(2)

(*b*) Calculate the total amount of money she gave over the 20-year period.

(3)

Kevin also gave money to charity over the same 20-year period.

He gave £A in Year 1 and the amounts of money he gave each year increased, forming an arithmetic sequence with common difference £30.

The total amount of money that Kevin gave over the 20-year period was **twice** the total amount of money that Jill gave.

(*c*) Calculate the value of A.

(4)

(C1, Q7 Jan 2010)

19. A farmer has a pay scheme to keep fruit pickers working throughout the 30 day season. He pays £a for their first day, £$(a + d)$ for their second day, £$(a + 2d)$ for their third day, and so on, thus increasing the daily payment by £d for each extra day they work.

A picker who works for all 30 days will earn £40.75 on the final day.

(a) Use this information to form an equation in a and d.

(2)

A picker who works for all 30 days will earn a total of £1005.

(b) Show that $15(a + 40.75) = 1005$.

(2)

(c) Hence find the value of a and the value of d. **(4)**

(C1, Q7 May 2010)

21. An arithmetic sequence has first term a and common difference d. The sum of the first 10 terms of the sequence is 162.

(a) Show that $10a + 45d = 162$. **(2)**

Given also that the sixth term of the sequence is 17,

(b) write down a second equation in a and d, **(1)**

(c) find the value of a and the value of d. **(4)**

(C1, Q6 Jan 2011)

24. A company offers two salary schemes for a 10-year period, Year 1 to Year 10 inclusive.

Scheme 1: Salary in Year 1 is £P.

Salary increases by £$(2T)$ each year, forming an arithmetic sequence.

Scheme 2: Salary in Year 1 is £$(P + 1800)$.

Salary increases by £T each year, forming an arithmetic sequence.

(a) Show that the total earned under Salary Scheme 1 for the 10-year period is

$$£(10P + 90T).$$

(2)

For the 10-year period, the total earned is the same for both salary schemes.

(b) Find the value of T. **(4)**

For this value of T, the salary in Year 10 under Salary Scheme 2 is £29 850.

(c) Find the value of P.

(3)

(C1, Q4 Jan 2012)

26. A boy saves some money over a period of 60 weeks. He saves 10p in week 1, 15p in week 2, 20p in week 3 and so on until week 60. His weekly savings form an arithmetic sequence.

(*a*) Find how much he saves in week 15. **(2)**

(*b*) Calculate the total amount he saves over the 60 week period. **(3)**

The boy's sister also saves some money each week over a period of m weeks. She saves 10p in week 1, 20p in week 2, 30p in week 3 and so on so that her weekly savings form an arithmetic sequence. She saves a total of £63 in the m weeks.

(*c*) Show that

$$m(m + 1) = 35 \times 36.$$

(4)

(*d*) Hence write down the value of m.

(1)

(C1, Q6 May 2012)

28. Lewis played a game of space invaders. He scored points for each spaceship that he captured.

Lewis scored 140 points for capturing his first spaceship.

He scored 160 points for capturing his second spaceship, 180 points for capturing his third spaceship, and so on.

The number of points scored for capturing each successive spaceship formed an arithmetic sequence.

(*a*) Find the number of points that Lewis scored for capturing his 20th spaceship.

(2)

(*b*) Find the total number of points Lewis scored for capturing his first 20 spaceships.

(3)

Sian played an adventure game. She scored points for each dragon that she captured. The number of points that Sian scored for capturing each successive dragon formed an arithmetic sequence.

Sian captured n dragons and the total number of points that she scored for capturing all n dragons was 8500.

Given that Sian scored 300 points for capturing her first dragon and then 700 points for capturing her nth dragon,

(*c*) find the value of n.

(3)

(C1, Q7 Jan 2013)

30. A company, which is making 200 mobile phones each week, plans to increase its production.

The number of mobile phones produced is to be increased by 20 each week from 200 in week 1 to 220 in week 2, to 240 in week 3 and so on, until it is producing 600 in week N.

(*a*) Find the value of N.

(2)

The company then plans to continue to make 600 mobile phones each week.

(*b*) Find the total number of mobile phones that will be made in the first 52 weeks starting from and including week 1.

(5)

(C1, Q4 May 2013)

32. Each year, Abbie pays into a savings scheme. In the first year she pays in £500. Her payments then increase by £200 each year so that she pays £700 in the second year, £900 in the third year and so on.

(*a*) Find out how much Abbie pays into the savings scheme in the tenth year.

(2)

Abbie pays into the scheme for n years until she has paid a total of £67 200.

(*b*) Show that $n^2 + 4n - 24 \times 28 = 0$.

(5)

(*b*) Hence find the number of years that Abbie pays into the savings scheme.

