OCR Core Maths 4

Past paper questions Algebra

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- You must be able to simplify algebraic fractions; best tactic is always to factorise and cancel:

$$\frac{4x^5 - 10x^4 - 6x^3}{12x^6 - 18x^5 - 12x^4} = \frac{2x^3(2x^2 - 5x - 3)}{6x^4(2x^2 - 3x - 2)} = \frac{2x^3(2x+1)(x-3)}{6x^4(x-2)(2x+1)} = \frac{x-3}{3x(x-2)}.$$

- You must be able to divide a polynomial ($p(x)$) by a divisor ($a(x)$), finding the quotient ($q(x)$) and remainder ($r(x)$). If there is no remainder then $a(x)$ is a factor of $p(x)$. It is always such that

$$\frac{p(x)}{a(x)} = q(x) + \frac{r(x)}{a(x)} \qquad \Rightarrow \qquad p(x) = a(x)q(x) + r(x).$$

  The order of $q(x)$ is the order of $p(x)$ subtract the order of $a(x)$. The order of $r(x)$ is *at most* one less than $a(x)$. For example if you have a quintic (power 5 polynomial) divided by a quadratic you would expect

$$\frac{\text{quintic}}{\text{quadratic}} = \text{cubic} + \frac{\text{linear}}{\text{quadratic}},$$
$$\frac{\text{quintic}}{\text{quadratic}} = Ax^3 + Bx^2 + Cx + D + \frac{Ex + F}{\text{quadratic}}.$$

  Of course it *may* turn out that $Ex + F$ is just a constant or zero (if the cubic divides the quintic).

- Division is most easily done step-by-step working *down* the powers of $p(x)$. For example divide $2x^4 - x^3 + 3x^2 - 7x + 1$ by $x^2 + 2x + 3$:

$$\begin{aligned}
2x^4 - x^3 + 3x^2 - 7x + 1 &= (x^2 + 2x + 3)(\text{quadratic}) + (\text{remainder}) & \\
&= (x^2 + 2x + 3)(2x^2 + \dots) + (\text{remainder}) & x^4\checkmark \\
&= (x^2 + 2x + 3)(2x^2 - 5x + \dots) + (\text{remainder}) & x^3\checkmark \\
&= (x^2 + 2x + 3)(2x^2 - 5x + 7) + (\text{remainder}) & x^2\checkmark \\
&= (x^2 + 2x + 3)(2x^2 - 5x + 7) - 6x + \text{const.} & x\checkmark \\
&= (x^2 + 2x + 3)(2x^2 - 5x + 7) - 6x - 20 & \text{const.}\checkmark
\end{aligned}$$

  Therefore $q(x) = 2x^2 - 5x + 7$ and $r(x) = -6x - 20$.

- Partial fractions is effectively the reverse of combining together two algebraic fractions. For example

$$\frac{1}{x+1}+\frac{1}{x+2} \quad \begin{matrix}\longrightarrow \\ \longleftarrow\end{matrix} \quad \begin{matrix}\text{Algebraic Fractions} \\ \text{Partial Fractions}\end{matrix} \quad \begin{matrix}\longrightarrow \\ \longleftarrow\end{matrix} \quad \frac{2x+3}{(x+1)(x+2)}.$$

  You can use partial fractions provided the order of the top line is less than the order of the bottom line.

- If you simply have a product of linear factors on the bottom line then you split out into that many terms with constants placed on top (usually denoted by A, B, C, etc.). Place this equal (in an identity "$\equiv$") to the original expression and then multiply through to get rid of the denominators. For example:

$$\begin{aligned} \frac{7x-1}{(2x+1)(x-1)} &\equiv \frac{A}{2x+1}+\frac{B}{x-1} \\ \Rightarrow \qquad 7x-1 &\equiv (x-1)A+(2x+1)B. \end{aligned}$$

  Because this is an identity we can choose any value of x we fancy to help us discover A and B. In this case letting $x=1$ is a good choice because one of the brackets become zero. Similarly $x=-\frac{1}{2}$ is another good choice[1]; we put these into the identity.

$$\begin{aligned} x=1 \quad &\Rightarrow \quad 7-1\equiv 3B \quad \Rightarrow \quad \underline{B=2}, \\ x=-\tfrac{1}{2} \quad &\Rightarrow \quad -\tfrac{7}{2}-1\equiv -\tfrac{3}{2}A \quad \Rightarrow \quad \underline{A=3}. \end{aligned}$$

  Therefore $\dfrac{7x-1}{(2x+1)(x-1)} \equiv \dfrac{3}{2x+1}+\dfrac{2}{x-1}$.