(2)

(C1, Q6 May 2013_R)

34. In the year 2000 a shop sold 150 computers. Each year the shop sold 10 more computers than the year before, so that the shop sold 160 computers in 2001, 170 computers in 2002, and so on forming an arithmetic sequence.

(*a*) Show that the shop sold 220 computers in 2007.

(2)

(*b*) Calculate the total number of computers the shop sold from 2000 to 2013 inclusive.

(3)

In the year 2000, the selling price of each computer was £900. The selling price fell by £20 each year, so that in 2001 the selling price was £880, in 2002 the selling price was £860, and so on forming an arithmetic sequence.

(*c*) In a particular year, the selling price of each computer in £s was equal to three times the number of computers the shop sold in that year. By forming and solving an equation, find the year in which this occurred.

(4)

(C1, Q8 May 2014)

36. Xin has been given a 14 day training schedule by her coach.

Xin will run for A minutes on day 1, where A is a constant.

She will then increase her running time by $(d + 1)$ minutes each day, where d is a constant.

(a) Show that on day 14, Xin will run for

$$(A + 13d + 13) \text{ minutes.}$$ **(2)**

Yi has also been given a 14 day training schedule by her coach.

Yi will run for $(A - 13)$ minutes on day 1.

She will then increase her running time by $(2d - 1)$ minutes each day.

Given that Yi and Xin will run for the same length of time on day 14,

(b) find the value of d. **(3)**

Given that Xin runs for a total time of 784 minutes over the 14 days,

(c) find the value of A. **(3)**

(C1, Q10 May 2014_R)

37. Jess started work 20 years ago. In year 1 her annual salary was £17 000. Her annual salary increased by £1500 each year, so that her annual salary in year 2 was £18 500, in year 3 it was £20 000 and so on, forming an arithmetic sequence. This continued until she reached her maximum annual salary of £32000 in year k. Her annual salary then remained at £32 000.

(a) Find the value of the constant k.

(2)

(b) Calculate the total amount that Jess has earned in the 20 years.

(5)

(C1, Q9 May 2015)

38. On John's 10th birthday he received the first of an annual birthday gift of money from his uncle. This first gift was £60 and on each subsequent birthday the gift was £15 more than the year before. The amounts of these gifts form an arithmetic sequence.

(a) Show that, immediately after his 12th birthday, the total of these gifts was £225.

(1)

(b) Find the amount that John received from his uncle as a birthday gift on his 18th birthday.

(2)

(c) Find the total of these birthday gifts that John had received from his uncle up to and including his 21st birthday.

(3)

When John had received n of these birthday gifts, the total money that he had received from these gifts was £3375.

(d) Show that $n^2 + 7n = 25 \times 18$.

(3)

(e) Find the value of n, when he had received £3375 in total, and so determine John's age at this time.

(2)

(C1, Q9 May 2016)

39. A company, which is making 140 bicycles each week, plans to increase its production. The number of bicycles produced is to be increased by d each week, starting from 140 in week 1, to $140 + d$ in week 2, to $140 + 2d$ in week 3 and so on, until the company is producing 206 in week 12.

(a) Find the value of d.

(2)

After week 12 the company plans to continue making 206 bicycles each week.

(b) Find the total number of bicycles that would be made in the first 52 weeks starting from and including week 1.

(5)

(C1, Q4 May 2017)

40. Shelim starts his new job on a salary of £14 000. He will receive a rise of £1500 a year for each full year that he works, so that he will have a salary of £15 500 in year 2, a salary of £17 000 in year 3 and so on. When Shelim's salary reaches £26 000, he will receive no more rises. His salary will remain at £26 000.

(a) Show that Shelim will have a salary of £26 000 in year 9.

(2)

(b) Find the total amount that Shelim will earn in his job in the first 9 years.

(2)

Anna starts her new job at the same time as Shelim on a salary of £A. She receives a rise of £1000 a year for each full year that she works, so that she has a salary of £$(A + 1000)$ in year 2, £$(A + 2000)$ in year 3 and so on. The maximum salary for her job, which is reached in year 10, is also £26 000.

(c) Find the difference in the total amount earned by Shelim and Anna in the first 10 years.

(6)

(IAL C1, Q7, Jan 2014)

41. The first three terms of an arithmetic series are 60, $4p$ and $2p – 6$ respectively.

(a) Show that $p = 9$.

(2)

(b) Find the value of the 20th term of this series.

(3)

(c) Prove that the sum of the first n terms of this series is given by the expression

$$12n\,(6 – n)$$

(3)

(IAL C12, Q11, Jan 2014)

42. A 25-year programme for building new houses began in Core Town in the year 1986 and finished in the year 2010.

The number of houses built each year form an arithmetic sequence. Given that 238 houses were built in the year 2000 and 108 were built in the year 2010, find

(*a*) the number of houses built in 1986, the first year of the building programme,

(5)

(*b*) the total number of houses built in the 25 years of the programme.

(2)

(IAL C12, Q11, May 2015)

43. The 4th term of an arithmetic sequence is 3 and the sum of the first 6 terms is 27.

Find the first term and the common difference of this sequence.

(6)

(IAL C12, Q4, Jan 2016)

44. In a large theatre there are 20 rows of seats.

The number of seats in the first row is a, where a is a constant.

In the second row the number of seats is $(a + d)$, where d is a constant. In the third row the number of seats is $(a + 2d)$, and on each subsequent row there are d more seats than on the previous row. The number of seats in each row forms an arithmetic sequence.

The **total** number of seats in the first 10 rows is 395.

(*a*) Use this information to show that $10a + 45d = 395$

(1)

The **total** number of seats in the first 18 rows is 927.

(*b*) Use this information to write down a second simplified equation relating a and d.

(2)

(*c*) Solve these equations to find the value of a and the value of d.

(3)

(*d*) Find the number of seats in the 20th row of the theatre.

(2)

(IAL C12, Q9, Oct2016)

45. An arithmetic series has first term a and common difference d.

Given that the sum of the first 9 terms is 54,

(*a*) show that

$$a + 4d = 6$$

(2)

Given also that the 8th term is half the 7th term,

(*b*) find the values of a and d.

(4)

(IAL C12, Q4, Jan 2017)

46. An arithmetic sequence has first term 6 and common difference 10

Find

(*a*) the 15th term of the sequence,

(2)

(*b*) the sum of the first 20 terms of the sequence.

(2)

(IAL C12, Q1, May 2017)

47. Each year Lin pays into a savings scheme. In year 1 she pays in £600.
Her payments then increase by £80 a year, so that she pays £680 into the savings scheme in year 2, £760 in year 3 and so on. In year N, Lin pays £1000 into the savings scheme.

(*a*) Find the value of N.

(2)

(*b*) Find the total amount that Lin pays into the savings scheme from year 1 to year 15 inclusive.

(2)

Saima starts paying into a different savings scheme at the same time as Lin starts paying into her savings scheme.

In year 1 she pays in £A. Her payments increase by £A each year so that she pays £2A in year 2, £3A in year 3 and so on.

Given that Saima and Lin have each paid, in total, the same amount of money into their savings schemes after 15 years,

(*c*) find the value of A.

(3)

(IAL C12, Q6, Oct 2017)

48. Each year, Andy pays into a savings scheme. In year one he pays in £600. His payments increase by £120 each year so that he pays £720 in year two, £840 in year three and so on, so that his payments form an arithmetic sequence.

(*a*) Find out how much Andy pays into the savings scheme in year ten.

(2)

Kim starts paying money into a different savings scheme at the same time as Andy. In year one she pays in £130. Her payments increase each year so that she pays £210 in year two, £290 in year three and so on, so that her payments form a different arithmetic sequence.

At the end of year N, Andy has paid, in total, twice as much money into his savings scheme as Kim has paid, in total, into her savings scheme.

(*b*) Find the value of N.

(5)

(C1, Q4 May 2018)

49. For 25 years, Chloe gave money to a charity.

She gave £80 in the first year, £100 in the second year, £120 in the third year and so on, so that the amounts she gave each year form an arithmetic sequence.

(*a*) Find the amount of money that she gave to the charity in the 17th year.

(2)

(*b*) Calculate the total amount that she gave over the 25 years.

(2)

Jack also gave money to the same charity over the same 25 years.

He gave £16 in the first year and increased the amount he gave each year so that he gave £$(16 + d)$ in the second year, £$(16+ 2d)$ in the third year and so on. The amounts he gave each year form an arithmetic sequence with common difference £d.

The total amount that he gave over the 25 years was £4000

(*c*) Calculate how much he gave to the charity in the 17th year.

(5)

(C1, Q8 May 2019)

50. A car manufacturer currently makes 1000 cars each week.
The manufacturer plans to increase the number of cars it makes each week.
The number of cars made will be increased by 20 each week from 1000 in week 1, to 1020 in week 2, to 1040 in week 3 and so on, until 1500 cars are made in week N.

(*a*) Find the value of N.

(2)

The car manufacturer then plans to continue to make 1500 cars each week.

(*b*) Find the total number of cars that will be made in the first 50 weeks starting from and including week 1.

(5)

(IAL C12, Q9, Oct 2018)