- If you have a repeated factor in the denominator then you deal with it as follows (notice the top line is a quadratic and the bottom a cubic, so partial fractions are fine):

$$\begin{aligned} \frac{5x^2-10x+1}{(x-3)(x-1)^2} &\equiv \frac{A}{x-3}+\frac{B}{x-1}+\frac{C}{(x-1)^2} \\ \Rightarrow \qquad 5x^2-10x+1 &\equiv (x-1)^2A+(x-3)(x-1)B+(x-3)C. \end{aligned}$$

  Similarly, good values of x to choose are $x=1$ and $x=3$:

$$\begin{aligned} x=1 \quad &\Rightarrow \quad 5-10+1\equiv -2C \quad \Rightarrow \quad \underline{C=2}, \\ x=3 \quad &\Rightarrow \quad 45-30+1\equiv 4A \quad \Rightarrow \quad \underline{A=4}. \end{aligned}$$

  This tells us that

$$5x^2-10x+1 \equiv 4(x-1)^2+(x-3)(x-1)B+2(x-3),$$

  but it hasn't told us B. I would consider $x=0$, here, and sub in to discover

$$0-0+1\equiv 4+3B-6 \Rightarrow \underline{B=1}.$$

  Therefore $\dfrac{5x^2-10x+1}{(x-3)(x-1)^2} \equiv \dfrac{4}{x-3}+\dfrac{1}{x-1}+\dfrac{2}{(x-1)^2}$.

1.

Find the quotient and the remainder when $x^4 + 3x^3 + 5x^2 + 4x - 1$ is divided by $x^2 + x + 1$. [4]

Q1 June 2005

2.

Simplify $\frac{x^3 - 3x^2}{x^2 - 9}$. [3]

Q1 Jan 2006

3.

(i) Find the quotient and the remainder when $3x^3 - 2x^2 + x + 7$ is divided by $x^2 - 2x + 5$. [4]

(ii) Hence, or otherwise, determine the values of the constants a and b such that, when $3x^3 - 2x^2 + ax + b$ is divided by $x^2 - 2x + 5$, there is no remainder. [2]

Q3 Jan 2006

4.

It is given that

$$f(x) = \frac{x^2 + 2x - 24}{x^2 - 4x} \quad \text{for } x \neq 0,\ x \neq 4.$$

Express $f(x)$ in its simplest form. [3]

Q1 Jan 2007

5.

The equation of a curve is $y = f(x)$, where $f(x) = \frac{3x + 1}{(x + 2)(x - 3)}$.

(i) Express $f(x)$ in partial fractions. [2]

(ii) Hence find $f'(x)$ and deduce that the gradient of the curve is negative at all points on the curve. [3]

Q1 June 2007

6.

When $x^4 - 2x^3 - 7x^2 + 7x + a$ is divided by $x^2 + 2x - 1$, the quotient is $x^2 + bx + 2$ and the remainder is $cx + 7$. Find the values of the constants a, b and c. [5]

Q3 Jan 2008

7.

(a) Simplify $\dfrac{(2x^2 - 7x - 4)(x + 1)}{(3x^2 + x - 2)(x - 4)}$. [2]

(b) Find the quotient and remainder when $x^3 + 2x^2 - 6x - 5$ is divided by $x^2 + 4x + 1$. [4]

Q1 June 2008

8.

Simplify $\dfrac{20 - 5x}{6x^2 - 24x}$. **[3]**

Q1 Jan 2009

9.

Find the quotient and the remainder when $3x^4 - x^3 - 3x^2 - 14x - 8$ is divided by $x^2 + x + 2$. **[4]**

Q1 June 2009

10.

Find the quotient and the remainder when $x^4 + 11x^3 + 28x^2 + 3x + 1$ is divided by $x^2 + 5x + 2$. **[4]**

Q1 Jan 2010

11.

Express $\dfrac{x^2}{(x - 1)^2(x - 2)}$ in partial fractions. **[5]**

Q3 June 2010

12.

Simplify $\dfrac{x^4 - 10x^2 + 9}{(x^2 - 2x - 3)(x^2 + 8x + 15)}$. **[4]**

Q1 June 2011

13.

When the polynomial f(x) is divided by $x^2 + 1$, the quotient is $x^2 + 4x + 2$ and the remainder is $x - 1$. Find f(x), simplifying your answer. **[3]**

Q1 Jan 2012

14.

Simplify

(i) $\dfrac{1-x}{x^2-3x+2}$, **[2]**

(ii) $\dfrac{(x+1)}{(x-1)(x-3)} - \dfrac{(x-5)}{(x-3)(x-4)}$. **[4]**

Q1 June 2012

15.

Express $\dfrac{(x-7)(x-2)}{(x+2)(x-1)^2}$ in partial fractions. **[5]**

Q1 June 2013

16.

Express $x+\dfrac{1}{1-x}+\dfrac{2}{1+x}$ as a single fraction, simplifying your answer. **[3]**

Q1 June 2014

17.

(i) Express $\dfrac{2}{3-x}+\dfrac{3}{1+x}$ as a single fraction in its simplest form. **[2]**

(ii) Hence express $\left(\dfrac{2}{3-x}+\dfrac{3}{1+x}\right)\times\dfrac{x^2+8x-33}{121-x^2}$ as a single fraction in its lowest terms. **[3]**

Q1 June 2015